Simultaneous Transformation of Neutrinos and Antineutrinos in the Supernova Environment

INFO ‘05

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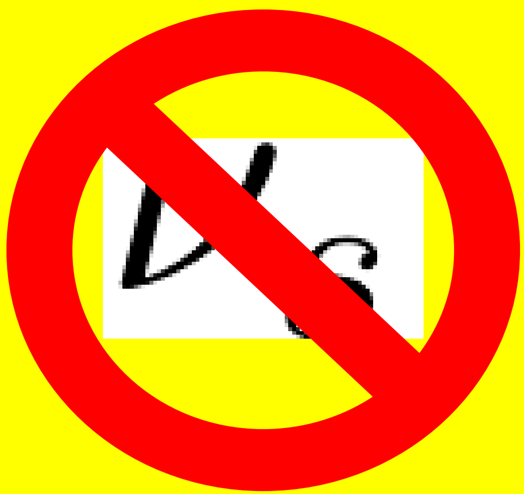
The Fun Is Over!



In this talk I will concentrate on the known, active neutrinos.

Active-sterile neutrino transformation can have dramatic effects in supernovae and the early universe, but active-active neutrino mixing is an experimental fact which must be incorporated into our astrophysical models.

Neutrinos Dominate the Energetics of Core Collapse Supernovae

Explosion
only ~1% of
neutrino energy

Total optical + kinetic energy, 10^{51} ergs

Total energy released in Neutrinos, 10^{53} ergs

10 % of star's rest mass!

$$E_{\mathrm{GRAV}} \approx \frac{3}{5}\frac{G\,M_{\mathrm{NS}}^2}{R_{\mathrm{NS}}} \approx 3\times10^{53}\,\mathrm{ergs}\left[\frac{M_{\mathrm{NS}}}{1.4\mathrm{M}_{\mathrm{sun}}}\right]^2\left[\frac{10\ \mathrm{km}}{R_{\mathrm{NS}}}\right]$$

Neutrino diffusion time, $\tau_\nu \approx 2\ \mathrm{s}$

$$L_\nu \approx \frac{1}{6}\frac{GM_{\mathrm{NS}}^2}{R_{\mathrm{NS}}}\frac{1}{\tau_\nu} \approx 4\times10^{51}\,\mathrm{ergs\ s^{-1}}$$

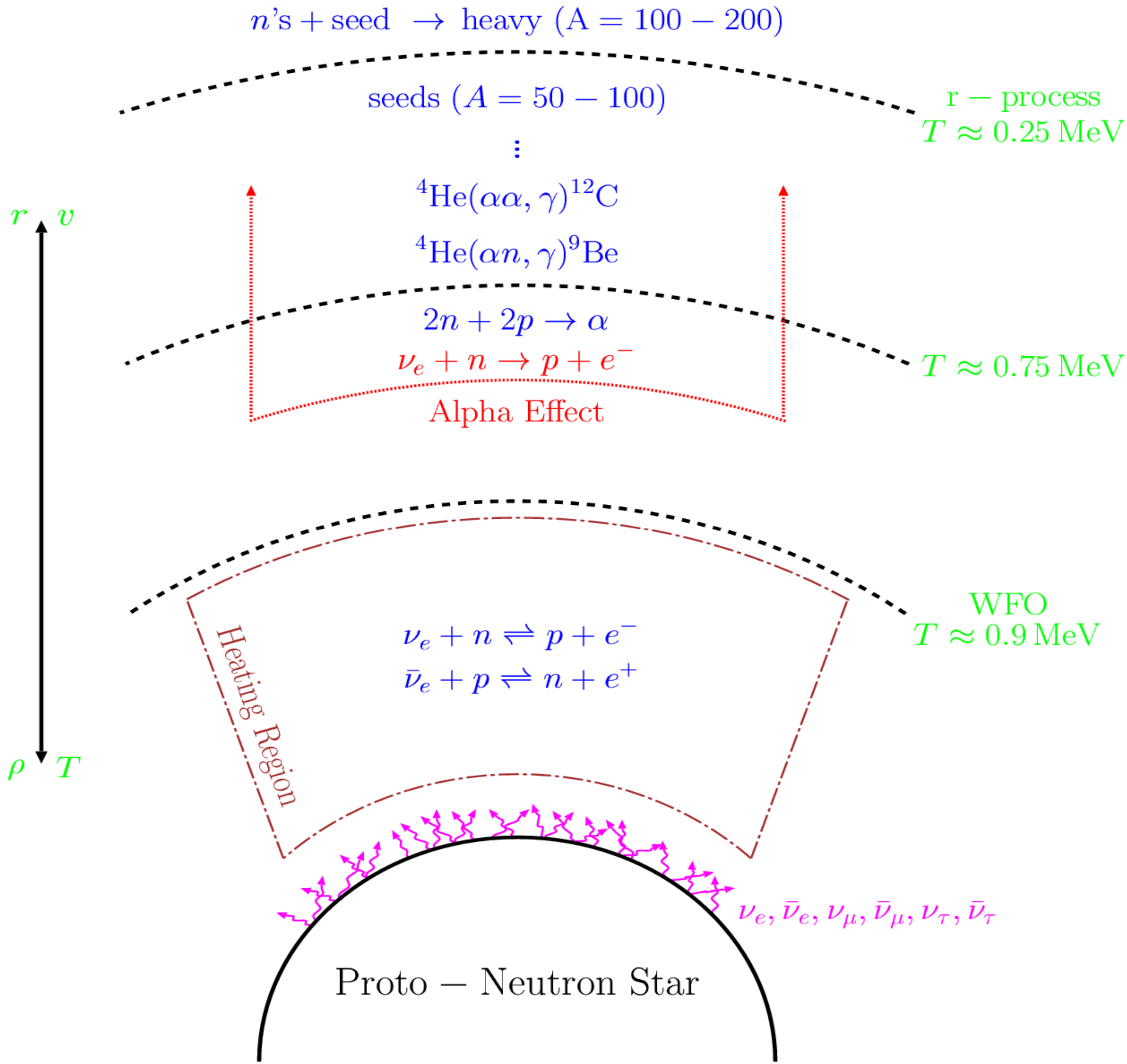
Neutron-to-proton ratio and energy deposition largely determined by these processes:

$$\boxed{\nu_e + n \longrightarrow p + e^-}$$

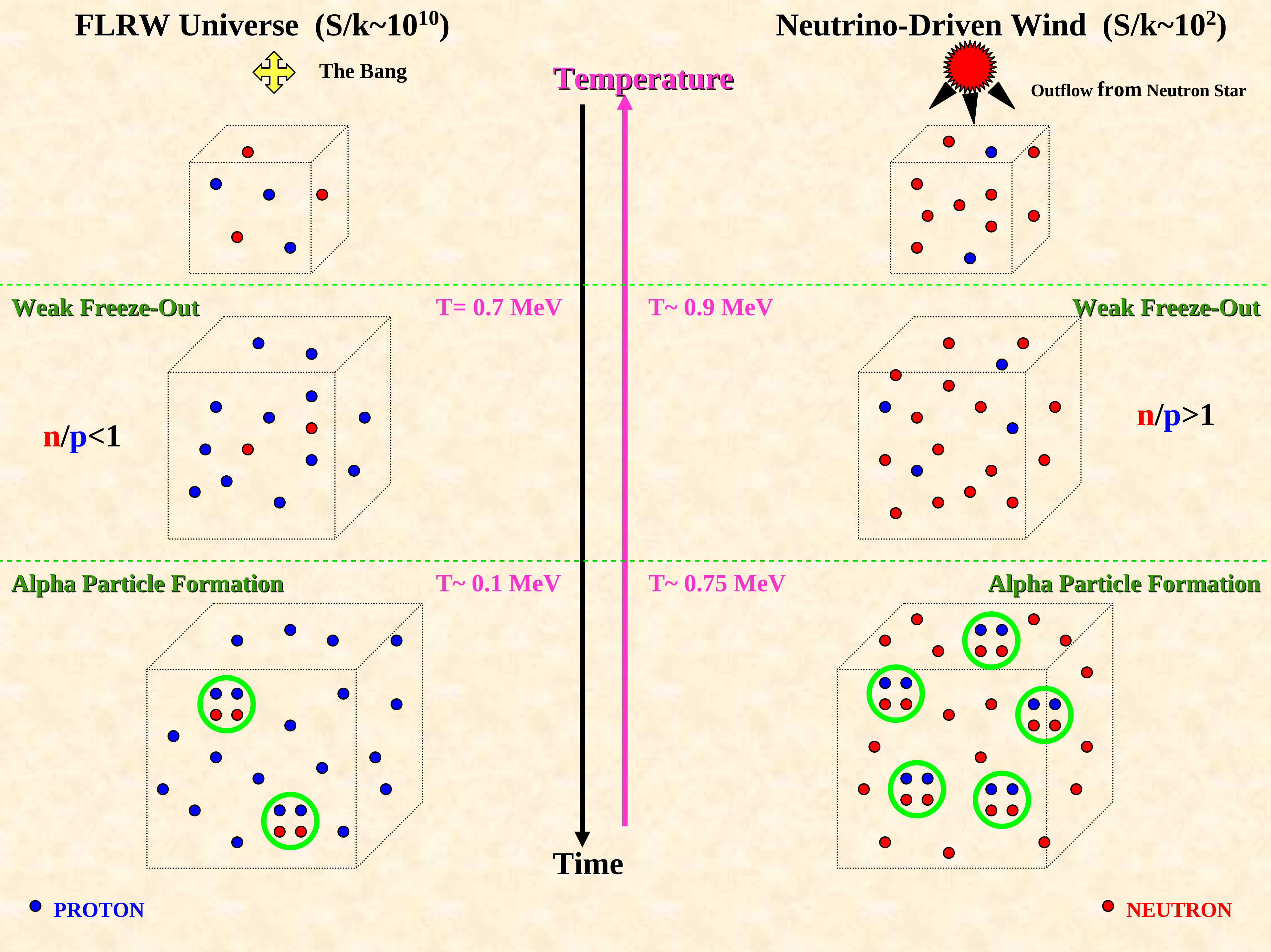
$$\boxed{\bar{\nu}_e + p \longrightarrow n + e^+}$$

Shock Propagation

R. Schirato & G. Fuller, astro-ph/0205390



n's + seed $\rightarrow$ heavy (A = 100 − 200)
seeds ($A = 50 - 100$)
r − process
$T \approx 0.25$ MeV
$^4\mathrm{He}(\alpha\alpha,\gamma)^{12}\mathrm{C}$
$^4\mathrm{He}(\alpha n,\gamma)^{9}\mathrm{Be}$
r v
$2n + 2p \rightarrow \alpha$
$\nu_e + n \rightarrow p + e^-$
$T \approx 0.75$ MeV
Alpha Effect
WFO
$T \approx 0.9$ MeV
$\nu_e + n \rightleftharpoons p + e^-$
$\bar{\nu}_e + p \rightleftharpoons n + e^+$
Heating Region
ρ T
$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$
Proto − Neutron Star



FLRW Universe (S/k~10^10)
Neutrino-Driven Wind (S/k~10^2)
The Bang
Temperature
Outflow from Neutron Star
Weak Freeze-Out
T= 0.7 MeV
T~ 0.9 MeV
Weak Freeze-Out
n/p<1
n/p>1
Alpha Particle Formation
T~ 0.1 MeV
T~ 0.75 MeV
Alpha Particle Formation
Time
PROTON
NEUTRON

The Alpha Effect

The paradox of neutrino-heated *r*-Process nucleosynthesis

Require neutrino interactions on free nucleons to give enough energy to each baryon to overcome the gravitational binding energy near the neutron star (~100 MeV per baryon). Since the average energies of neutrinos are ~ 10 MeV, we need some ~10 neutrino and antineutrino captures per nucleon to ensure ejection of the material.

