# Neutrino Flavor Transformation in SNe
## —A Quasi-static Picture

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# Outline

- Neutrino oscillations in SNe: general introduction
- Numerical simulations
- Neutrino flavor isospin
- Quasi-static picture
- Stepwise spectral swapping
- Conclusions

# 2x2 approximation

$$\left.\begin{array}{r} \delta m^2_{\rm atm} \gg \delta m^2_{\odot} \\ \text{equally mixed } \nu_\mu, \nu_\tau \\ \text{similar } \nu_\mu, \nu_\tau \text{ spectra} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \nu_e \leftrightarrow \nu_{\tau^*},\ |\delta m^2| \simeq \delta m^2_{\rm atm} \\ \\ \nu_e \leftrightarrow \nu_{\mu^*},\ \delta m^2 \simeq \delta m^2_{\odot} \end{array}\right.$$

normal mass hierarchy: $\delta m^2 \simeq \delta m^2_{\rm atm},\ \theta_{\rm v} \simeq \theta_{13}$

inverted mass hierarchy: $\delta m^2 \simeq -\delta m^2_{\rm atm},\ \theta_{\rm v} \simeq \theta_{13}$

or equivalently:

$$\delta m^2 \simeq \delta m^2_{\rm atm},\ \theta_{\rm v} \simeq \frac{\pi}{2} - \theta_{13}$$

# Coherent limit

Coherent, 2x2: $$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} a_{\nu_e} \\ a_{\nu_\tau} \end{pmatrix} = \mathcal{H}\begin{pmatrix} a_{\nu_e} \\ a_{\nu_\tau} \end{pmatrix}$$

Vacuum mass differences:

$$\mathcal{H}_{\mathrm{vac}} \equiv \frac{\delta m^2}{4E_\nu}\begin{pmatrix} -\cos 2\theta_{\mathrm{v}} & \sin 2\theta_{\mathrm{v}} \\ \sin 2\theta_{\mathrm{v}} & \cos 2\theta_{\mathrm{v}} \end{pmatrix}$$

Charge current, forward scattering results in

different refractive indices: $\mathcal{H}_e \equiv \frac{G_{\mathrm{F}} n_e}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

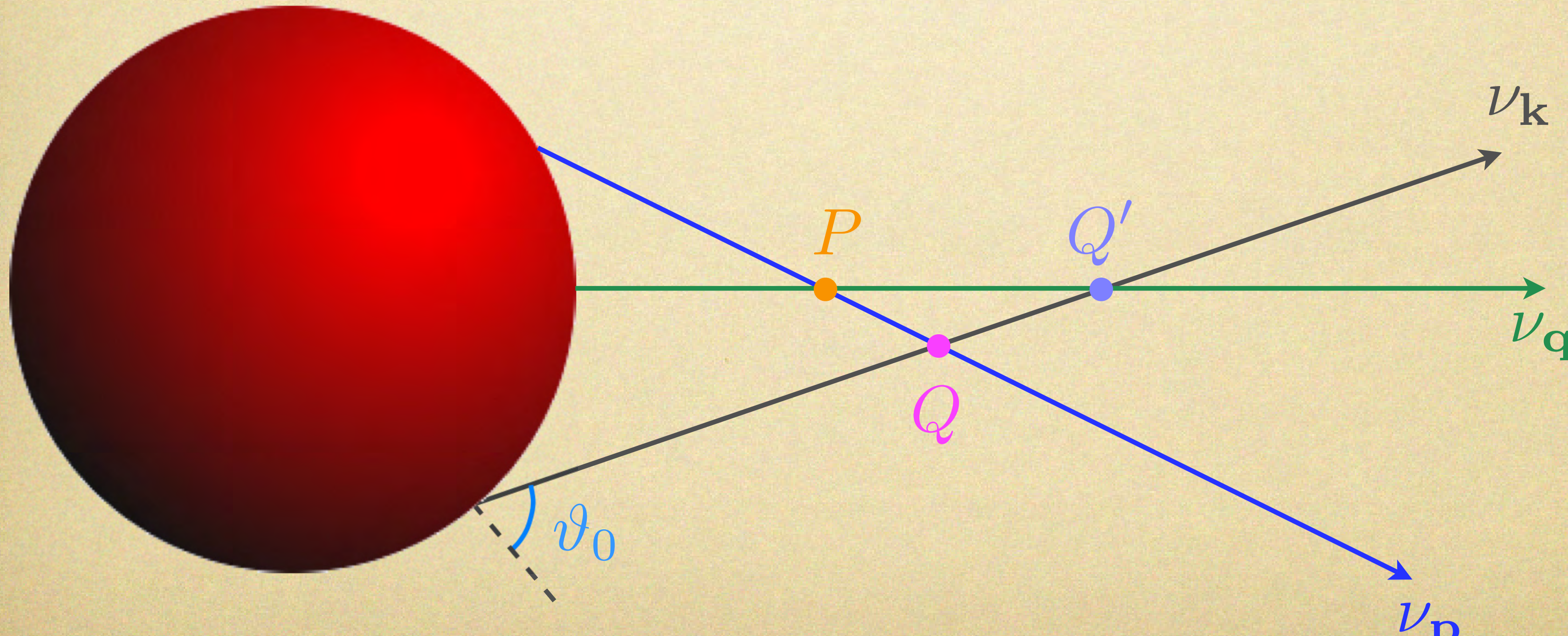
# Neutrino-neutrino coupling

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\psi_{\nu,i} = (\mathcal{H}_{\mathrm{vac},i} + \mathcal{H}_e + \mathcal{H}_{\nu\nu,i})\psi_{\nu,i}$$

$$\begin{aligned}\mathcal{H}_{\nu\nu,i} \equiv\; & \sqrt{2}G_\mathrm{F}\sum_j (1-\hat{\mathbf{k}}_i\cdot\hat{\mathbf{k}}_j)\underline{n_{\nu,j}}\,\psi_{\nu,j}\psi^\dagger_{\nu,j} \\ & -\sqrt{2}G_\mathrm{F}\sum_j (1-\hat{\mathbf{k}}_i\cdot\hat{\mathbf{k}}_j)\underline{n_{\bar{\nu},j}}\,(\psi_{\bar{\nu},j}\psi^\dagger_{\bar{\nu},j})^*\end{aligned}$$

