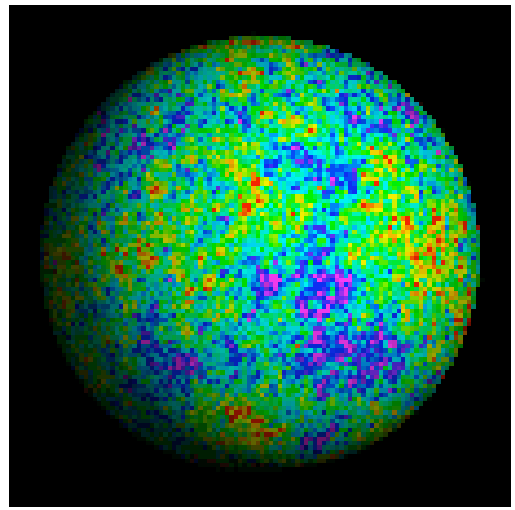
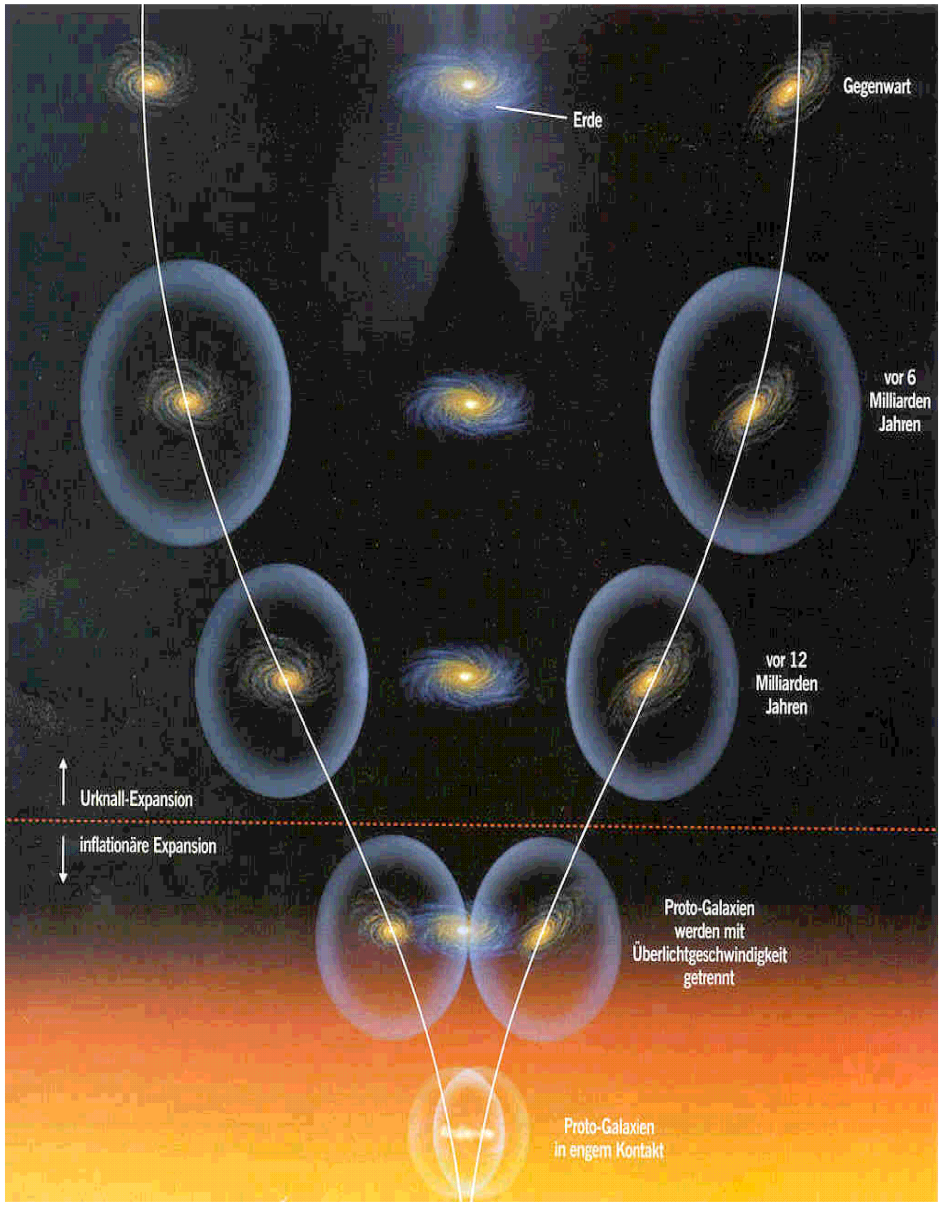
Characterizing Inflationary Perturbations