However, formation of alpha particles incorporates all protons thereby isolating some free neutrons. These can capture electron neutrinos to become protons, which are immediately incorporated into alpha particles. Each reaction $\nu_e + n \rightarrow p + e^-$ takes out two neutrons!

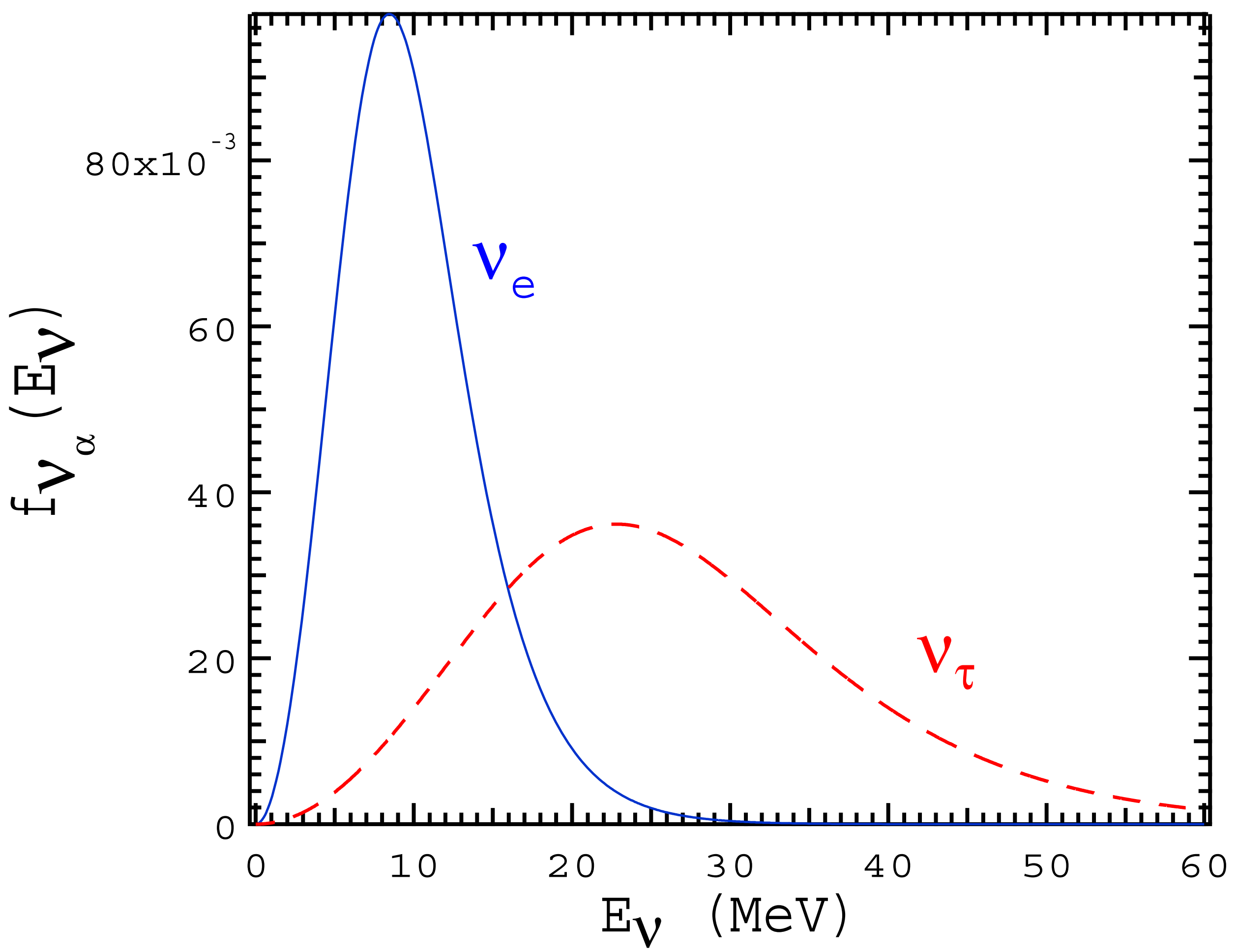
In short order there are not enough neutrons to make the *r*-Process

(Fuller, G. McLaughlin, B. Meyer)

Neutrino Distribution Functions f_ν

At late times (t_{pb} > 10 s) we expect an average energy hierarchy:

$$\langle E_{\nu_{\mu,\tau}} \rangle > \langle E_{\bar{\nu}_e} \rangle > \langle E_{\nu_e} \rangle$$



So, what happens when the active neutrinos transform among themselves ?

Shock re-heating *may* be enhanced,
neutrino nucleosynthesis and signal affected.
(depending on where the transformation happens
and on the neutrino energy spectra)

R-Process/Alpha-Effect problems get worse!

The weak interaction, or flavor basis is not coincident with the energy eigenstate, or mass basis.

These bases are related through a unitary transformation,

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle$$

where the flavors are $\alpha = e, \mu, \tau, s, s', \ldots$

and where the mass states are $i = 1, 2, 3, 4, \ldots$

$U_{\alpha i}$ **is parameterized by vacuum mixing angles and CP-violating phases, in general.**

If we consider only two-by-two neutrino mixing then the unitary transformation is parameterized by a single vacuum mixing angle:

$$|\nu_\alpha\rangle = \cos\theta |\nu_1\rangle + \sin\theta |\nu_2\rangle$$

$$|\nu_\beta\rangle = -\sin\theta |\nu_1\rangle + \cos\theta |\nu_2\rangle$$

Difference of the squares of the neutrino mass eigenvalues:

$$\delta m^2 = m_2^2 - m_1^2$$

Atmospheric Neutrinos

$$\delta m_{23}^2 \approx 2.5 \times 10^{-3}\ \text{eV}^2$$
$$\sin^2 2\theta_{23} \approx 1.0$$

"Solar"/KamLaND Neutrinos

$$\delta m_{\text{sol}}^2 \approx 7 \times 10^{-5}\ \text{eV}^2$$
$$\tan^2 \theta_{12} \approx 0.42 \ \leftrightarrow \ 0.45$$

Chooz

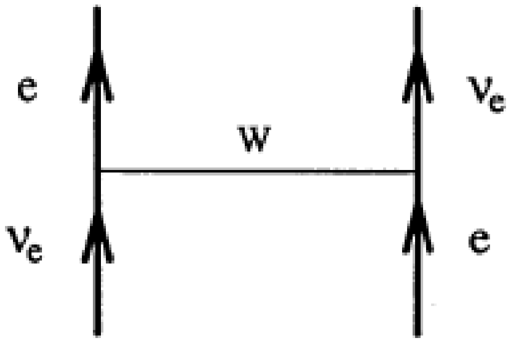
Chooz limit on $\theta_{13} \Rightarrow$

$\left|U_{e3}\right|^2 < 2.5\%$ or $\sin^2 2\theta_{13} < 0.1$ $(\theta_{13} < \frac{\pi}{20} \approx 9^\circ)$