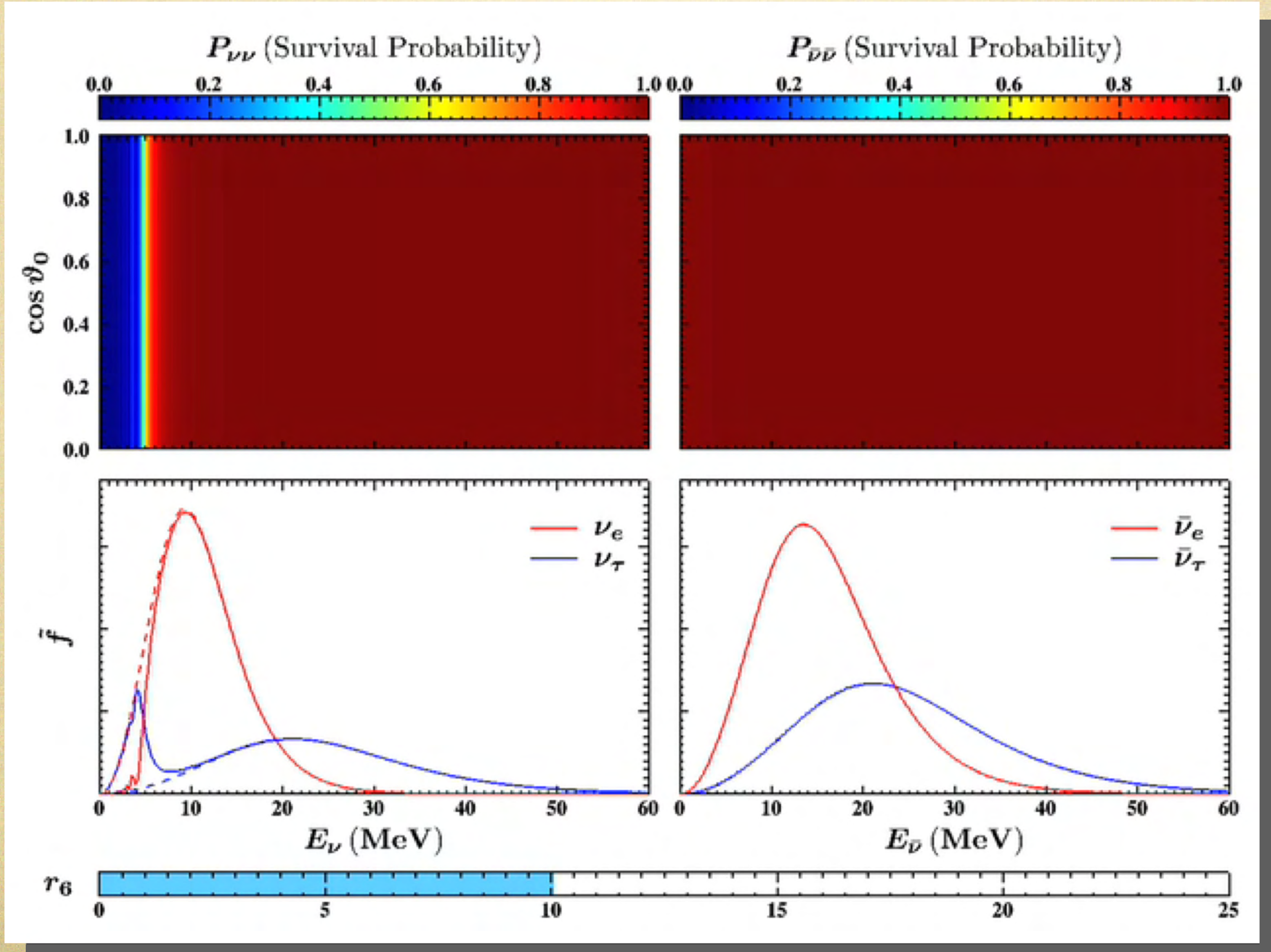
# Neutrino-neutrino coupling

- Anisotropic, nonlinear coupling of all neutrino flavor evolution histories



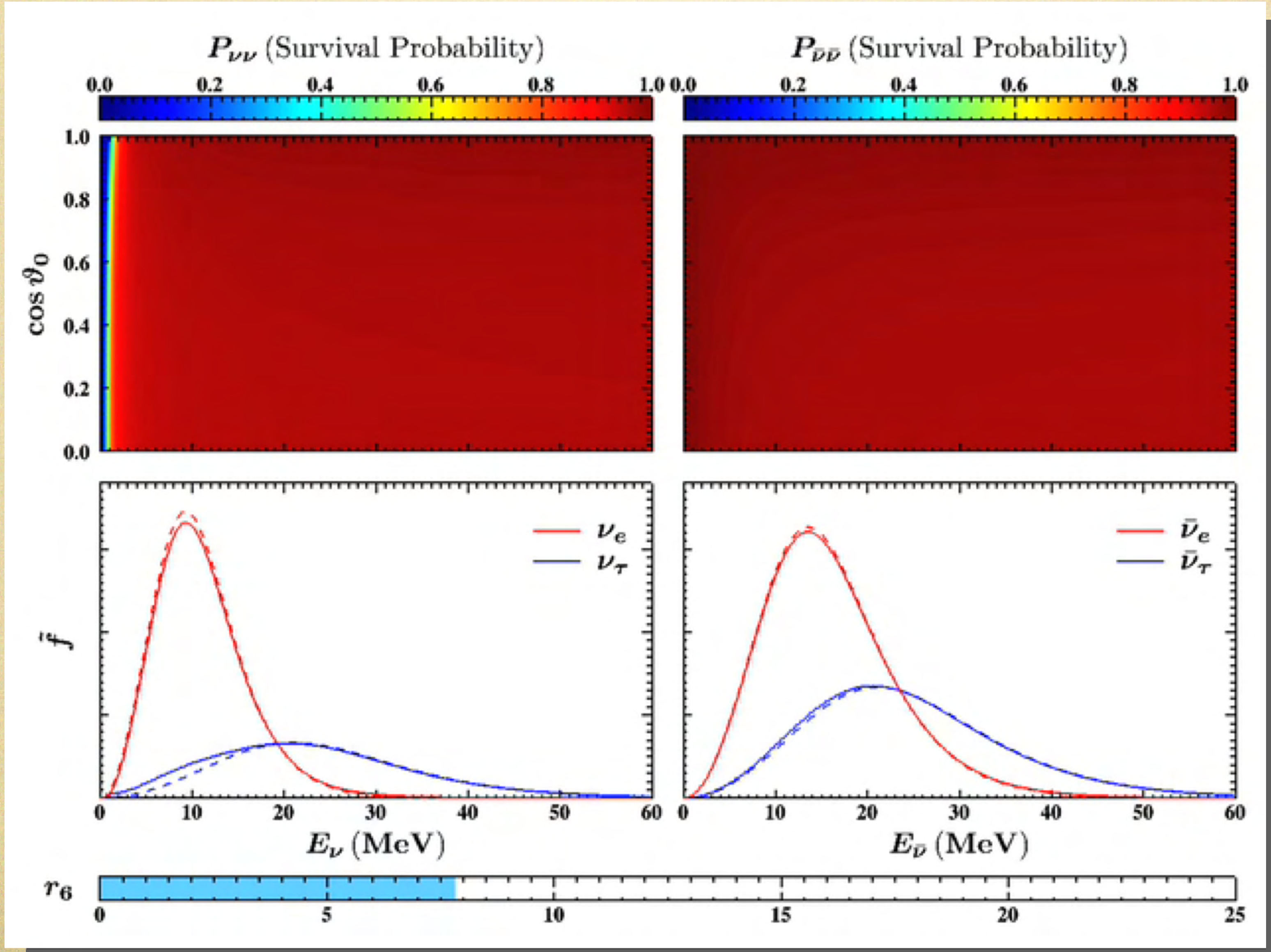
Numerical simulations

$$\delta m^2 = 3\times 10^{-3}\,\mathrm{eV}^2 \simeq \delta m^2_{\mathrm{atm}},\ \theta_{\mathrm{v}} = 0.1,\ L_\nu = 0$$