Salman Habib (T-8), Andreas Heinen (Dortmund), Katrin Heitmann (ISR-1), Gerard Jungman (T-6), Carmen Molina-Paris (Leeds)

PRL 89, 281301 (2002), PRD 70, 083507 (2004), PRL 94, 061303 (2005), PRD 71, 043518 (2005)

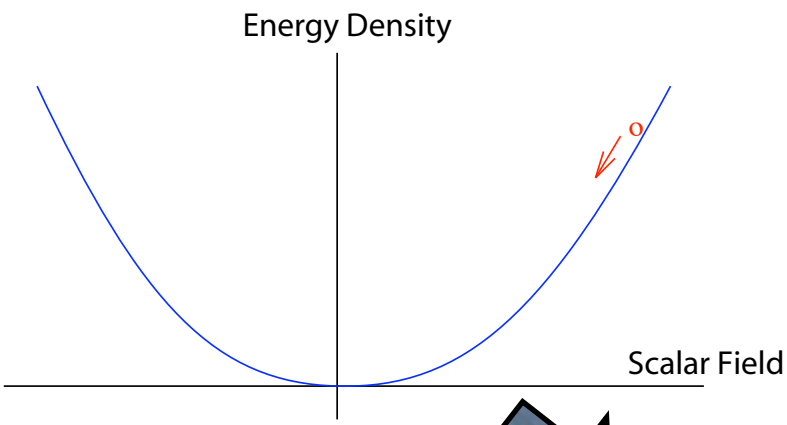


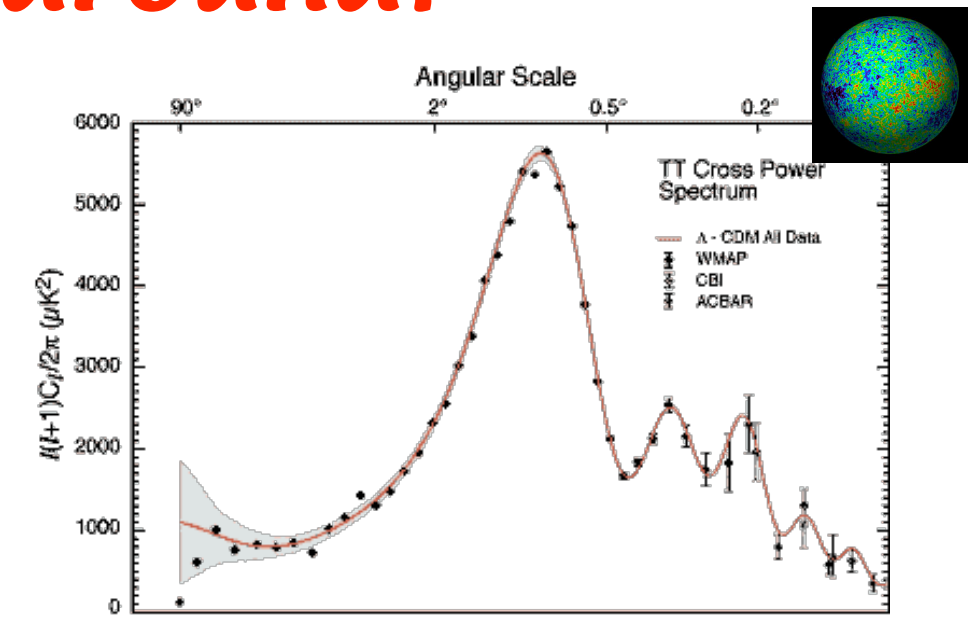
The Inflationary Universe



- *Accelerated expansion* ***phase in the very early Universe***
- ***Explains*** *tiny ripples* ***in the CMBR across the sky and the*** *large scale structure* ***we observe today***
- ***Solves*** *homogeneity* ***problem***
- ***Solves*** *monopole* ***problem***