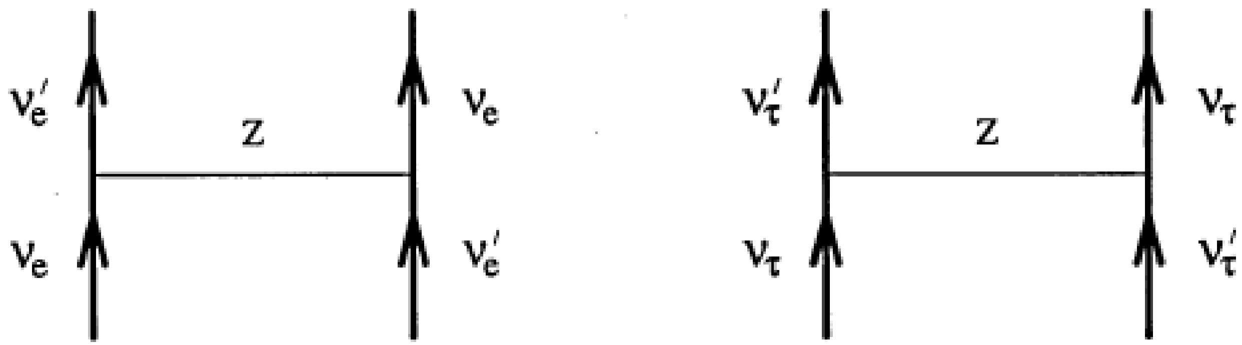
plus KamLaND $\Rightarrow$

$\sin^2 2\theta_{13} < 6.65 \times 10^{-2}$ $(< 0.2$ at $3\sigma)$

The A potential arises from charged current forward exchange

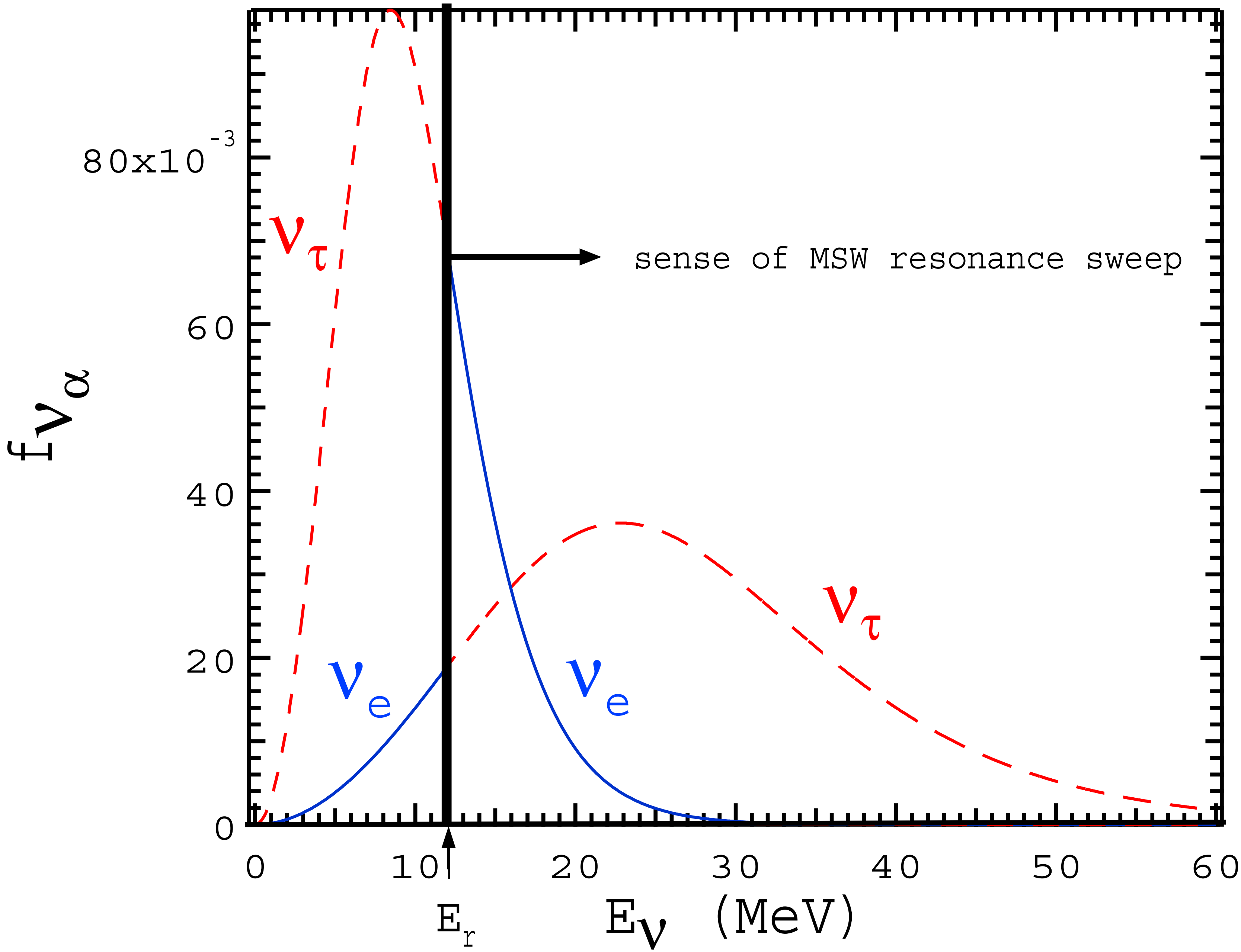


The neutrino "background" potentials arise from neutral current forward exchange scattering, e.g.,



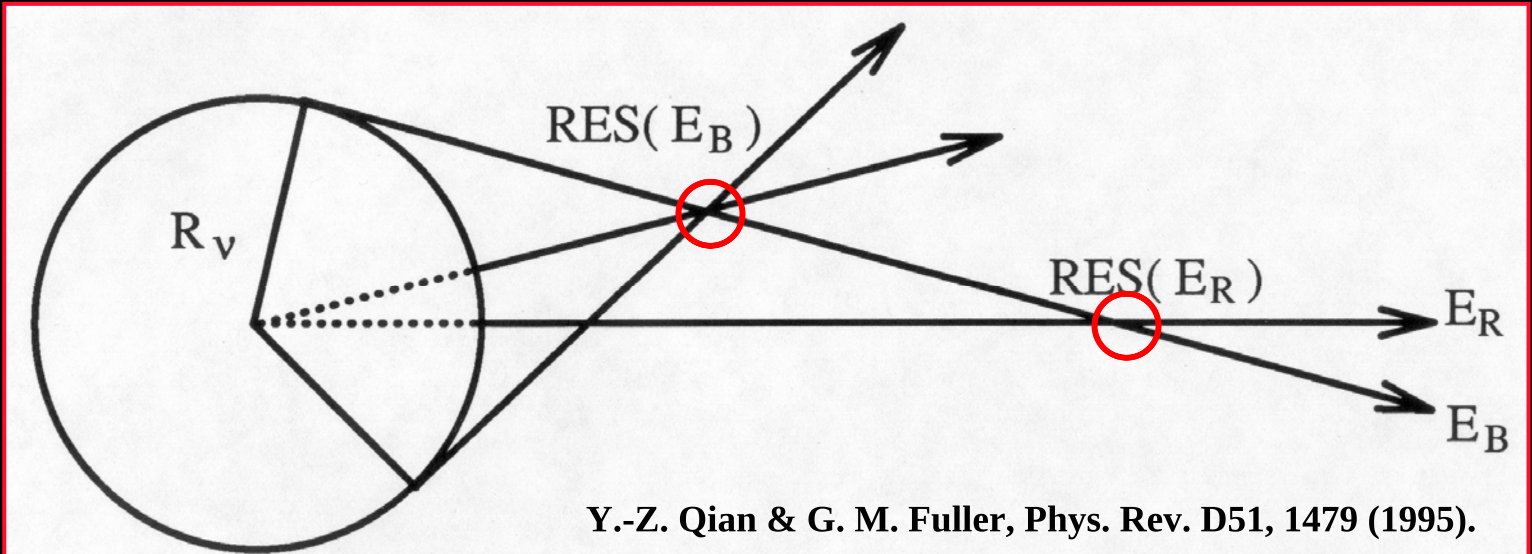
flavor diagonal potential B

flavor off-diagonal potential $B_{e\tau}$



80x10^-3
60
40
20
0
f_{ν_α}
ν_τ
ν_τ
ν_e
ν_e
sense of MSW resonance sweep
0
10
20
30
40
50
60
E_r
E_ν (MeV)

The flavor amplitude evolution history of a given neutrino depends on the prior amplitude evolution histories of the background neutrinos which intersect its world line.



Y.-Z. Qian & G. M. Fuller, Phys. Rev. D51, 1479 (1995).

So, this cannot be followed with a simple *one dimensional* mean-field Schroedinger equation.

Instead we must face a *computationally daunting problem*, one where the geometric entangling of histories is treated adequately.

Each forward scattering event results in a quantum mechanical “entanglement” of the flavor histories of the two neutrinos!

In light of this, one could legitimately ask about the efficacy of a mean field Schroedinger equation treatment for this problem.

A. Friedland & C. Lunardini PRD 68, 013007 (2003);
JHEP 43, 0310 (2003).

N. Bell, A. Rawlinson, & R. F. Sawyer Phys. Lett. B573, 86 (2003).

But first . . . Make an approximation that all neutrinos evolve just like a radially-propagating neutrino, i.e., solve numerically a simple "one-dimensional" Schroedinger mean-field picture.

Neutrino-Electron forward scattering potential

$$\hat{H}_{e\nu}(t) = A(t)|\nu_e\rangle\langle\nu_e|$$

Neutrino-Neutrino forward scattering potential

$$\hat{H}_{\nu\nu}(t) = \sqrt{2}G_F \int (1 - \cos\theta_{\mathbf{pq}}) \left[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)\right] d^3\mathbf{q}$$

Flavor Basis Evolution $\left|\Psi_{\nu_\alpha}\right\rangle$ **neutrino born as ν_α (α=e,τ) at neutrino sphere**

$$\Psi_f \equiv \begin{bmatrix} a_{e\alpha}(t) \\ a_{\tau\alpha}(t) \end{bmatrix}$$

$$a_{e\alpha}(t) \equiv \langle \nu_e | \Psi_{\nu_\alpha}(t) \rangle$$

$$a_{\tau\alpha}(t) \equiv \langle \nu_\tau | \Psi_{\nu_\alpha}(t) \rangle$$

$$\Delta \equiv \frac{\delta m^2}{2E_\nu}$$

$$i\frac{\partial \Psi_f}{\partial t} \approx \left[\left(p + \frac{m_1^2 + m_2^2}{4p} + \frac{A}{2} + \alpha_\nu\right)\hat{I} + \frac{1}{2}\begin{pmatrix} A + B - \Delta\cos 2\theta & \Delta\sin 2\theta + B_{e\tau} \\ \Delta\sin 2\theta + B_{\tau e} & \Delta\cos 2\theta - A - B \end{pmatrix}\right]\Psi_f$$

Potentials

$$A = \sqrt{2}G_{\mathrm{F}}\left(n_{e^-} - n_{e^+}\right)$$

$$B = \sqrt{2}G_{\mathrm{F}}\int (1-\cos\theta_{\mathbf{pq}})\left(\left[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)\right]_{ee} - \left[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)\right]_{\tau\tau}\right)d^3\mathbf{q}$$

$$B_{e\tau} = 2\sqrt{2}G_{\mathrm{F}}\int (1-\cos\theta_{\mathbf{pq}})\left[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)\right]_{e\tau} d^3\mathbf{q}$$

Density Operators

$$\hat{\rho}_{\mathbf{p}}(t)\, d^3\mathbf{p} \equiv \sum_\alpha dn_{\nu_\alpha} |\Psi_{\nu_\alpha}(t)\rangle\langle\Psi_{\nu_\alpha}(t)|$$

$$\hat{\bar{\rho}}_{\mathbf{p}}(t)\, d^3\mathbf{p} \equiv \sum_\alpha dn_{\bar{\nu}_\alpha} |\Psi_{\bar{\nu}_\alpha}(t)\rangle\langle\Psi_{\bar{\nu}_\alpha}(t)|$$

e.g., number of neutrinos of alpha flavor in a pencil of directions and energy

$$dn_{\nu_\alpha} \approx \frac{L_{\nu_\alpha}}{\pi R_\nu^2}\frac{1}{\langle E_{\nu_\alpha}\rangle}\left(\frac{d\Omega_\nu}{4\pi}\right) f_{\nu_\alpha}(E_\nu)\, dE_\nu$$