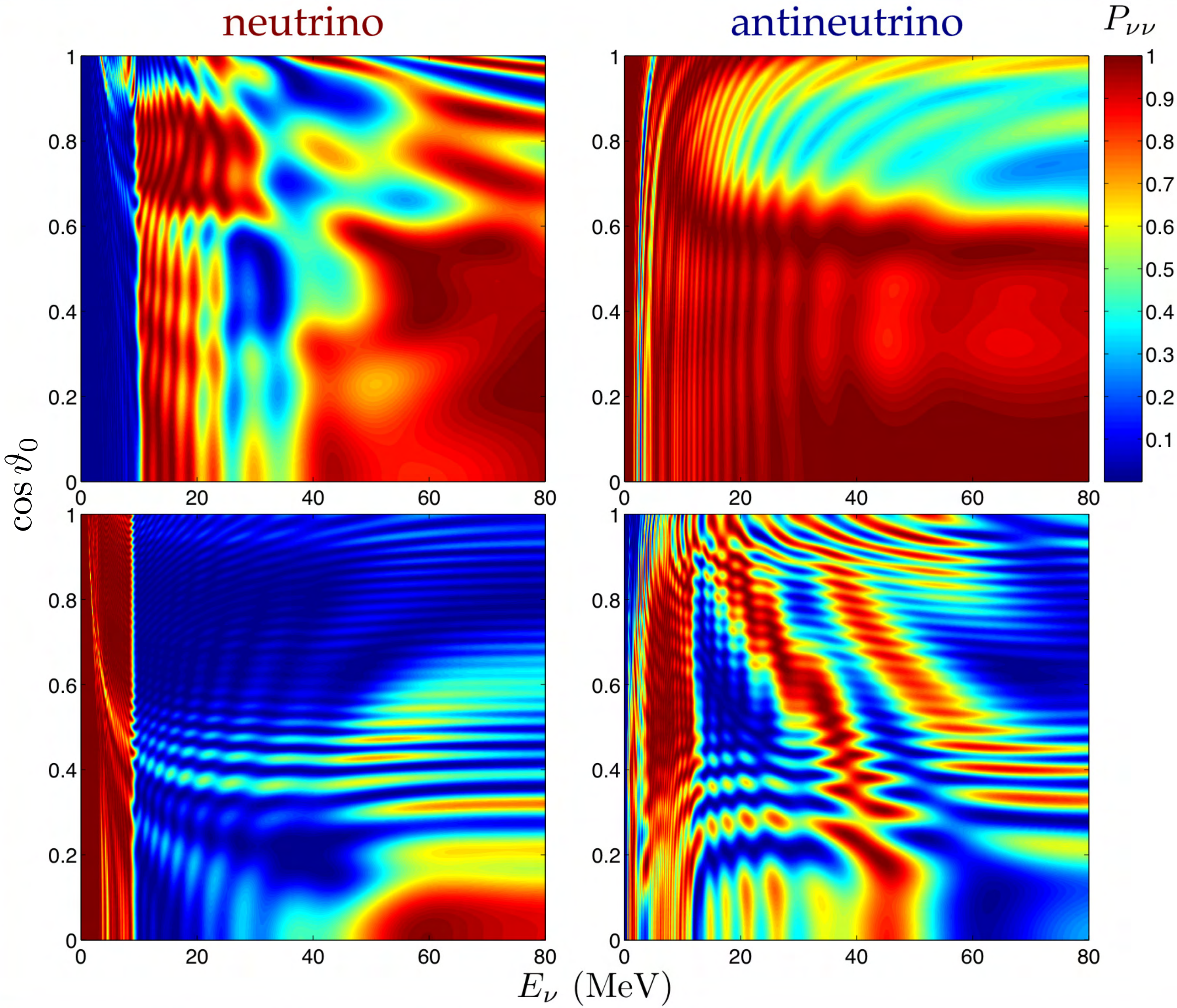


Numerical simulations

$$\delta m^2 = 3\times 10^{-3}\,\mathrm{eV}^2 \simeq \delta m^2_{\mathrm{atm}},\ \theta_{\mathrm{v}} = 0.1,\ L_\nu = 10^{51}\,\mathrm{erg/s}$$



Numerical simulations




normal mass hierarchy

inverted mass hierarchy

# Neutrino flavor isospin

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\psi_\nu = \mathcal{H}\,\psi_\nu$$

$$= -\mathbf{H}\cdot\frac{\boldsymbol{\sigma}}{2}\psi_\nu$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{s} = \mathbf{s}\times\mathbf{H}$$

$$\begin{bmatrix} e^- \\ \nu_e \end{bmatrix} \quad \begin{bmatrix} \mu/\tau \\ \nu_\mu/\nu_\tau \end{bmatrix}$$

Weak Isospin

Flavor Isospin

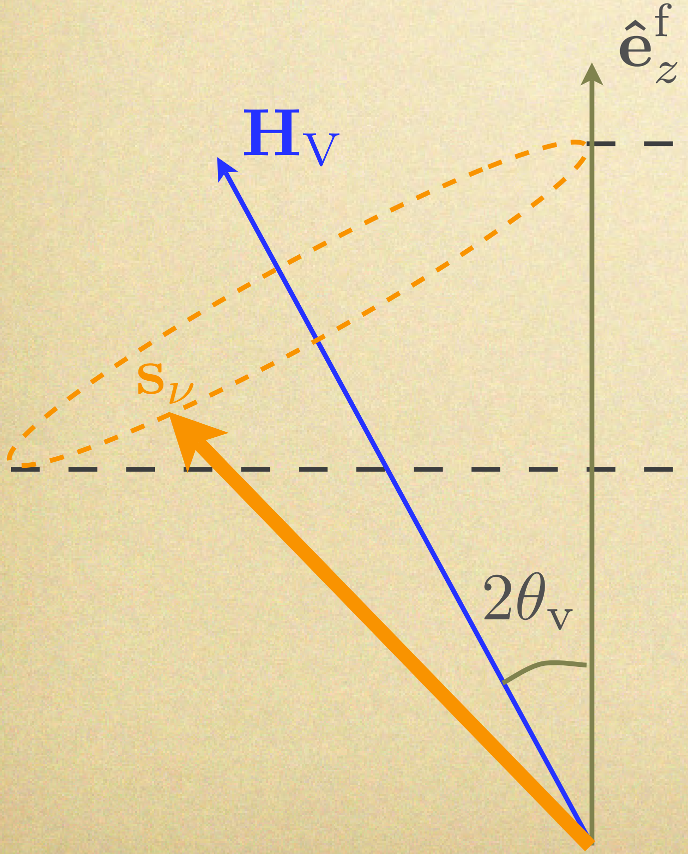
| | $e$-flavor | $\tau$-flavor | maximally mixed |
|---|---|---|---|
| $\mathbf{s}_\nu \equiv \psi_\nu^\dagger \frac{\boldsymbol{\sigma}}{2}\psi_\nu$ | $\uparrow$ | $\downarrow$ | $\rightarrow$ |
| $\mathbf{s}_{\bar\nu} \equiv (\sigma_y\psi_{\bar\nu})^\dagger \frac{\boldsymbol{\sigma}}{2}(\sigma_y\psi_{\bar\nu})$ | $\downarrow$ | $\uparrow$ | $\rightarrow$ |

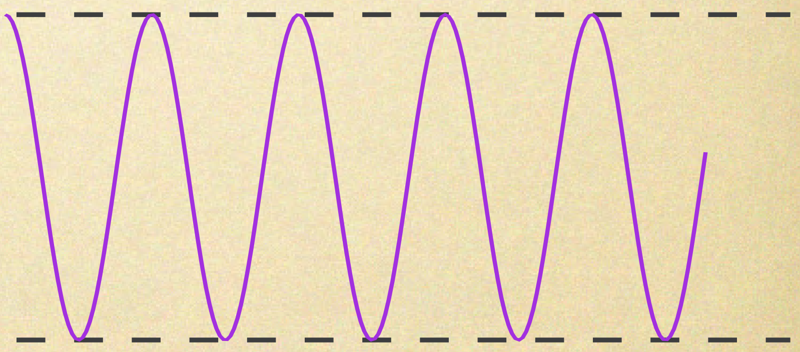
# Vacuum oscillations

$$\mathbf{H} = \omega \mathbf{H}_{\mathrm{V}}$$

$$\mathbf{H}_{\mathrm{V}} \equiv -\hat{\mathbf{e}}_x^{\mathrm{f}} \sin 2\theta_{\mathrm{v}} + \hat{\mathbf{e}}_z^{\mathrm{f}} \cos 2\theta_{\mathrm{v}}$$