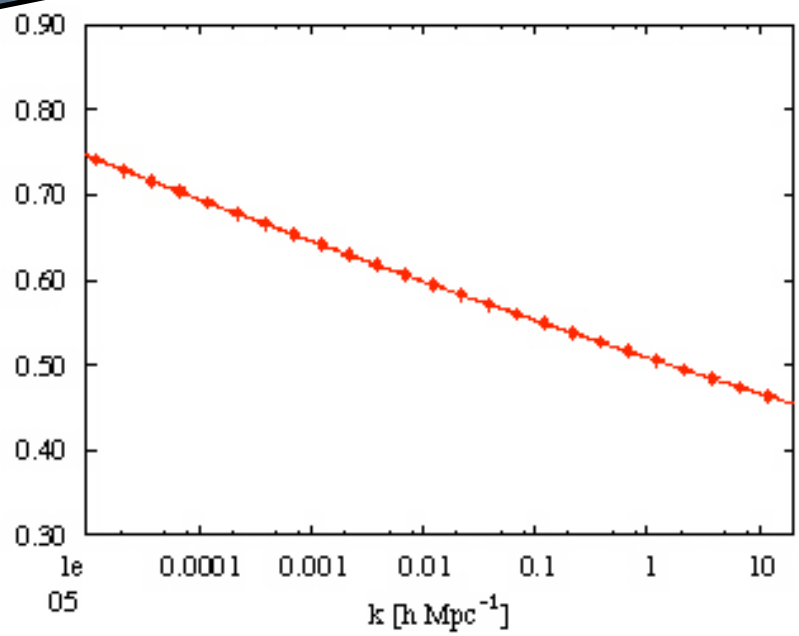
From Theory to Observation – or the Other Way around?





WMAP

* pick your favorite inflation model, V(Φ)



* Calculate the back-ground evolution and the primordial power spectrum (this talk!)

* read primordial P(k) into a Boltzmann solver

* process P(k) to compare with measurements today

* OR: the other way around (next talk!)

The Primordial Power Spectrum

Background Equations:

$$\ddot{\phi} + 3H\dot{\phi} + 3V'(\phi) = 0$$

$$\dot{H} = -\dot{\phi}^2/2$$

Fluctuation Equations:

