Reconstructing Inflationary Expansion

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Inverse Scattering Theory and the Density Perturbations from Inflation

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Density Perturbations from Inflation

Metric/Matter Perturbations: $$\boxed{\phi_k'' + \left(k^2 c_s(\eta)^2 - \frac{z(\eta)''}{z(\eta)}\right)\phi_k = 0}$$

$\eta =$ conformal time

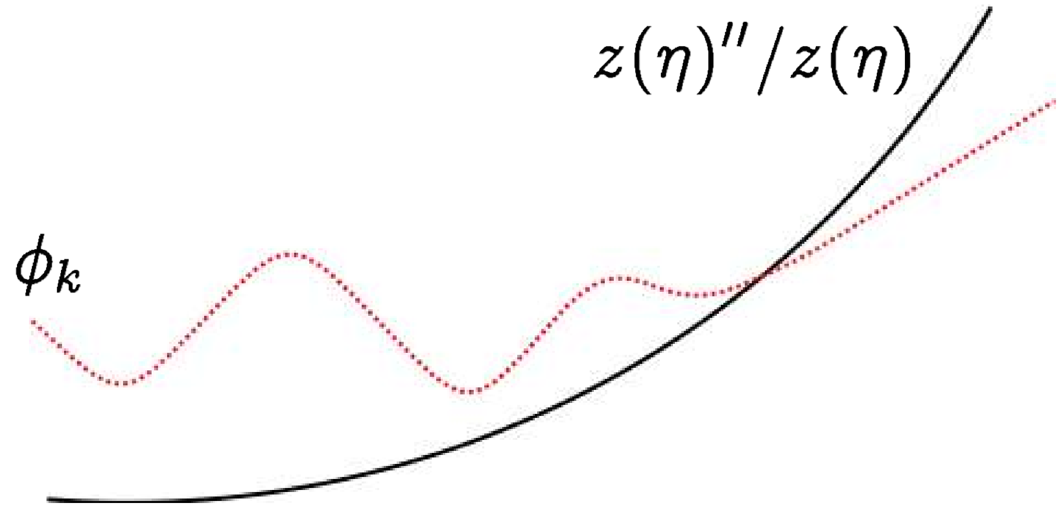
$$d\eta = \frac{dt}{a(t)}$$

$$z(\eta) = \frac{a(\eta)}{h(\eta)}\left[h(\eta)^2 - h(\eta)'\right]^{1/2}$$

$$h(\eta) = \frac{a(\eta)'}{a(\eta)}$$

Inflationary Expansion $\longrightarrow a(\eta) \longrightarrow z(\eta)$

Growth of Perturbations During Inflation



$$\phi_k \sim A_k z(\eta)$$

$$\phi_k \sim (2k)^{-1/2}\, e^{-ik\eta + i\gamma}$$

$$-\infty \leftarrow \eta$$

$$\boxed{P(k) = \frac{k^3}{2\pi^2} |A_k|^2}$$

Questions

Q: What does $P(k)_{\texttt{observed}}$ tell us about inflation?

A: Inverse scattering theory reconstructs $\dfrac{z(\eta)''}{z(\eta)}$ uniquely.

Q: What do we learn in a model-independent way?

A: $\dfrac{z(\eta)''}{z(\eta)}$; no more or less.

Scattering Theory

$$\phi'' + \left(k^2 - \frac{\ell(\ell+1)}{\eta^2} - V(\eta)\right)\phi = 0$$

input: positive-energy ("right-moving") wave
output: ϕ_k for $\eta \to 0^-$

Jost Function: $F_\ell(k) = \lim_{\eta \to 0^-} \left[\frac{e^{-i\pi\ell}\Gamma(1/2)}{\Gamma(\ell+1/2)} \left(\frac{-k\eta}{2}\right)^\ell \phi_{\ell,k}(\eta)\right]$

$$\boxed{P(k) = \frac{\Gamma(\ell+1/2)^2}{4\pi^3} k^2 \left(\frac{-k\eta_0}{2}\right)^{-2\ell} \left|\frac{F_\ell(k)}{F_\ell(0)}\right|^2}$$