$$\omega \equiv \pm \frac{\delta m^2}{2E}$$

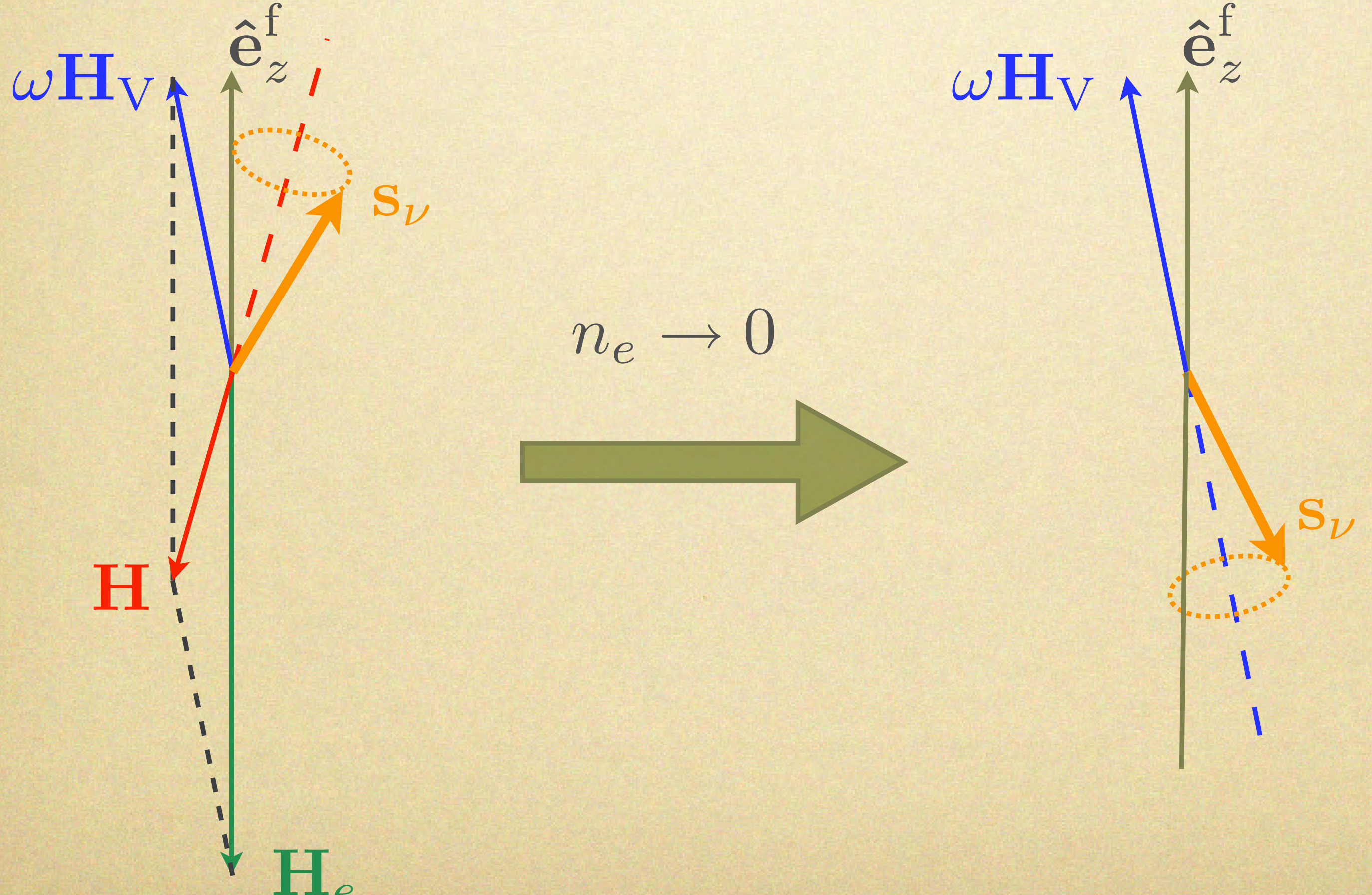


$$s_z^{\mathrm{f}} = P_{\nu_e \nu_e} - \frac{1}{2}$$

# MSW mechanism

$$\mathbf{H} = \omega\mathbf{H}_{\mathrm{V}} + \mathbf{H}_e \qquad \mathbf{H}_e \equiv -\hat{\mathbf{e}}_z^{\mathrm{f}}\sqrt{2}G_{\mathrm{F}}n_e$$



# Adiabaticity

- NFIS **s** aligned (anti-aligned) with its effective field **H** corresponds to instantaneous mass light (heavy) mass eigenstate.

$$\varepsilon = -\mathbf{s} \cdot \mathbf{H}$$

- If NFIS **s** remains aligned or anti-aligned with its effective field **H**, the flavor evolution is adiabatic.

# Single-Angle Approximation

- Flavor evolution histories of all neutrinos are the same as those propagating along the radial trajectories:

$$\sum_j (1 - \hat{\mathbf{k}}_i \cdot \hat{\mathbf{k}}_j) n_j \longrightarrow \sum_{j'} n_{j'}^{\text{eff}}$$

- Single-angle simulations capture many qualitative features in multi-angle simulations, including "*Stepwise Spectral Swapping*" of neutrino flavors.

# Dense neutrino gases

Equation of motion: $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{s}_i = \mathbf{s}_i \times \mathbf{H}_i$

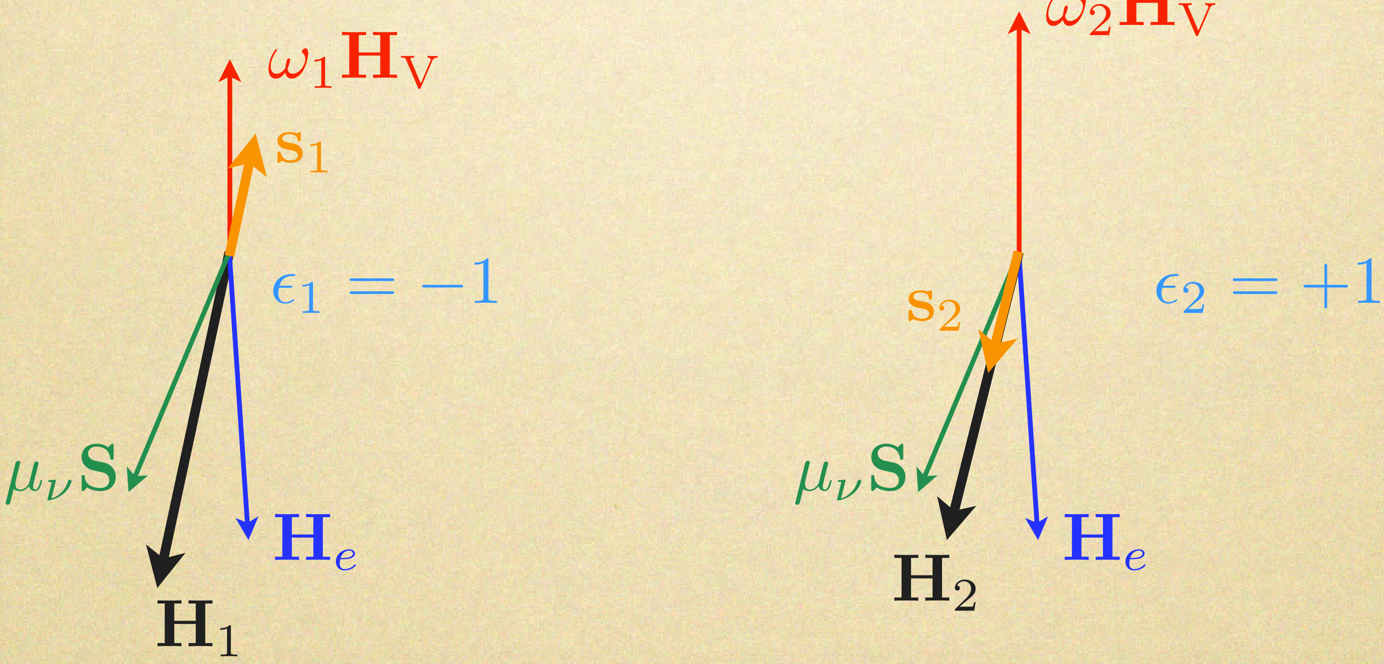
Effective field: $\mathbf{H}_i = \omega_i \mathbf{H}_\mathrm{V} + \mathbf{H}_e + \mu_\nu \mathbf{S}$