$$u_k'' + (k^2 - z''/z)u_k = 0$$

$$z = a\dot{\phi}/H$$

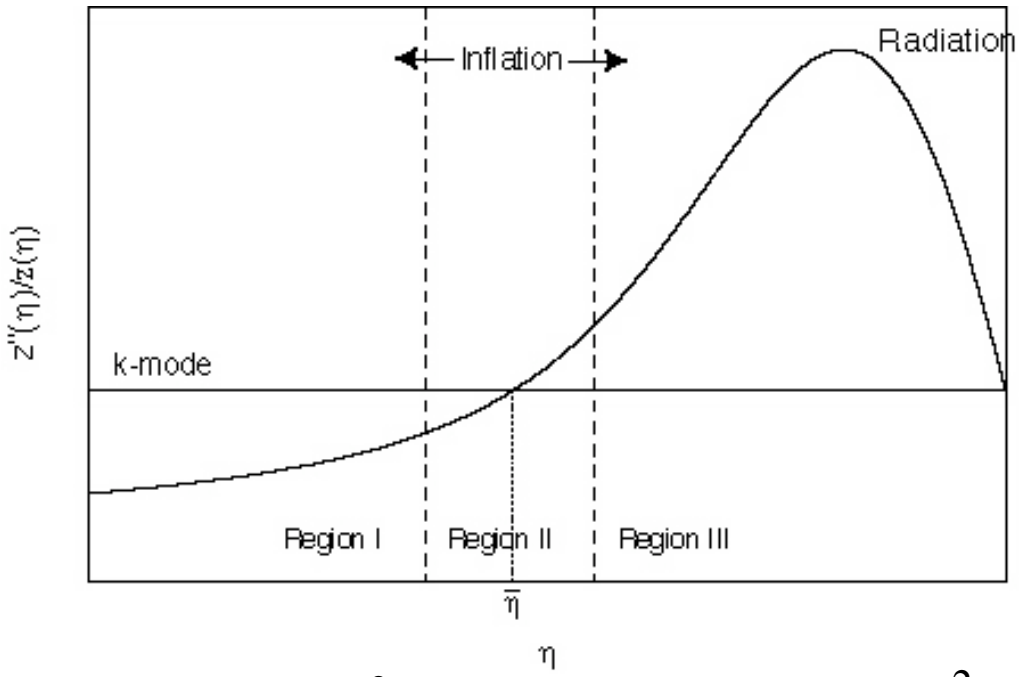
Power Spectrum

$$P(k) = \frac{k^3}{2\pi^2}\left|\frac{u_k}{z}\right|^2$$

$$n_S(k) = \frac{d\ln P(k)}{d\ln k}$$

So far...

$$u_k'' + (k^2 - z''/z)u_k = 0$$



Simplest Approximation:

* **limit:** $k^2 >> z''/z \Rightarrow u_k = e^{-ik\eta}/\sqrt{2k}$
* **limit:** $k^2 << z''/z \Rightarrow u_k = A_k z$

* **match them at k=aH to fix A_k**

$$\Rightarrow A_k z = e^{-ik\eta}/\sqrt{2k} \Rightarrow |A_k| = 1/\sqrt{2k}z \Rightarrow P(k) = \frac{k^3}{2\pi^2}\left|\frac{u_k}{z}\right|^2 = \frac{k^3}{2\pi^2}|A_k|^2 = \left(\frac{H^4}{4\pi^2\dot{\eta}^2}\right)_{k=aH}$$

Improved Approximation:

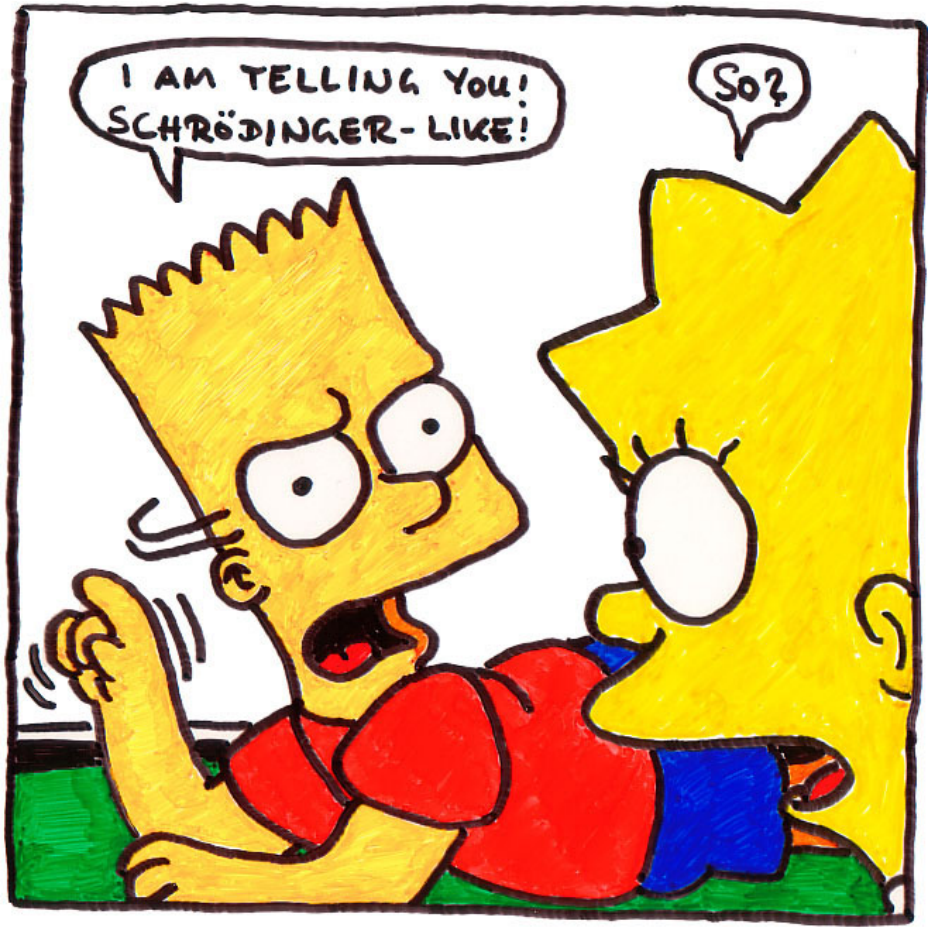
* **replace:** $z''/z \simeq C/\eta^2$
* **leads to Hankel solution**