Notation for the matrix elements of density operators:

$$\boxed{\left[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)\right]_{ee} d^3\mathbf{q} \equiv \underbrace{\langle \nu_e | \hat{\rho}_{\mathbf{q}}(t) d^3\mathbf{q} | \nu_e \rangle}_{\text{number of electron neutrinos}} - \underbrace{\langle \bar{\nu}_e | \hat{\bar{\rho}}_{\mathbf{q}}(t) d^3\mathbf{q} | \bar{\nu}_e \rangle}_{\text{number of electron antineutrinos}}}$$

$$\boxed{\left[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)\right]_{\tau\tau} d^3\mathbf{q} \equiv \langle \nu_\tau | \hat{\rho}_{\mathbf{q}}(t) d^3\mathbf{q} | \nu_\tau \rangle - \langle \bar{\nu}_\tau | \hat{\bar{\rho}}_{\mathbf{q}}(t) d^3\mathbf{q} | \bar{\nu}_\tau \rangle}$$

$$\boxed{\left[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t)\right]_{e\tau} d^3\mathbf{q} \equiv \langle \nu_e | \hat{\rho}_{\mathbf{q}}(t) d^3\mathbf{q} | \nu_\tau \rangle - \langle \bar{\nu}_e | \hat{\bar{\rho}}_{\mathbf{q}}(t) d^3\mathbf{q} | \bar{\nu}_\tau \rangle}$$

instantaneous transformation between in-medium mass states and flavor states

$$\left|\nu_e\right\rangle = \cos\theta_{\mathrm{M}}\left(t\right)\left|\nu_1\left(t\right)\right\rangle + e^{-i\delta(t)}\sin\theta_{\mathrm{M}}\left(t\right)\left|\nu_2\left(t\right)\right\rangle$$

$$\left|\nu_\tau\right\rangle = -e^{i\delta(t)}\sin\theta_{\mathrm{M}}\left(t\right)\left|\nu_1\left(t\right)\right\rangle + \cos\theta_{\mathrm{M}}\left(t\right)\left|\nu_2\left(t\right)\right\rangle$$

$$\Delta_{\mathrm{eff}}\cos 2\theta_{\mathrm{M}} = \Delta\cos 2\theta - A - B$$

$$\Delta_{\mathrm{eff}}\, e^{i\delta}\sin 2\theta_{\mathrm{M}} = \Delta\sin 2\theta + B_{e\tau}$$

$$\Delta_{\mathrm{eff}} = \sqrt{\left(\Delta\cos 2\theta - A - B\right)^2 + \left|\Delta\sin 2\theta + B_{e\tau}\right|^2}$$

Restrict discussion to real amplitudes . . .

$$\Delta_{\mathrm{eff}} \sin 2\theta_M\,(t) \equiv \Delta \sin 2\theta + B_{e\tau}$$

$$\Delta_{\mathrm{eff}} \cos 2\theta_M\,(t) \equiv \Delta \cos 2\theta - A - B$$

$$\Delta_{\mathrm{eff}} = \sqrt{(\Delta \cos 2\theta - A - B)^2 + (\Delta \sin 2\theta + B_{e\tau})^2}$$

Consider active-active neutrino mixing:

in vacuum

$$\left|\nu_\alpha\right\rangle = \cos\theta\left|\nu_1\right\rangle + \sin\theta\left|\nu_2\right\rangle$$

$$\left|\nu_\beta\right\rangle = -\sin\theta\left|\nu_1\right\rangle + \cos\theta\left|\nu_2\right\rangle$$

here α,β = e,μ,τ

in "medium," in the supernova core or envelope

$$\left|\nu_\alpha\right\rangle = \cos\theta_M(t)\left|\nu_1(t)\right\rangle + \sin\theta_M(t)\left|\nu_2(t)\right\rangle$$

$$\left|\nu_\beta\right\rangle = -\sin\theta_M(t)\left|\nu_1(t)\right\rangle + \cos\theta_M(t)\left|\nu_2(t)\right\rangle$$

For the ultra-high density core/neutron star limit see for example Abazajian, Fuller, Patel, Phys. Rev. D64, 023501 (2001).

"mass basis" evolution

$$\Psi_M \equiv \begin{bmatrix} a_{1\alpha}(t) \\ a_{2\alpha}(t) \end{bmatrix}$$

$$a_{1\alpha}(t) \equiv \langle \nu_1(t) | \Psi_{\nu_\alpha}(t) \rangle$$

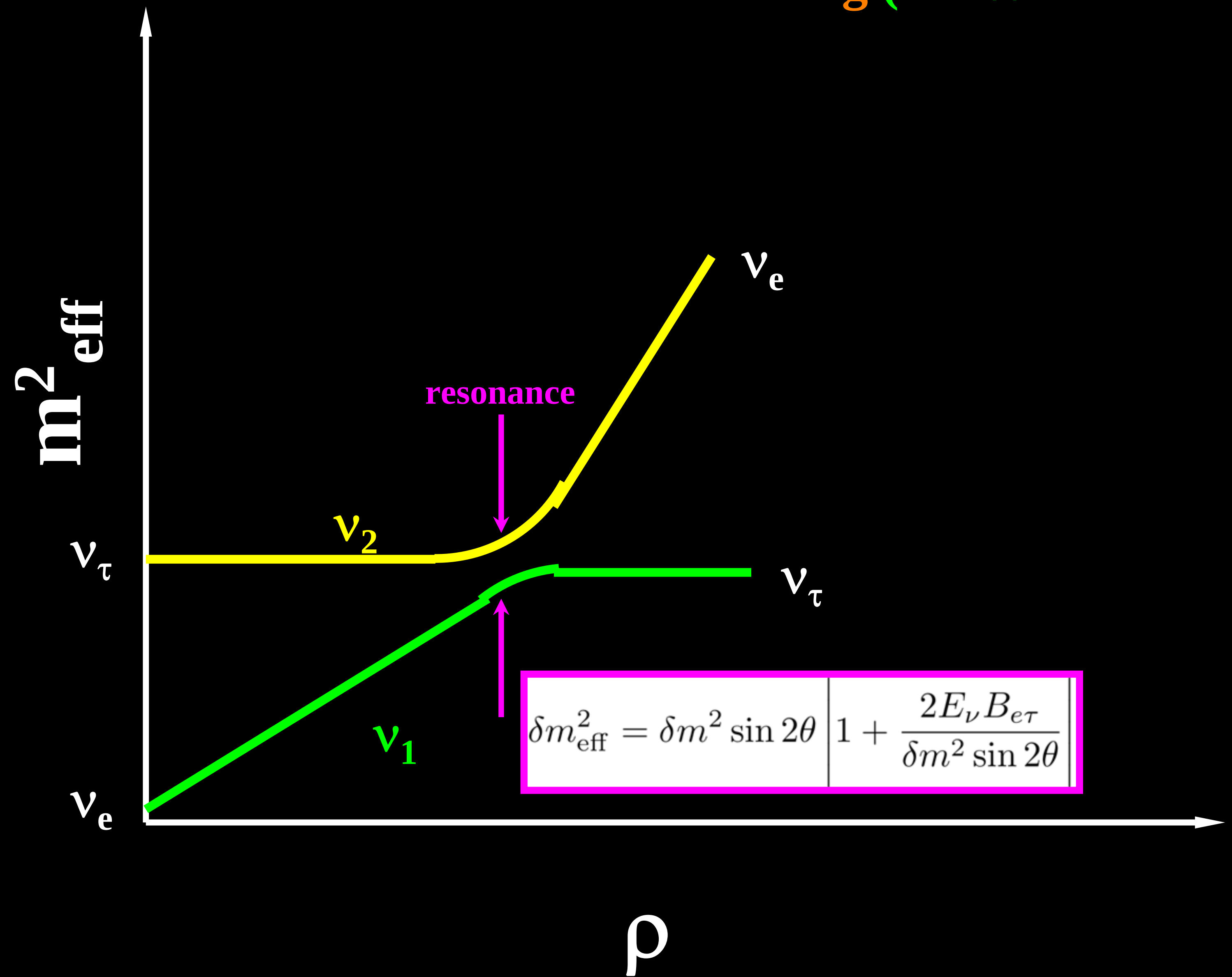
$$a_{2\alpha}(t) \equiv \langle \nu_2(t) | \Psi_{\nu_\alpha}(t) \rangle$$

$$i\frac{\partial \Psi_M}{\partial t} \approx \left[\left(p + \frac{m_1^2 + m_2^2}{4p} + \frac{A}{2} + \alpha_\nu \right) \hat{I} + \frac{\Delta_{\text{eff}}}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -\dot{\theta}_M(t) \\ \dot{\theta}_M(t) & 0 \end{pmatrix} \right] \Psi_M$$

$$\Delta_{\text{eff}} = \sqrt{(\Delta \cos 2\theta - A - B)^2 + (\Delta \sin 2\theta + B_{e\tau})^2}$$

MSW resonance when $\Delta \cos 2\theta = A + B$

Neutrino Mass Level Crossing (MSW Resonance)



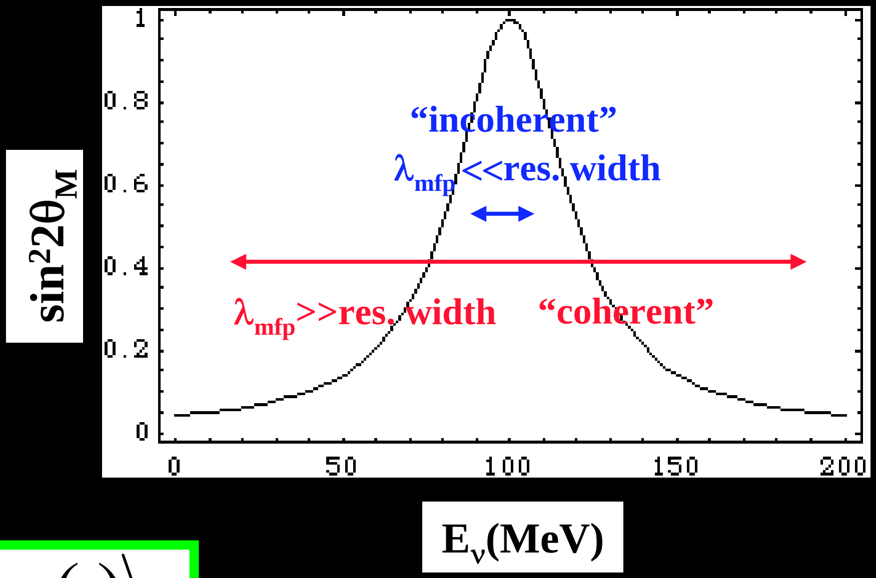
$$\delta m^2_{\rm eff} = \delta m^2 \sin 2\theta \left| 1 + \frac{2E_\nu B_{e\tau}}{\delta m^2 \sin 2\theta} \right|$$

ordinary MSW evolution of neutrino flavors

MSW resonance at neutrino energy

$$E_\nu = \frac{\delta m^2 \cos 2\theta}{2(A+B)} \approx (0.02\ \mathrm{MeV})\left(\frac{\delta m^2 \cos 2\theta}{3\times 10^{-3}\ \mathrm{eV}^2}\right)\left(\frac{10^6\ \mathrm{g\ cm}^{-3}}{\rho(\mathrm{Y_e}+\mathrm{Y}_\nu)}\right)$$