ν-ν coupling: $\mu_\nu \equiv -2\sqrt{2} G_\mathrm{F}$

Total NFIS: $\mathbf{S} \equiv \sum_j n_j \mathbf{s}_j$

# MSW-like solution

Assume perfect alignment: $\sum_i n_i \times \mathbf{s}_i = \epsilon_i \frac{\mathbf{H}_i}{2H_i}$



Quasi-static picture

# MSW-like solution

$$S_x = \frac{n_\nu^{\rm tot}}{2}(H_{e,x} + \mu_\nu S_x)\sum_i \frac{\epsilon_i \tilde{f}_i}{H_i}$$

$$\cancel{S_y = \frac{n_\nu^{\rm tot}}{2}\mu_\nu S_y \sum_i \frac{\epsilon_i \tilde{f}_i}{H_i}} \qquad S_y = 0 \text{ if } H_e \neq 0$$

$$S_z = \frac{n_\nu^{\rm tot}}{2}\sum_i \frac{\epsilon_i \tilde{f}_i}{H_i}(\omega_i + H_{e,z} + \mu_\nu S_z)$$

Vacuum mass basis:

$$\hat{\mathbf{e}}_x^{\rm v} = \hat{\mathbf{e}}_x^{\rm f}\cos 2\theta_{\rm v} + \hat{\mathbf{e}}_z^{\rm f}\sin 2\theta_{\rm v}$$

$$\hat{\mathbf{e}}_y^{\rm v} = \hat{\mathbf{e}}_y^{\rm f}$$

$$\hat{\mathbf{e}}_z^{\rm v} = \mathbf{H}_{\rm V} = -\hat{\mathbf{e}}_x^{\rm f}\sin 2\theta_{\rm v} + \hat{\mathbf{e}}_z^{\rm f}\cos 2\theta_{\rm v}$$

Total number flux: $n_\nu^{\rm tot} \equiv \sum_j n_j$

Distribution function: $\tilde{f}_i \equiv \dfrac{n_i}{n_\nu^{\rm tot}}$

# "Lepton number"

Quasi-static picture

In the absence of ordinary matter:

$$\sum_i \tilde{f}_i \mathbf{H}_\mathrm{V} \cdot \; \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{s}_i = \mathbf{s}_i \times \left(\omega_i \mathbf{H}_\mathrm{V} + \mu_\nu n_\nu^\mathrm{tot} \sum_j \tilde{f}_j \mathbf{s}_j\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{L} = 0$$

"Lepton number" in vacuum mass basis:

$$\mathcal{L} = 2\sum_i \tilde{f}_i \mathbf{s}_i \cdot \mathbf{H}_\mathrm{V} = \frac{2S_z}{n_\nu^\mathrm{tot}}$$

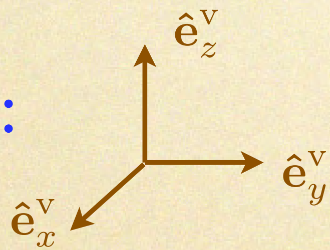
$$= \frac{(n_{\nu_1} + n_{\bar{\nu}_3}) - (n_{\bar{\nu}_1} + n_{\nu_3})}{n_\nu^\mathrm{tot}}$$

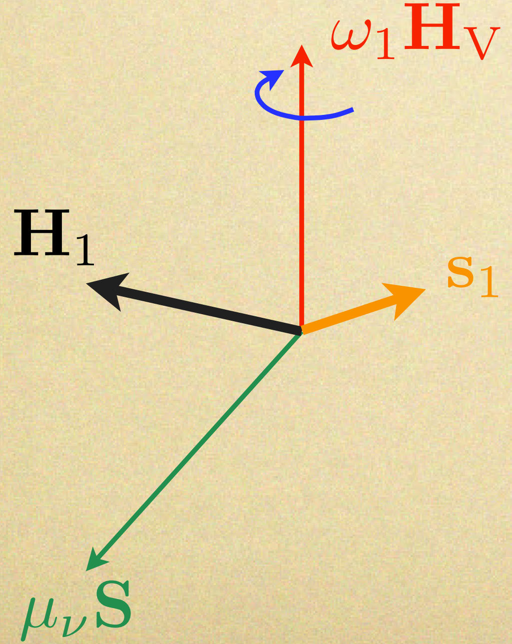
Quasi-static picture

# Precession solution

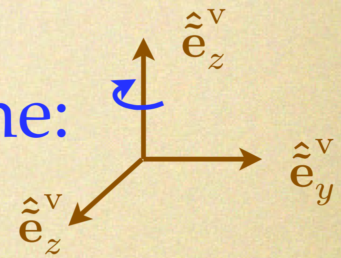
Assume steady precession around $\mathbf{H}_\mathrm{V}$ with $\omega_\mathrm{pr}$:

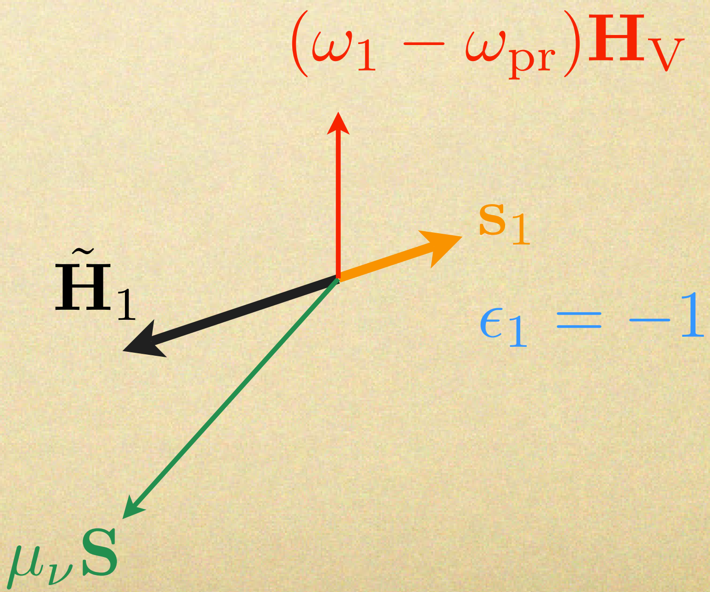
static frame:




corotating frame:



Quasi-static picture

# Precession solution

$$1 = \frac{\mu_\nu n_\nu^{\rm tot}}{2} \sum_i \frac{\epsilon_i \tilde{f}_i}{\sqrt{(\omega_i - \omega_{\rm pr} + \mu_\nu S_z)^2 + (\mu_\nu S_x)^2}}$$

$$\omega_{\rm pr} = \frac{\mu_\nu n_\nu^{\rm tot}}{2} \sum_i \frac{\epsilon_i \omega_i \tilde{f}_i}{\sqrt{(\omega_i - \omega_{\rm pr} + \mu_\nu S_z)^2 + (\mu_\nu S_x)^2}}$$