:-) improvement of estimate

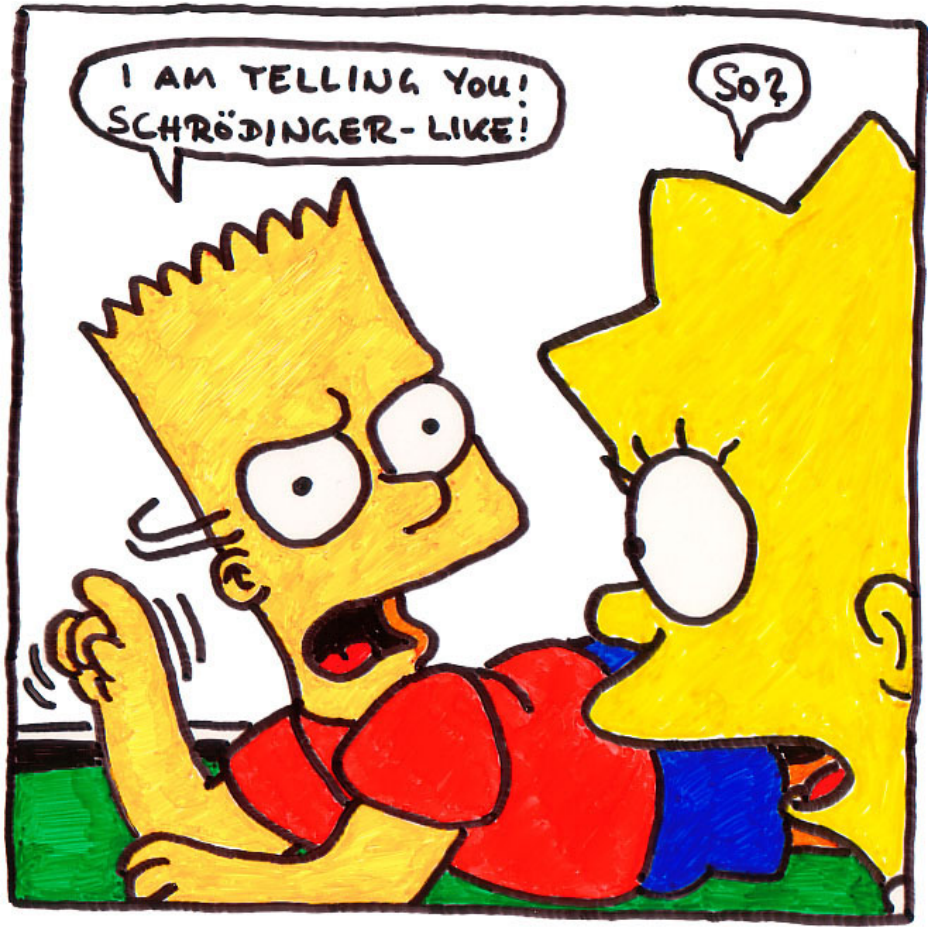
:-(no error control

:-(no way to improve it

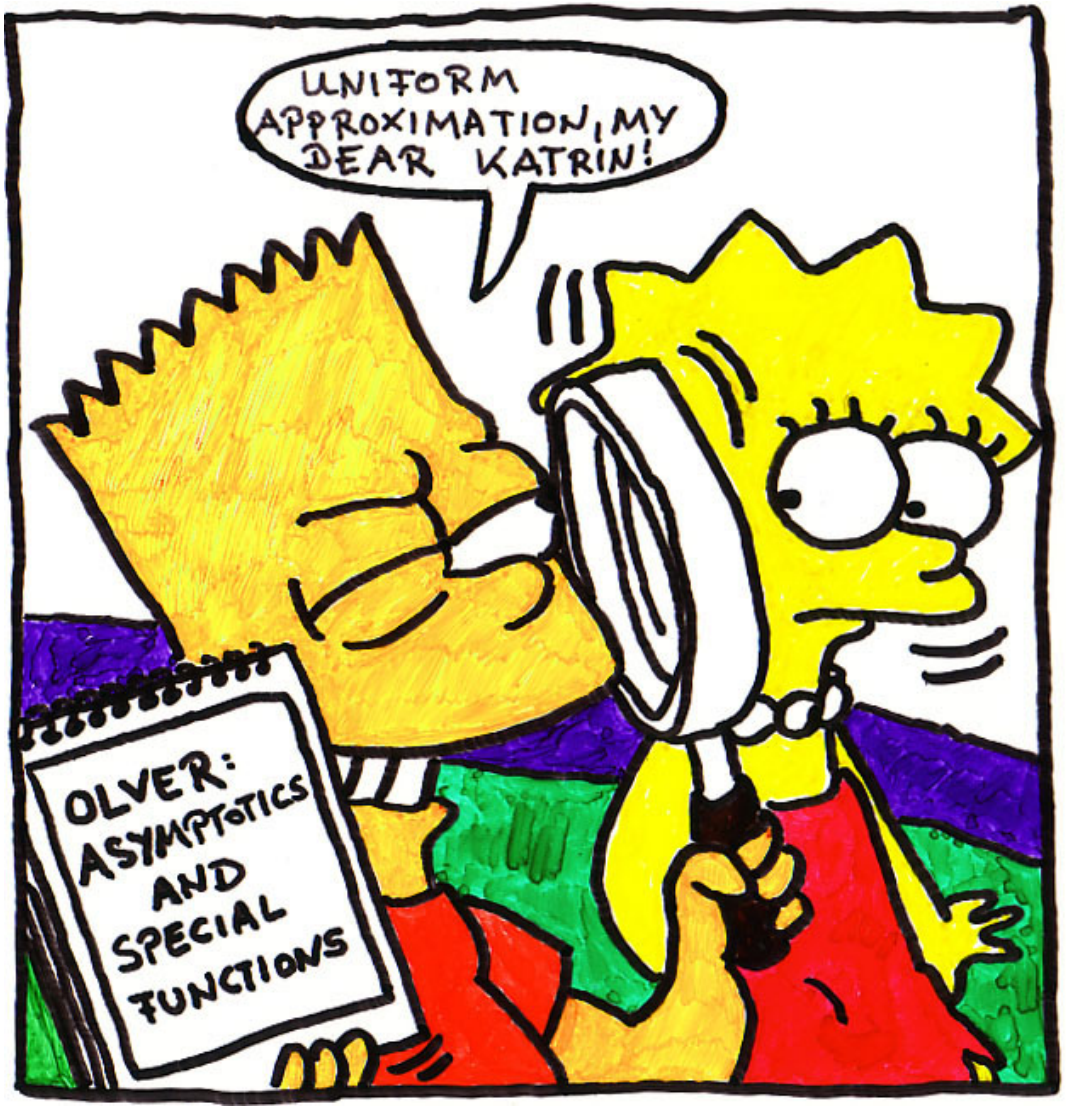