At a given location expect only neutrinos in a narrow energy range to experience efficient flavor conversion while anti-neutrino conversion is suppressed. With the small measured neutrino mass-squared differences we expect significant flavor conversion only at low densities.



time/position - dependent mixing angle and mass-states

$$|\nu_e\rangle = \cos\theta_M(t)|\nu_1(t)\rangle + \sin\theta_M(t)|\nu_2(t)\rangle$$

$$|\nu_\tau\rangle = -\sin\theta_M(t)|\nu_1(t)\rangle + \cos\theta_M(t)|\nu_2(t)\rangle$$

MSW Resonances in the Region Above the Neutron Star

(flavor diagonal potentials only - ignore neutrino background)

ρ

fixed neutrino energy

$\sin^2 2\theta_M$

$\bar{\theta}_M = 0$,
antineutrinos
do not transform

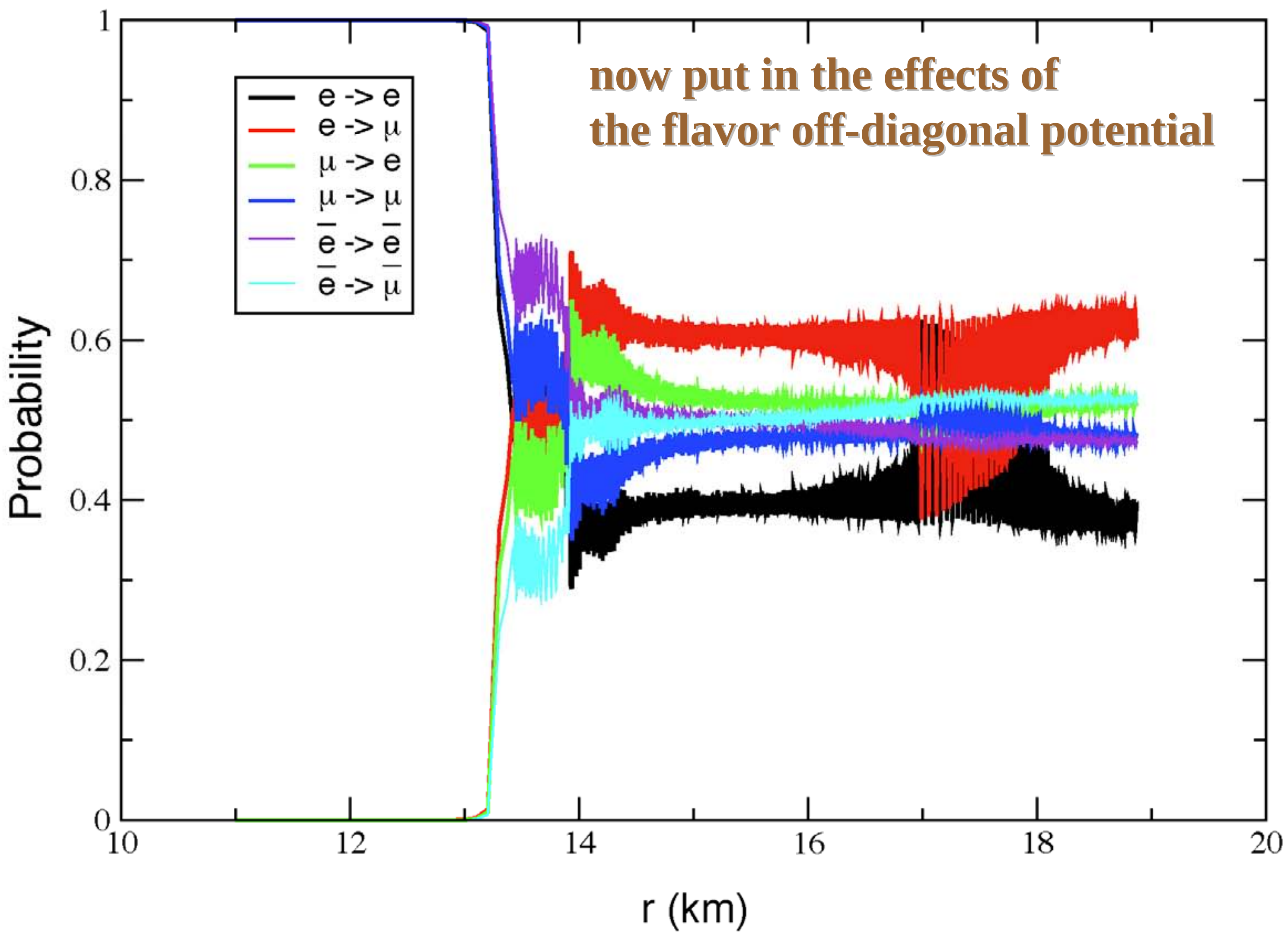
Schirato & Fuller

But what happens when we put in the flavor basis off-diagonal potential ?

Answer: **qualitatively very different with neutrino *and* antineutrino conversion**

J. Carlson (2004)

Flavor Conversion vs. Radius



So, the Flavor Off-Diagonal Potential $B_{e\tau}$ greatly complicates following neutrino flavor evolution numerically.

However, we have found that the problem simplifies in the limit where this potential is dominant.

And this has led to two surprising results:

(1) Neutrinos *and* antineutrinos could *simultaneously* and efficiently transform their flavors over a *broad* range of energy.

(2) This could happen even at high matter density, deep in the supernova environment even given the small measured neutrino mass-squared differences.

G. M. Fuller & Y.-Z. Qian

“Simultaneous Transformation of Neutrinos and Antineutrinos with Dominant Potentials from Neutrino-Neutrino Forward Scattering”

astro-ph/0505240

“ Background Dominant Solution”

$$\text{resonance condition}\quad \frac{\delta m^2 \cos 2\theta}{2E_{\mathrm{R}}} = A + B$$

In ordinary MSW, only neutrinos with energy E_R are “resonant” and have maximal mixing; antineutrino mixing is suppressed.

$$\sin 2\theta_{\mathrm{M}}(t_{\mathrm{RES}}) = \frac{\tan 2\theta\left[1 + \frac{2E_\nu B_{e\tau}}{\delta m^2 \sin 2\theta}\right]}{\sqrt{(1 - E_\nu / E_{\mathrm{R}})^2 + \tan^2 2\theta\left[1 + \frac{2E_\nu B_{e\tau}}{\delta m^2 \sin 2\theta}\right]^2}}$$

$$\approx 1 \text{ for } \frac{2E_\nu B_{e\tau}}{\delta m^2 \sin 2\theta} >> 1$$

$$\sin 2\overline{\theta}_{\mathrm{M}}(t_{\mathrm{RES}}) = \frac{\tan 2\theta\left[1 - \frac{2E_\nu B_{e\tau}}{\delta m^2 \sin 2\theta}\right]}{\sqrt{(1 + E_\nu / E_{\mathrm{R}})^2 + \tan^2 2\theta\left[1 - \frac{2E_\nu B_{e\tau}}{\delta m^2 \sin 2\theta}\right]^2}}$$

$$\approx -1 \text{ for } \frac{2E_\nu B_{e\tau}}{\delta m^2 \sin 2\theta} >> 1$$

neutrinos & antineutrinos mix maximally over a wide range of neutrino/antineutrino energy!!!

To get this limit, $B_{e\tau}$ need not be large if the mass-squared difference is small.

Large Off-Diagonal Potentials Increase Adiabaticity

…by decreasing neutrino oscillation length at resonance

$$L_{\rm osc}^{\rm res} = \frac{4\pi E_\nu}{\delta m^2_{\rm eff}} \approx \frac{4\pi E_\nu}{\delta m^2 \sin 2\theta}\left|1 + \frac{2E_\nu B_{e\tau}}{\delta m^2 \sin 2\theta}\right|^{-1}$$