$$\mathcal{L} = \frac{S_z}{n_\nu^{\rm tot}}$$

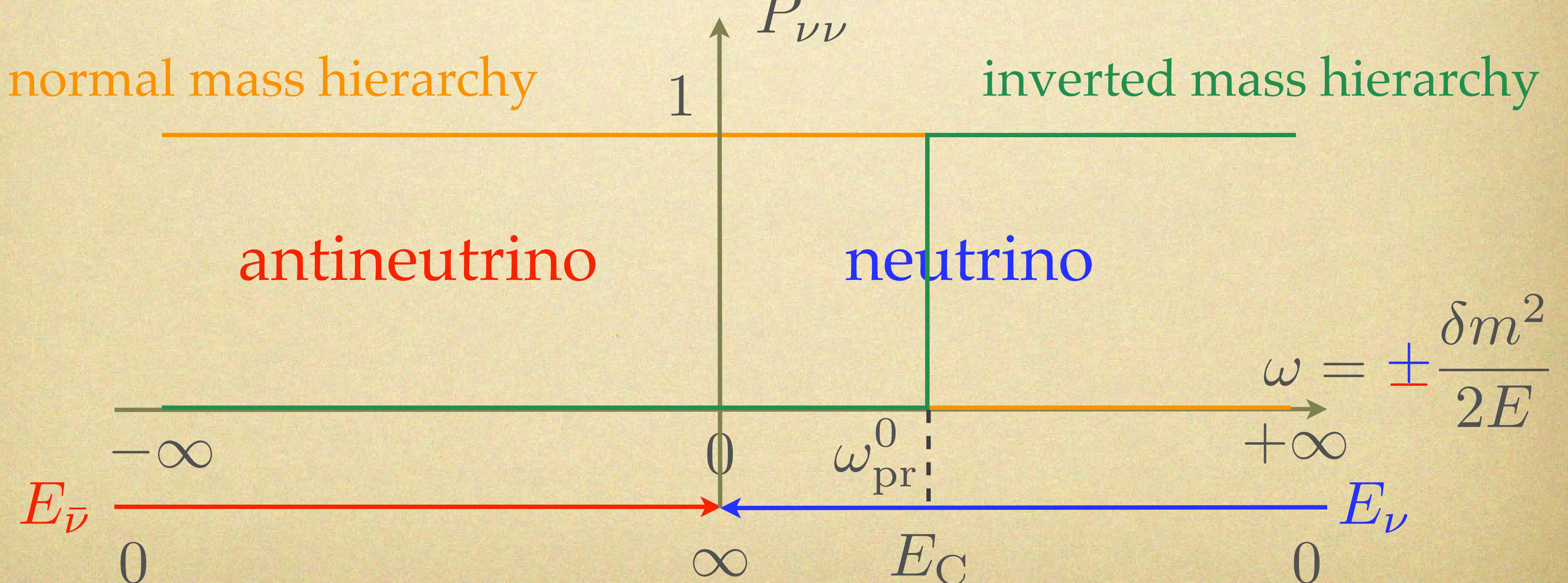
Raffelt & Smirnov, arXiv:0705.1830 [hep-ph]

Duan, Fuller & Qian, arXiv:0706.4293 [astro-ph]

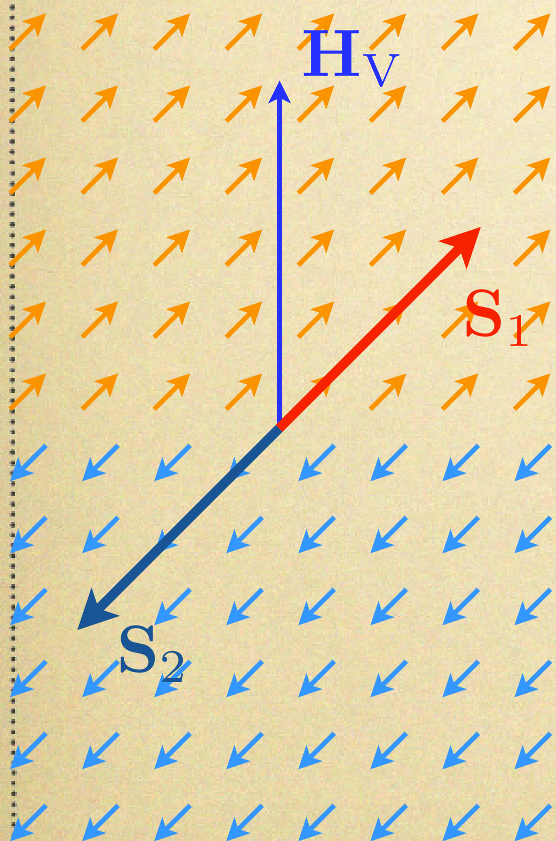
# Stepwise swapping

In the limit $n_\nu^{\text{tot}} \to 0$ : $\qquad \tilde{\mathbf{H}}_\omega = (\omega - \omega_{\text{pr}}^0)\mathbf{H}_{\text{V}}$

$$\mathbf{s}_\omega = \frac{\epsilon_\omega \mathbf{H}_{\text{V}}}{2}\,\text{sgn}(\omega - \omega_{\text{pr}}^0)$$



# Bipolar System



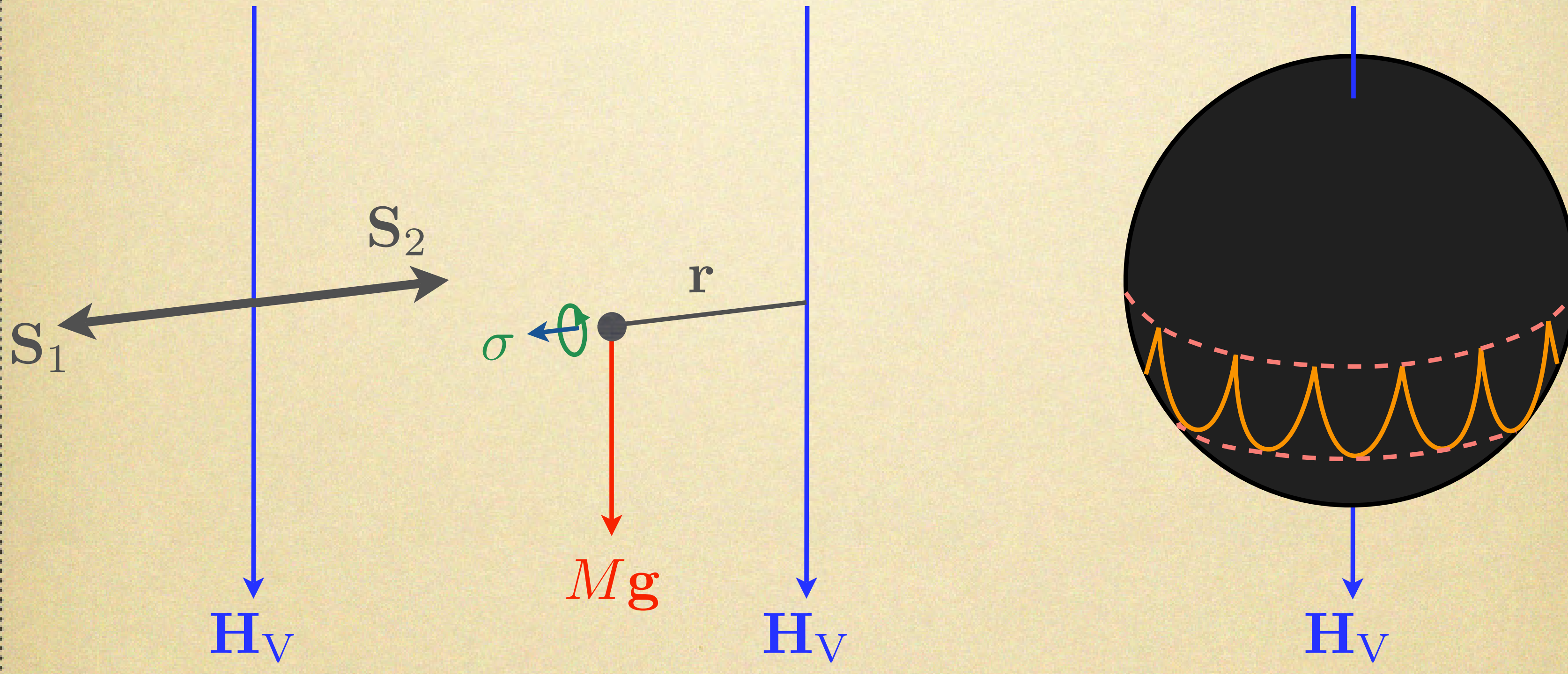

$$\mathbf{S}_1 \equiv n_{\nu,1}\mathbf{s}_1 \qquad \mathbf{S}_2 \equiv n_{\nu,2}\mathbf{s}_2$$

Neutrinos form a bipolar system when they leave the neutrino sphere:

$$\nu_e/\bar{\nu}_\tau : +\hat{\mathbf{e}}_z^{\rm f} \qquad \bar{\nu}_e/\nu_\tau : -\hat{\mathbf{e}}_z^{\rm f}$$

# Flavor Pendulum

Mono-energetic $\nu_e$-$\bar{\nu}_e$ gas, $n_{\nu,1} = n_\nu$ and $n_{\nu,2} = \alpha n_\nu$.