$$u_k'' + (k^2 - z''/z)u_k = 0$$

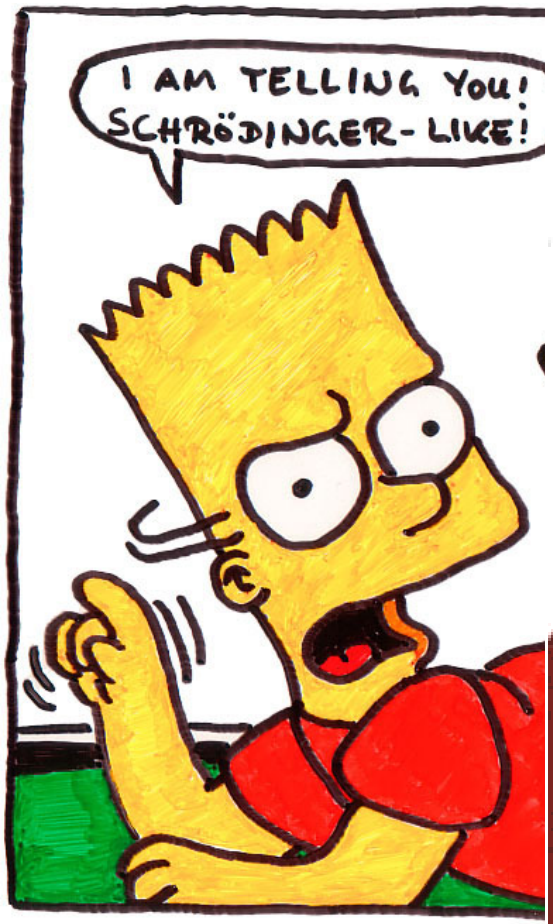


$$u_k'' + (k^2 - z''/z)u_k = 0$$


I AM TELLING YOU! SCHRÖDINGER-LIKE!
So?