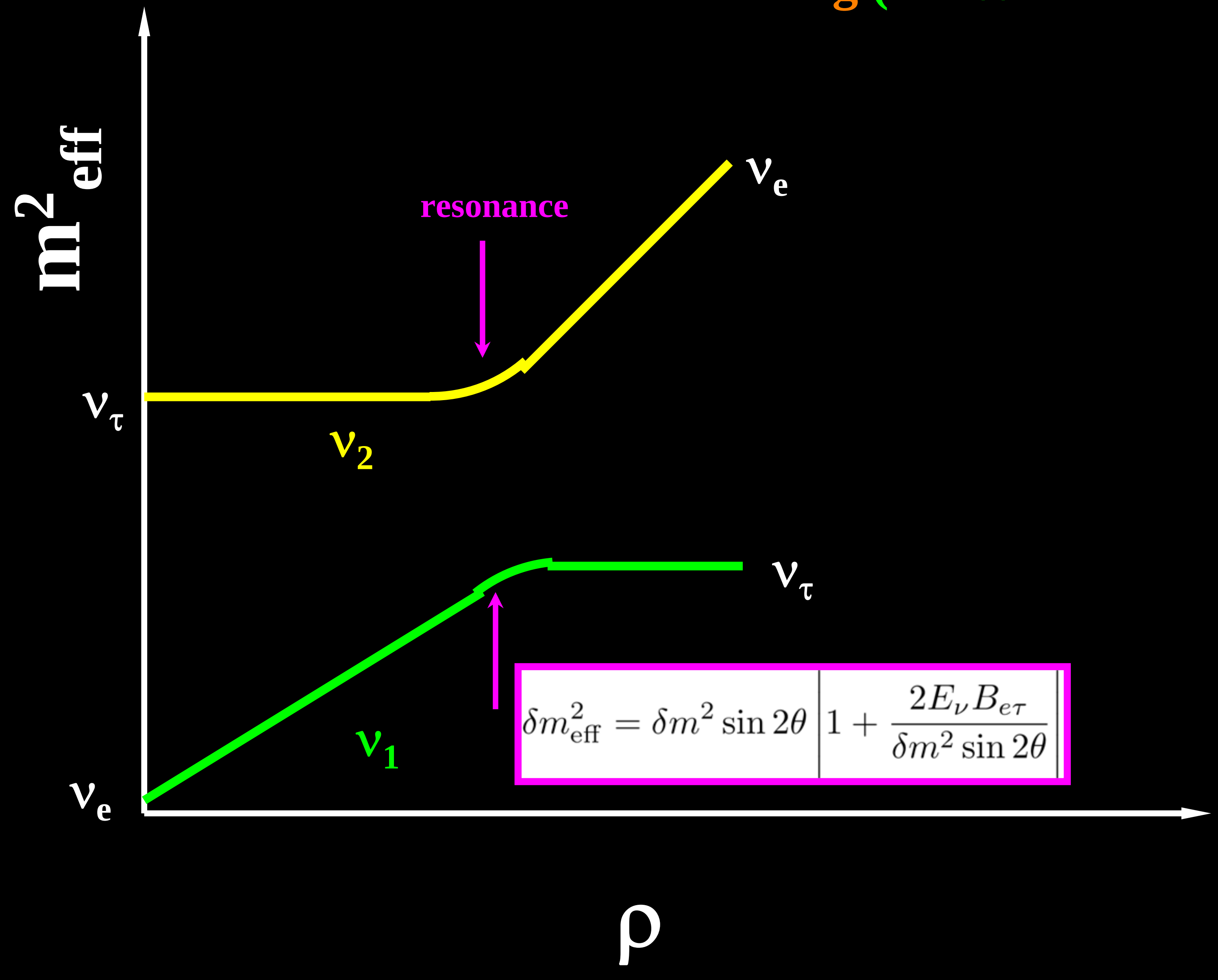
…and by increasing the resonance width $\Delta \equiv \delta m^2/2E_\nu$

$$\delta r = \frac{dr}{dV}\,\delta V = \underbrace{\left|\frac{1}{V}\frac{dV}{dr}\right|^{-1}}_{\text{Density scale height}} \Delta \sin 2\theta \left|1 + \frac{2E_\nu B_{e\tau}}{\delta m^2 \sin 2\theta}\right|$$

Adiabaticity parameter **(adiabatic if $\gamma >> 1$)**

$$\gamma = \frac{\delta r}{L_{\rm osc}^{\rm res}} \approx \frac{1}{2}\frac{\delta m^2 \mathcal{H}}{E_\nu}\frac{\sin^2 2\theta}{\cos 2\theta}\left|1 + \frac{2E_\nu B_{e\tau}}{\delta m^2 \sin 2\theta}\right|^2$$

Neutrino Mass Level Crossing (MSW Resonance)



The Background Dominant Solution (BDS) . . .

in the adiabatic limit

$$B_{e\tau} = \sqrt{2} G_F \int \left(1 - \cos\theta_{\mathbf{q}}\right) \left[\left(dn_{\nu_e} - dn_{\nu_\tau}\right) \sin 2\theta_M \left(t_{\mathrm{RES}}\right) + \left(dn_{\bar{\nu}_e} - dn_{\bar{\nu}_\tau}\right) \sin 2\bar{\theta}_M \left(t_{\mathrm{RES}}\right) \right]$$

Now employ the maximal mixing of the BDS . . .

$$B_{e\tau}^{\mathrm{BDS}} \approx \left(7 \times 10^{-12}\ \mathrm{MeV}\right) \frac{\left[1 - \sqrt{1 - \left(R_6^{\nu\mathrm{sp}}\right)^2 / r_6^2}\right]^2}{\left(R_6^{\nu\mathrm{sp}}\right)^2} \left\{ L_{\nu_e}^{51} \frac{10\ \mathrm{MeV}}{\langle E_{\nu_e} \rangle} - L_{\bar{\nu}_e}^{51} \frac{15\ \mathrm{MeV}}{\langle E_{\bar{\nu}_e} \rangle} \right\}$$

$$\approx 1.8 \times 10^{-15}\ \mathrm{MeV} \quad \text{at } r_6 \approx 5.6 \text{ with } \tan 2\theta = 0.1$$

Whereas, for 20 MeV neutrinos with the atmospheric mass-squared scale . . .

$$\left[\frac{\delta m^2}{2E_\nu} \sin 2\theta\right]_{\mathrm{ATMOS}} \approx 7.5 \times 10^{-18}\ \mathrm{MeV}$$

$<< B_{e\tau}^{\mathrm{BDS}}$ consistent with the BDS and ensuring adiabaticity

Background Dominant Solution for

$$|B_{e\tau}| \gg A$$

$$\left.\begin{aligned} \cos 2\theta_M &\to 0 \\ \sin 2\theta_M &\to 1 \\ \cos 2\bar{\theta}_M &\to 0 \\ \sin 2\bar{\theta}_M &\to -1 \end{aligned}\right\} \quad \begin{aligned} \theta_M &\to \frac{\pi}{4} \\ \bar{\theta}_M &\to \frac{3\pi}{4} \end{aligned}$$

for real, positive $B_{e\tau}$

(1) Hydrostatic equilibrium:

$$TS \approx \frac{G\, M_{\rm NS}\, m_p}{r} \qquad\qquad r_6 \approx \frac{22.5}{T_9\ S_{100}}$$

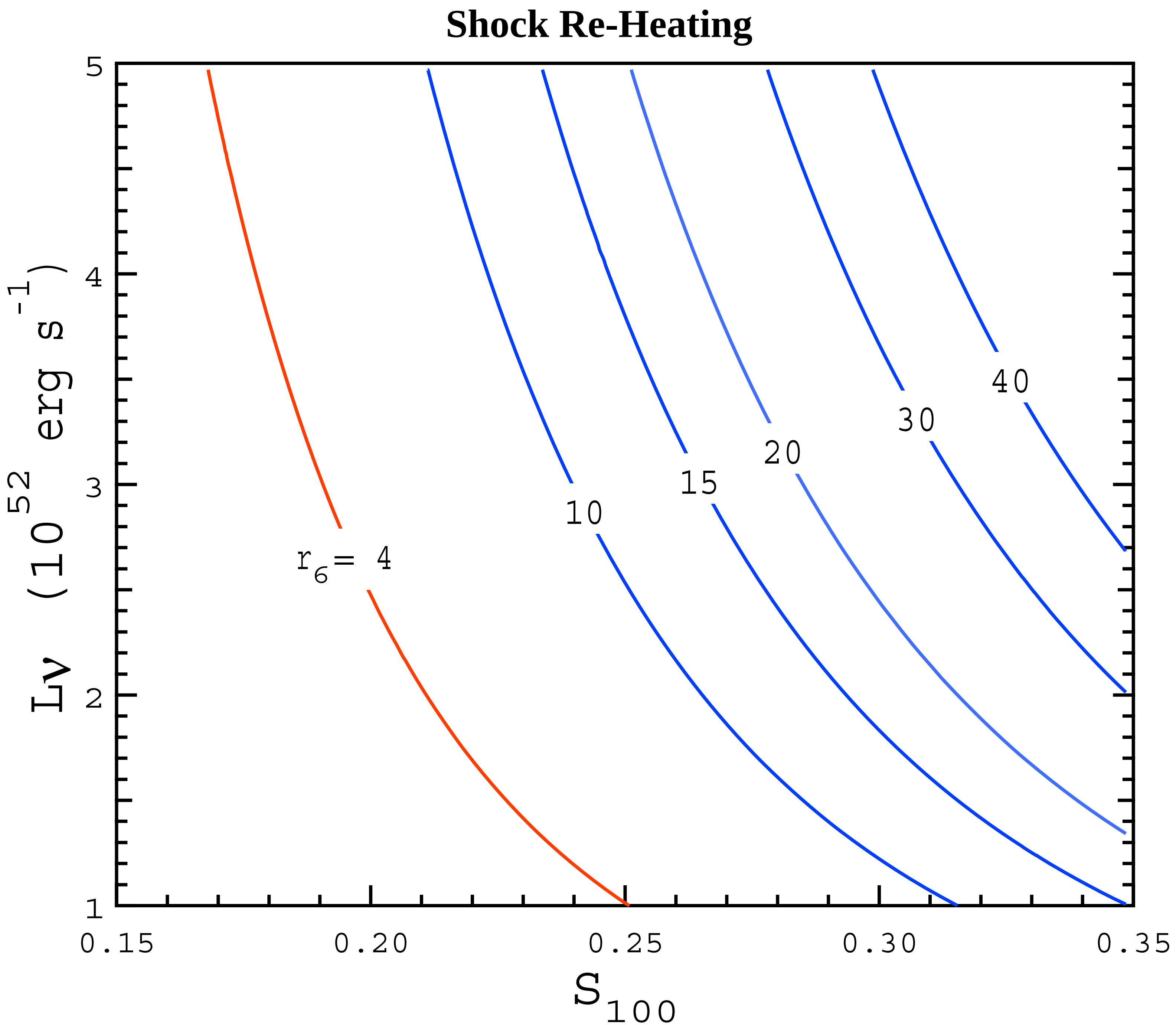
(2) Isentropic (constant entropy) flow & entropy in relativistic particles

$$S \approx \frac{2\pi^2}{45} g_s \frac{T^3}{n_b}$$

constant

(1) + (2)

$$\boxed{n_b \approx \frac{2\pi^2}{45} g_s \left(\frac{M_{\rm NS}\ m_p}{m_{\rm pl}^2} \right)^3 S^{-4} r^{-3}}$$