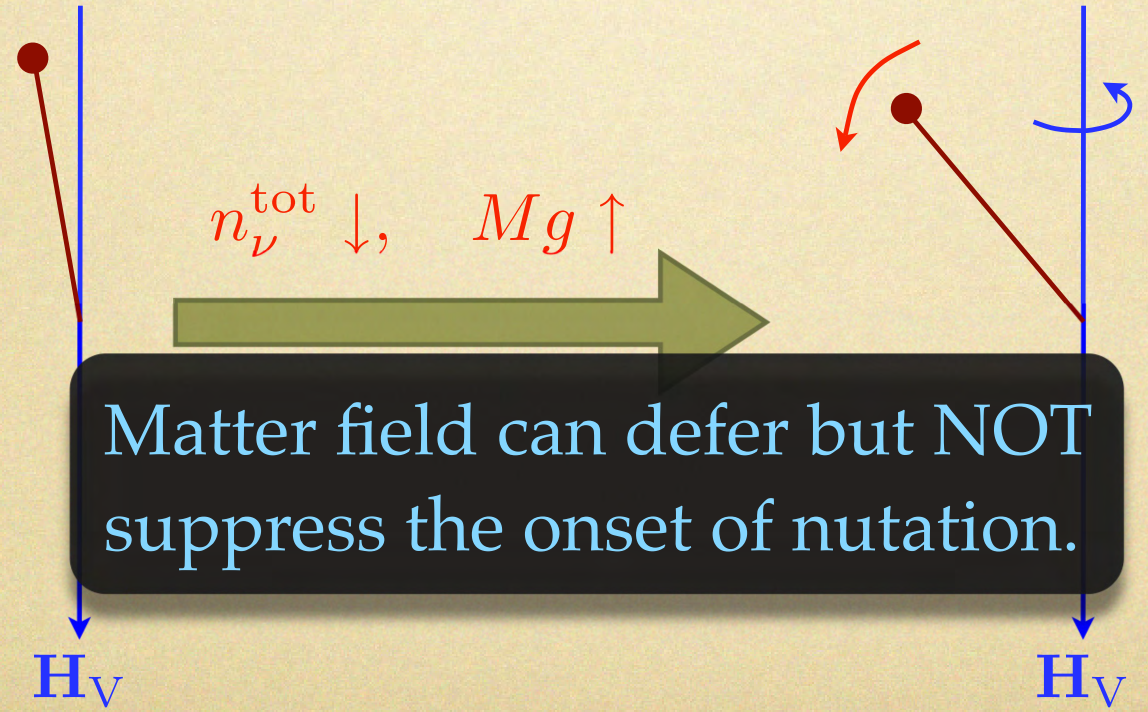



$$\sigma \sim \frac{n_{\nu_e} - n_{\bar{\nu}_e}}{n_{\nu_e} + n_{\bar{\nu}_e}} \qquad M\mathbf{g} \sim \frac{\mathbf{H}_\mathrm{V}}{n_{\nu_e} + n_{\bar{\nu}_e}}$$

Stepwise spectral swapping

Stepwise spectral swapping

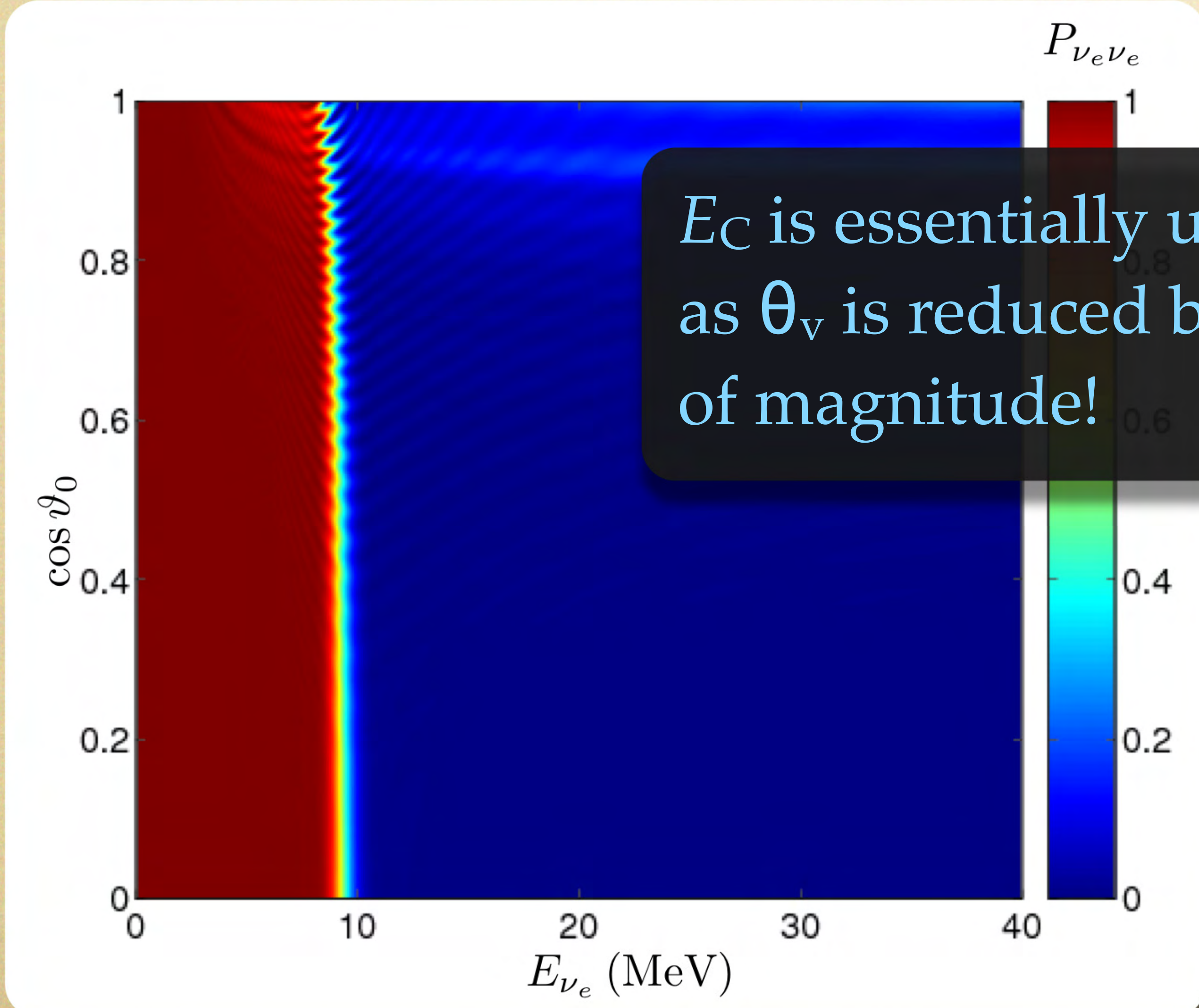
# Inverted mass hierarchy ($\theta_V \approx \pi/2$)




Matter field can defer but NOT suppress the onset of nutation.