$$u_k'' + (k^2 - z''/z)u_k = 0$$


UNIFORM APPROXIMATION, MY DEAR KATRIN!
OLVER: ASYMPTOTICS AND SPECIAL FUNCTIONS



AKP CLASSICS

Asymptotics and Special Functions

Frank W. J. Olver

$$''/z)u_k = 0$$

The Uniform Approximation

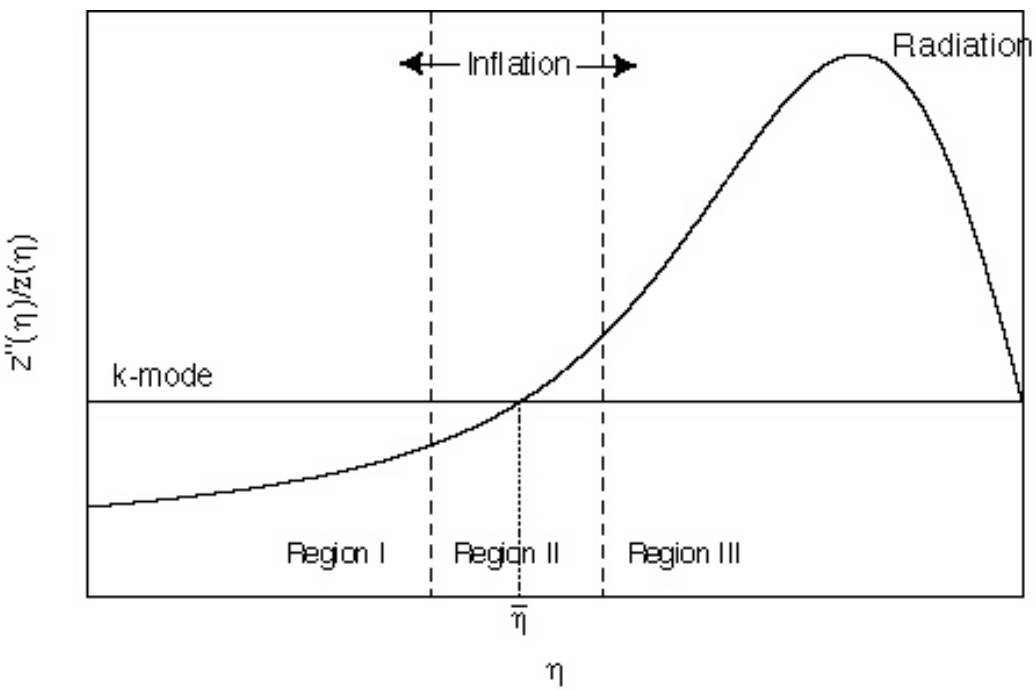
:-) systematically improvable to any order

:-) well-defined error bounds

:-) allows time dependence in z”/z

:-) next-to/improved leading order: errors < 0.1%

:-) provides initial conditions for exact numerical solution

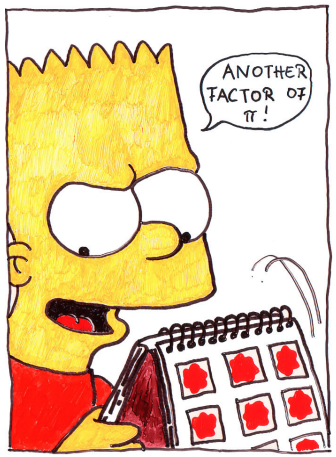


* $z''/z \simeq [\square^2(\square) - 1/4]/\square^2$

* leads to Airy-functions

* normalize at $k \rightarrow$

* obtain P(k)