Cosmological Inversion

$V(\eta)$ encodes the deviation from scale-invariance of expansion

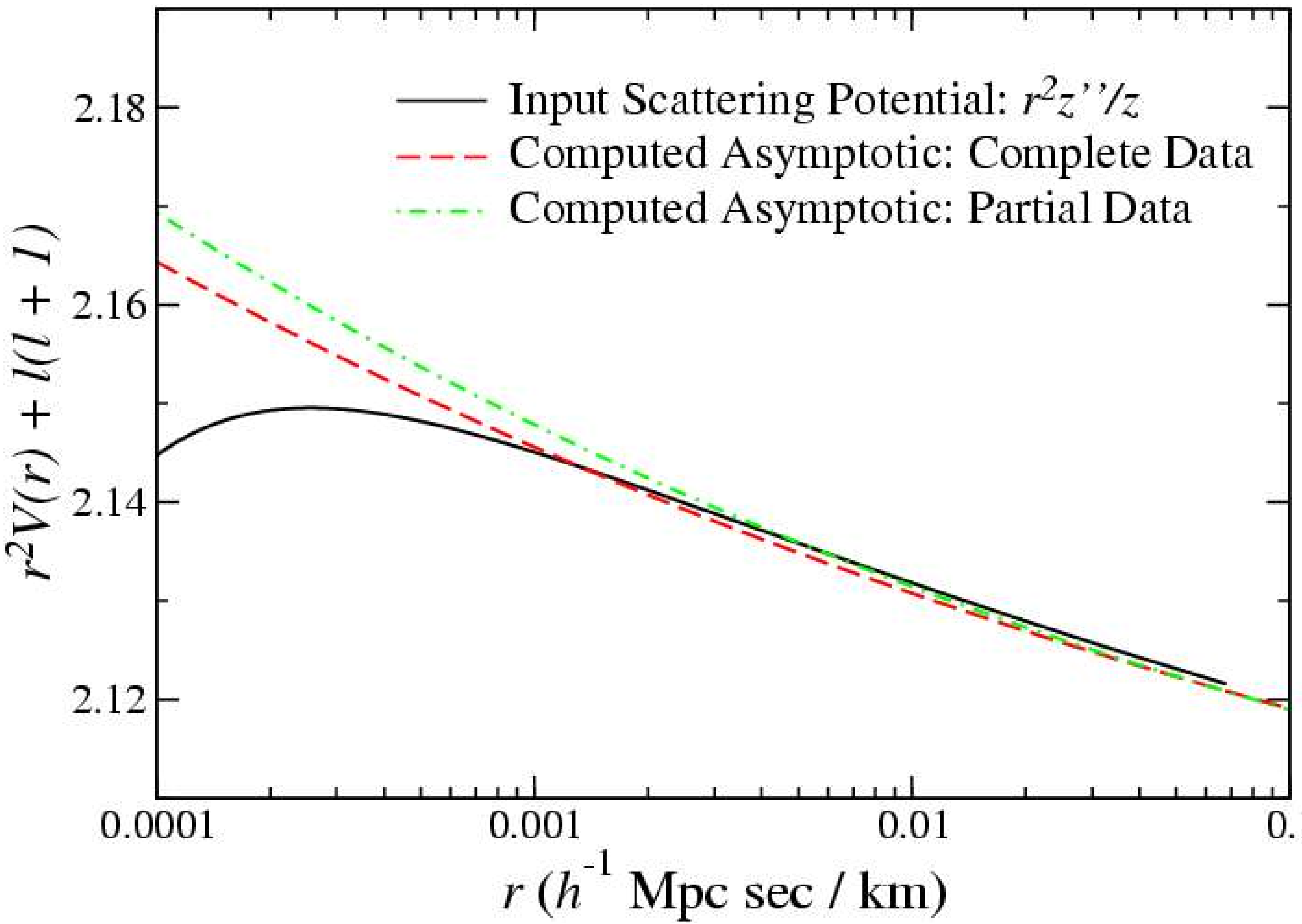
$$P(k)_{\mathrm{observed}} \longrightarrow \ell, \frac{F_\ell(k)}{F_\ell(0)} \longrightarrow \ell, V(\eta) \longrightarrow a(t)$$

Example:

$$H(t) = pt^{-1}\left[1 + \left(\frac{t}{t_0}\right)^b + \cdots\right] \qquad t \to 0$$

$$V(\eta) = \frac{1}{\eta^2}\left(\frac{\eta_1}{\eta}\right)^{\frac{b}{p-1}}(1 + \cdots) \qquad \eta \to -\infty$$

Inversion Example



Summary

- Primordial peturbations satisfy a *wave equation* (acoustic waves).
- $P(k)$ is actually *scattering data*, related to the Jost function.
- *Inverse scattering* allows to reconstruct the potential (geometry).

In progress:

- Full machinery (Gelfand-Levitan) for analysis of $P(k)$.
- Characterization of inflation.