Stepwise spectral swapping

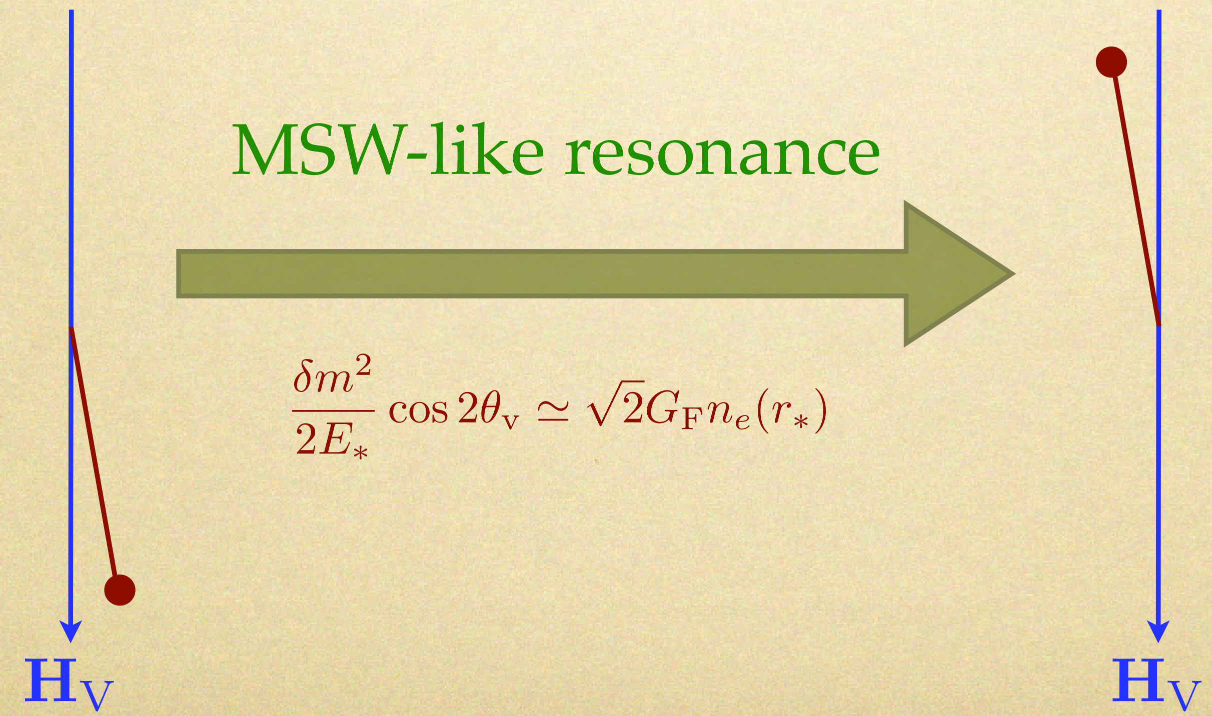
$$\delta m^2 = 3 \times 10^{-3}\,\mathrm{eV}^2 \simeq \delta m^2_{\mathrm{atm}},\ \theta_{\mathrm{v}} = \pi/2 - 10^{-9},\ L_\nu = 10^{51}\,\mathrm{erg/s}$$




$E_C$ is essentially unchanged as $\theta_{\mathrm{v}}$ is reduced by 8 orders of magnitude!

Stepwise spectral swapping

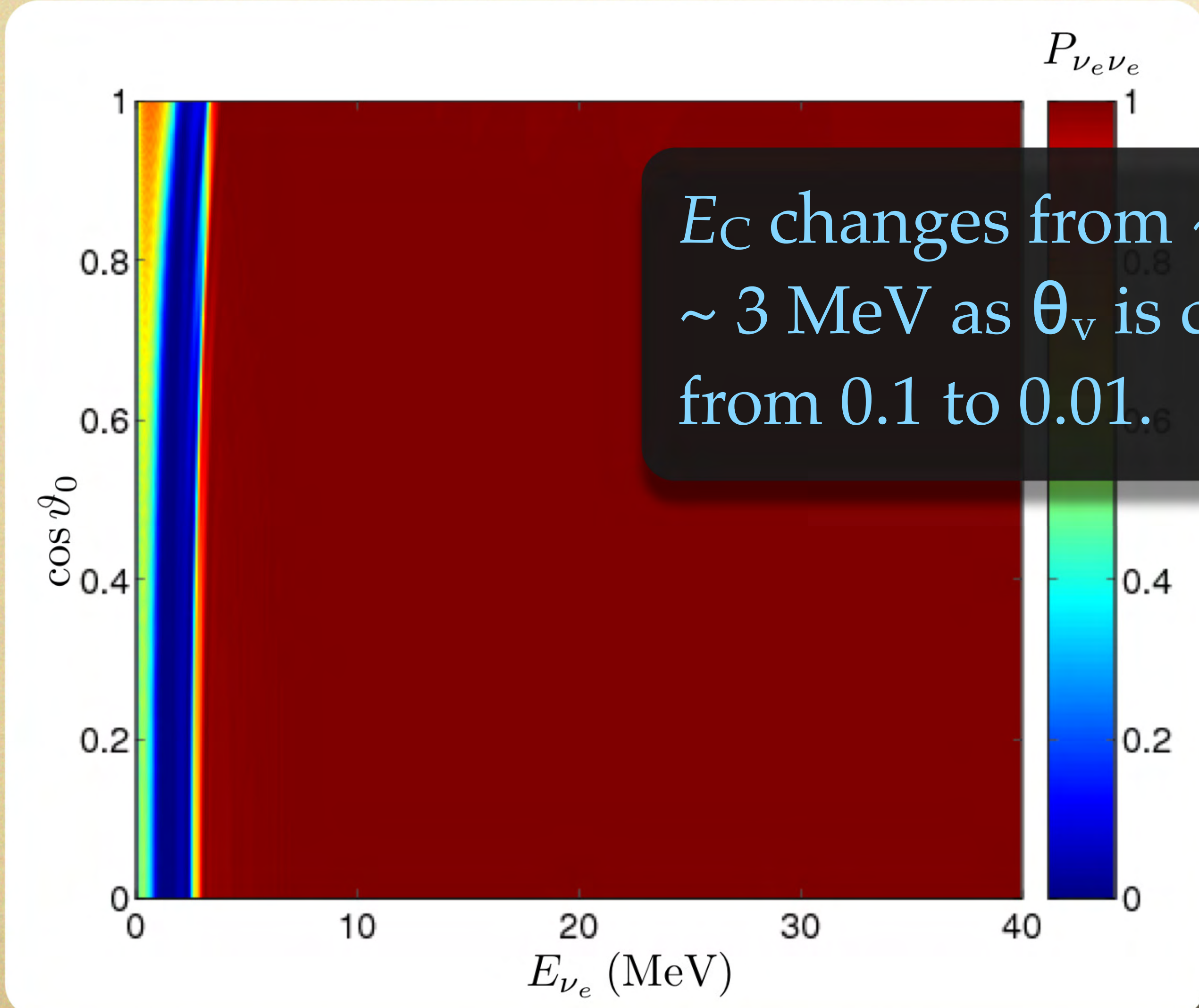
# Normal mass hierarchy ($\theta_V \approx 0$)




$$\frac{\delta m^2}{2E_*}\cos 2\theta_\mathrm{v} \simeq \sqrt{2} G_\mathrm{F} n_e(r_*)$$

Stepwise spectral swapping

$$\delta m^2 = 3\times 10^{-3}\,\mathrm{eV}^2 \simeq \delta m^2_{\mathrm{atm}},\ \theta_{\mathrm{v}} = 0.01,\ L_\nu = 10^{51}\,\mathrm{erg/s}$$




$E_C$ changes from ~10 MeV to ~ 3 MeV as $\theta_{\mathrm{v}}$ is decreased from 0.1 to 0.01.

# Summary

- Neutrino-neutrino forward scattering can be important for neutrino oscillations in SNe, which may invalidate the conventional MSW mechanism.

- Two quasi-static solutions have been found for flavor evolution in isotropic neutrino gases.

- Stepwise spectral swapping of supernova neutrino flavors can be used to probe neutrino mass hierarchy (see arXiv:0707.0290 [astro-ph]).

# References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- Duan, Fuller & Qian, arXiv:0706.4293 [astro-ph]
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