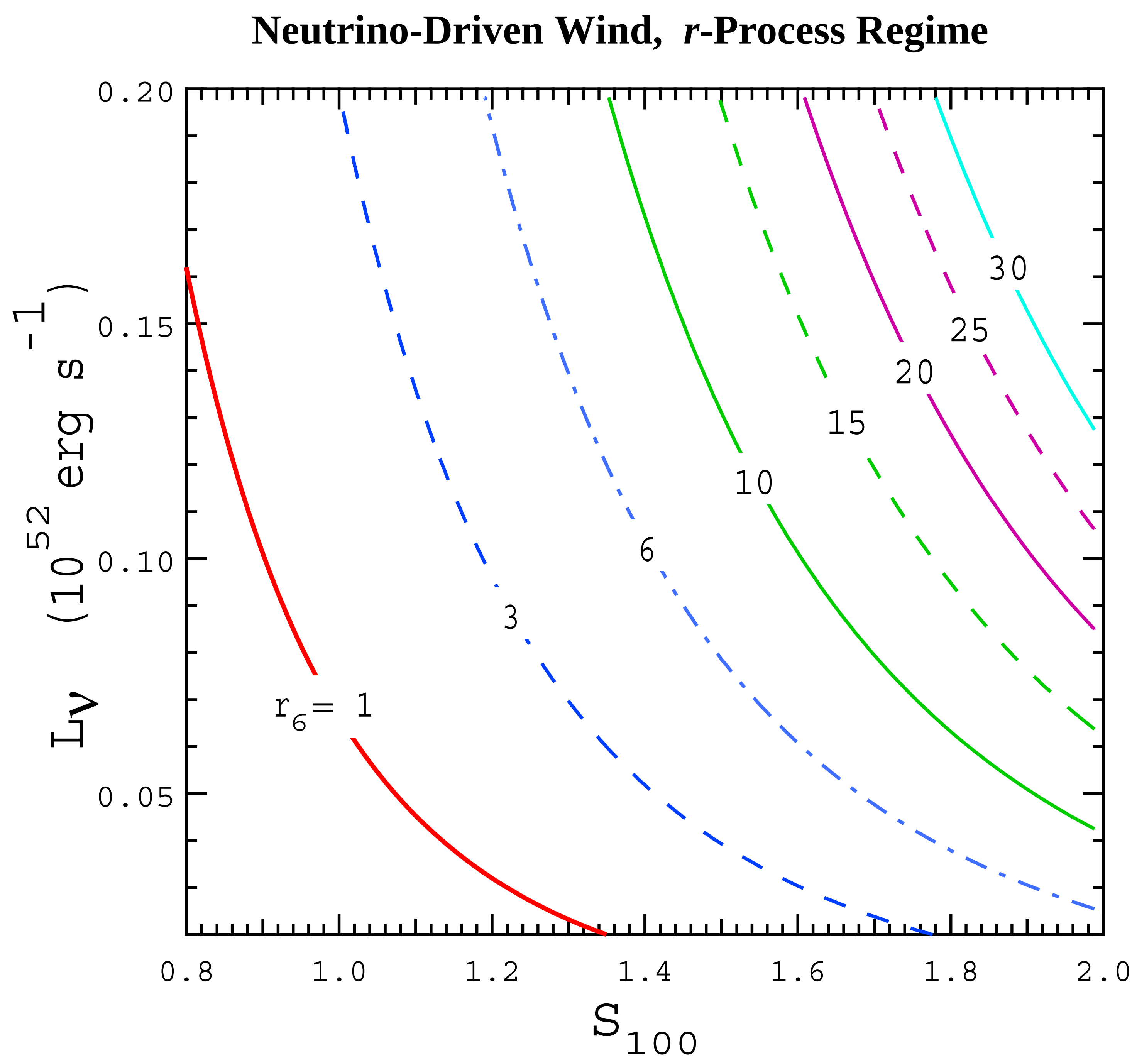


Shock Re-Heating
5
4
3
2
1
0.15
0.20
0.25
0.30
0.35
S_{100}
L_ν $(10^{52}$ erg s$^{-1})$
r_6 = 4
10
15
20
30
40

Note that the average electron neutrino and antineutrino energies may be quite similar during much of the shock re-heating epoch, but the luminosities for electron neutrinos can be significantly larger than those for electron antineutrinos.

This is especially true for shock break-out through the neutrino sphere, the so-called "neutronization burst." For a time span of ~ 10 ms we could have $L_{\nu e} \sim 10^{53}$ ergs s^{-1} with the electron antineutrino luminosity an order of magnitude smaller.

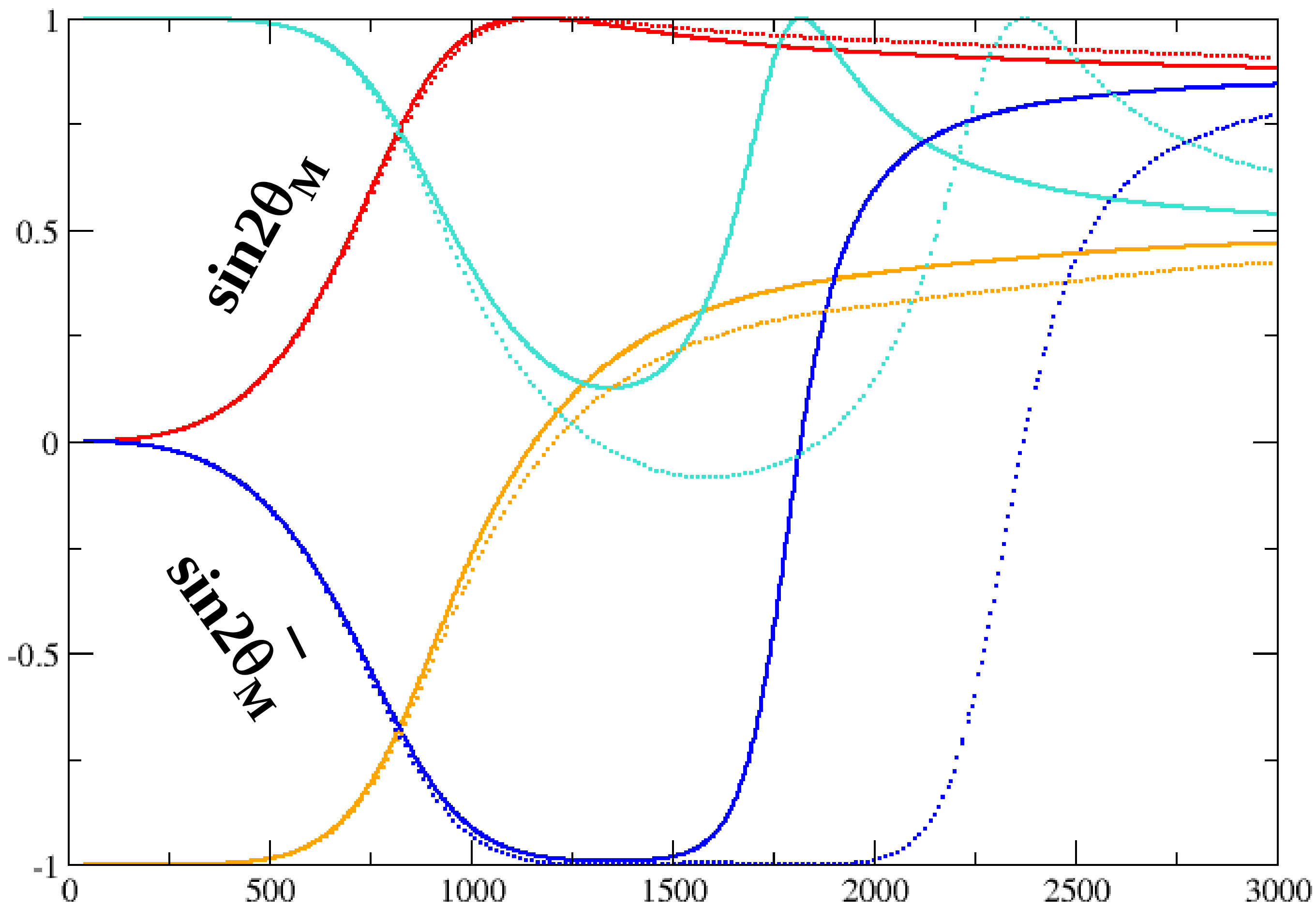
Since neutrino flavor mixing in the coherent limit is a phase effect, the 10 ms duration of this high-luminosity burst may be enough to "kick" the system into the BDS (Background Dominated Solution).


Neutrino-Driven Wind, r-Process Regime
0.20
0.15
0.10
0.05
0.8
1.0
1.2
1.4
1.6
1.8
2.0
L_ν (10^{52} erg s^{-1})
S_{100}
r_6 = 1
3
6
10
15
20
25
30

So now the question is:

Does nature ever find this solution?

Hasan Yuksel, A. B. Balantekin (collapsed angles, 1-D)



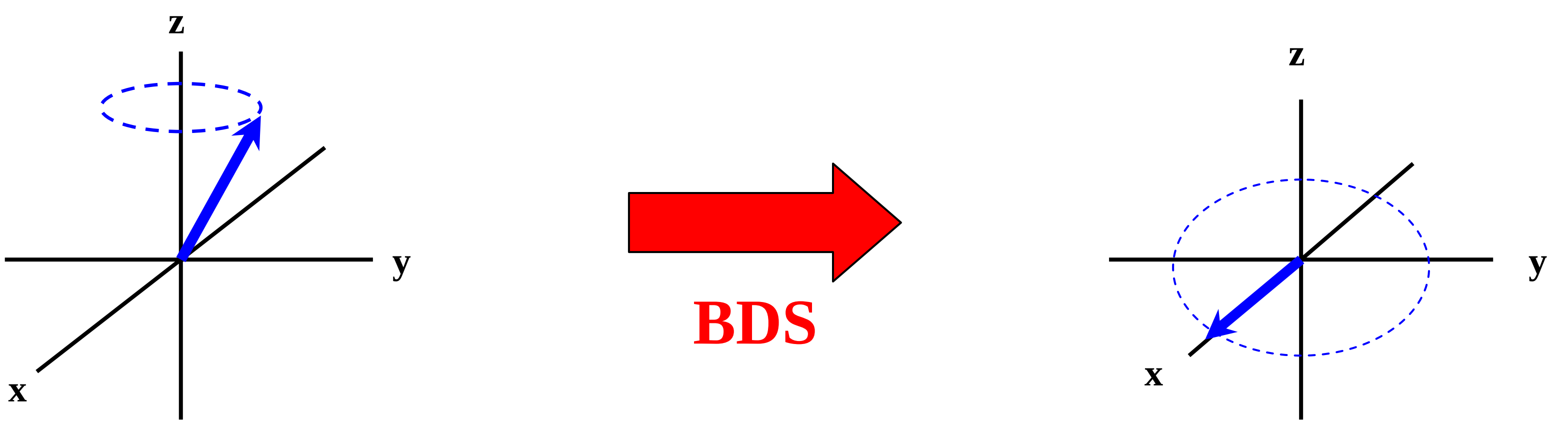
a measure of radius/time

Spin Polarization Analogy: Spin-One representation of SU(2)