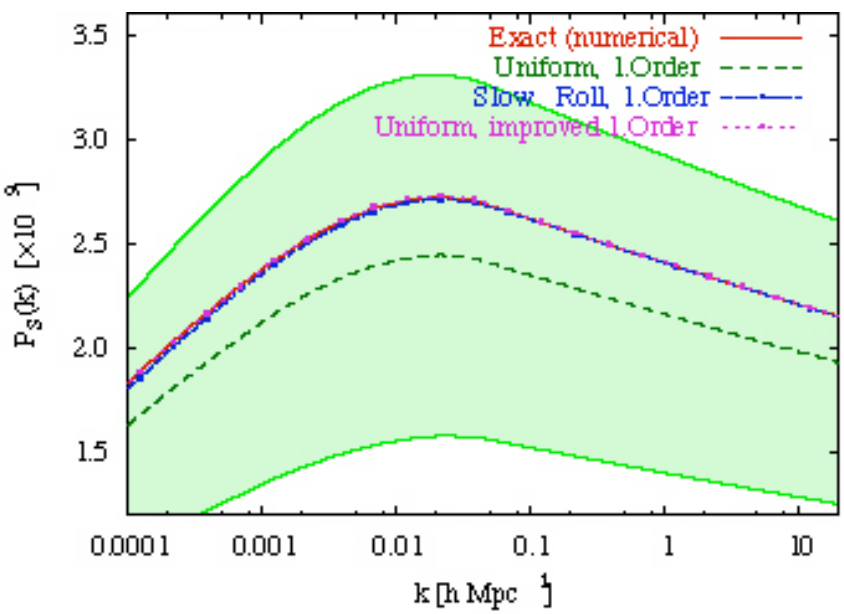


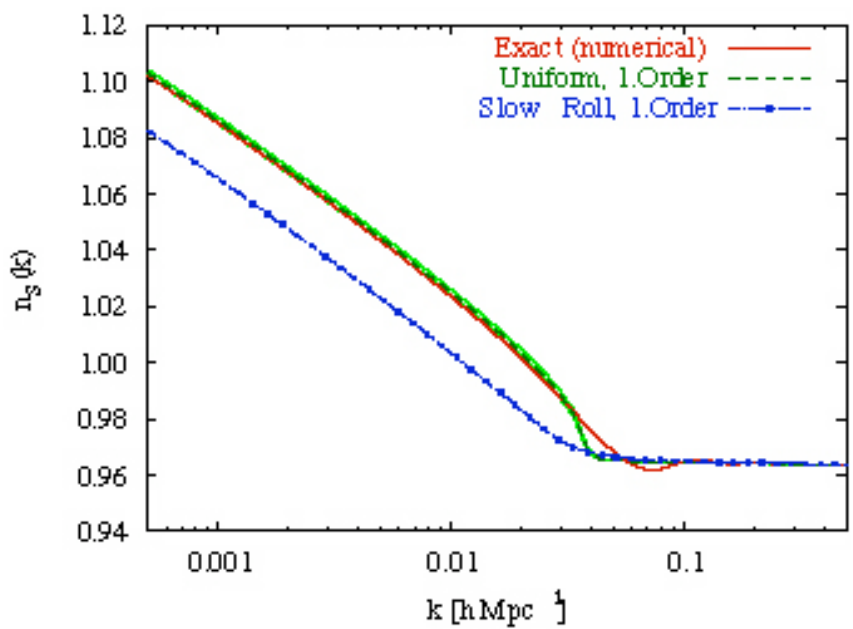
An Example

$$V_{>}(\phi) = -101m^2\phi_*^2/4 - 202m^2\phi\phi_* - 50m^2\phi^2 - 101m^2\phi^4/12\phi_*^2$$

$$V_{<}(\phi) = m^2\phi^2/2$$

- *potential is glued together at special point*
- *third derivative does not exist at this point*
- *unsteadiness in z”/z, first part of potential steeper*





Conclusions and Outlook

- *Inflation is a great idea* to solve many problems, e.g., monopole problem, horizon problem
- Ongoing and future CMB surveys hold the promise to *constrain the inflationary model space*
- *Efficient ways* to calculate power spectra and spectral indices are mandatory
- Here: *first full numerical implementation* as well as development of very *powerful uniform approximation* (error controlled and improvable to any desired order)
- Our results are *accurate to ~0.1%* !
- Future: *coupling* to CMBFAST and large scale structure code