Write flavor basis density operator in terms of Pauli spin matrices:

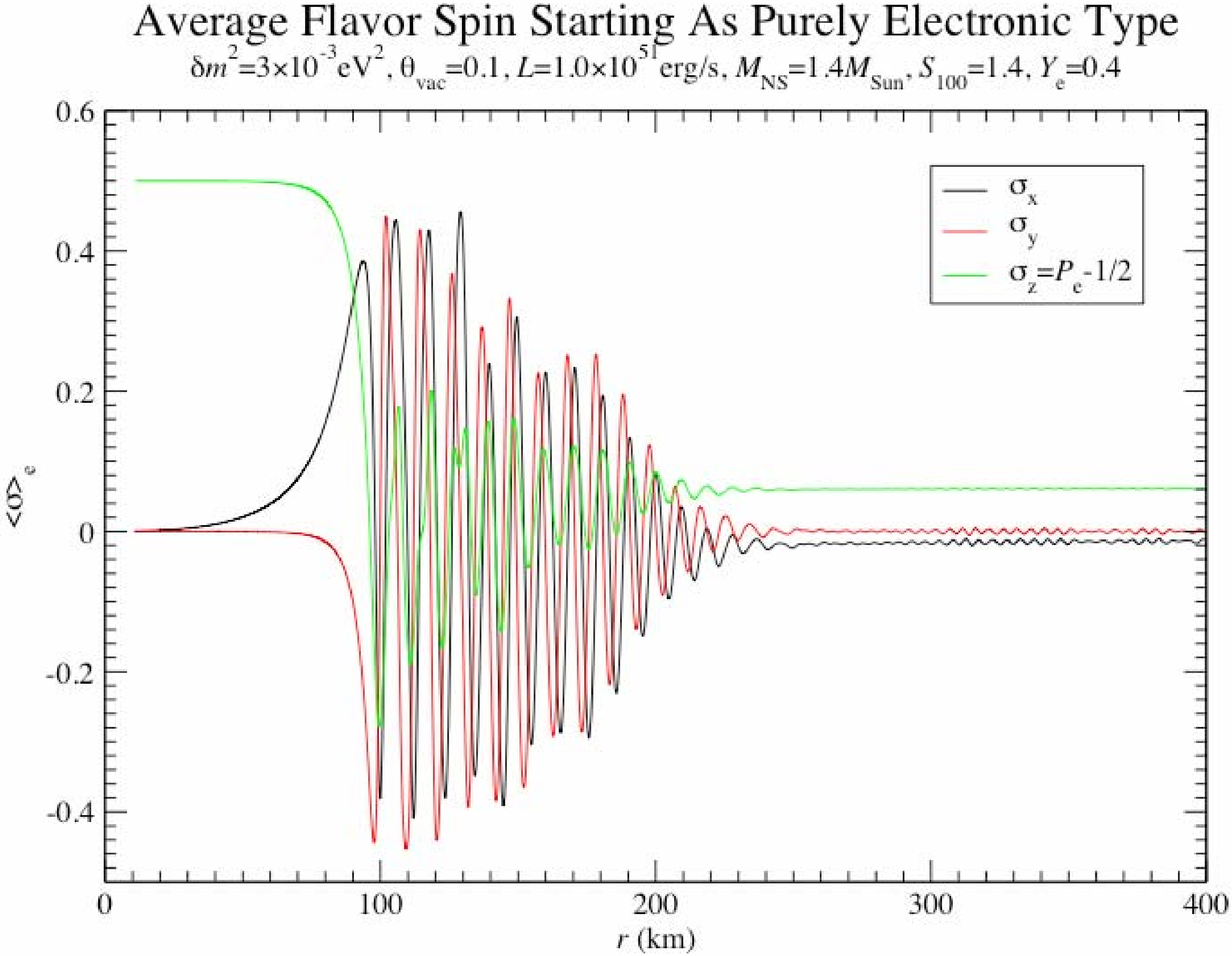
$$\begin{pmatrix} \rho_{ee} & \rho_{e\tau} \\ \rho_{\tau e} & \rho_{\tau\tau} \end{pmatrix} = \frac{P_0 \hat{I} + P_x\sigma_x + P_y\sigma_y + P_z\sigma_z}{2} = \frac{1}{2}\begin{pmatrix} P_z + P_0 & P_x - iP_y \\ P_x + iP_y & P_0 - P_z \end{pmatrix}$$

Polarization vector $\mathbf{P} \Rightarrow \{P_x, P_y, P_z\}$ **and** P_0



precession in xy-plane

H. Duan (2005)

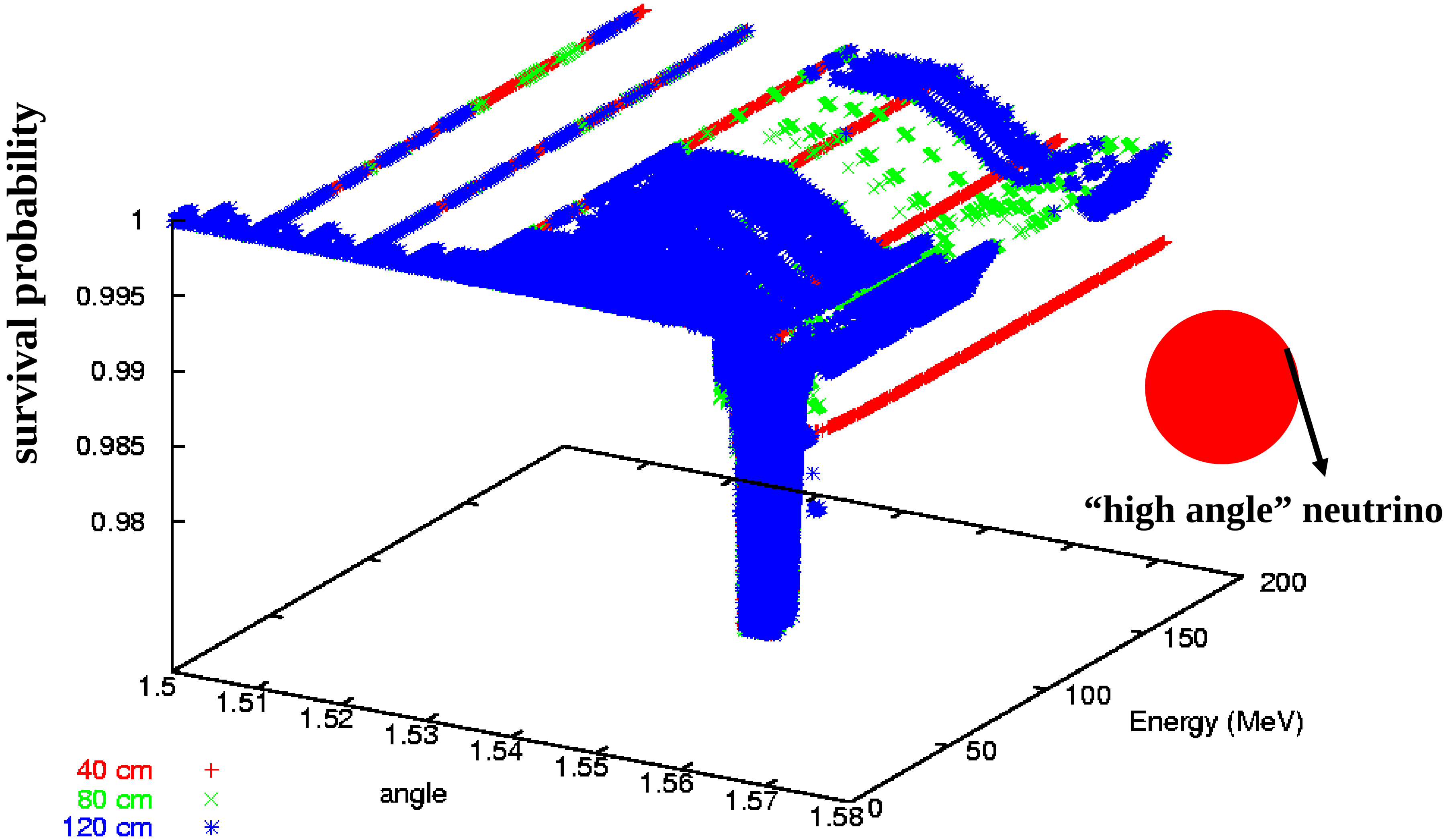


Average Flavor Spin Starting As Purely Electronic Type
δm²=3×10⁻³eV², θvac=0.1, L=1.0×10⁵¹erg/s, MNS=1.4MSun, S100=1.4, Ye=0.4
σx
σy
σz=Pe-1/2
<σ>e
r (km)
0.6
0.4
0.2
0
-0.2
-0.4
0
100
200
300
400

Now attempt to treat the full geometry,
following flavor evolution on all neutrino world lines
(at all trajectory angles) self consistently:

Adaptive Mesh Refinement preliminary calculations (Landry & Fuller) show significant differences from the "one-dimensional" treatment. However, these calculations are explicit and cannot follow flavor evolution beyond a few meters(!). We need to go to 100's kilometers. For this we need new numerical schemes (e.g., implicit schemes).

Low energy, high angle neutrinos transform first, bringing up B_{OFF} and thereby causing lower angle neutrinos to transform adiabatically. How far will this go?



Adaptive Mesh Refinement

Landry & Fuller 2004

TSI & UCSD/LANL

Computation of Neutrino Flavor Evolution in Compact Objects/SN/Early Universe

H. Duan, J. Hidaka, J. Carlson, A. Friedland, K. Abazajian, S. Reddy
P. Amanik, C. Smith, C. Kishimoto, Y. Qian, A. B. Balantekin, H. Yuksel,
A. Mezzacappa, C. Cardall, S. Bruenn, GMF

Conclusion

Active-active neutrino/antineutrino flavor transformation *may* occur deep in the supernova environment even though the measured mass-squared differences are small.

Need full and complete numerical simulations which correctly and self consistently treat the neutrino background and all trajectories (angles).