NOTES ON THERMAL HISTORY OF THE UNIVERSE

- THE UNIVERSE AS A COMPLEX, NONEQUILIBRIUM, DYNAMICAL SYSTEM
- IS THE EXPANSION OF THE UNIVERSE REAL?
- BASIC THERMODYNAMICS (FLAT SPACE)
- THERMODYNAMICS IN AN EXPANDING UNIVERSE
- ENTROPY CONSERVATION
- DECOUPLING
- OVERVIEW OF THERMAL HISTORY

ANALYSIS OF 'COSMIC SOUP' CARRIED OUT USING GENERAL RELATIVITY AND 'COSMOLOGICAL CHEMISTRY': KINETICS OF REACTIONS OF ELEMENTARY PARTICLES AT VERY HIGH ENERGIES.

COSMOLOGICAL PROBES OF PARTICLE PHYSICS

TIME & ENERGY SCALES

EVENT	TIME	z	kT	
PLANCK TIME GRAVITON DECOUPLING	10^{-43} s	10^{37}	10^{19} GeV (10^{31} K)	EQLBM
GUT / INFLATION / BARYOGENESIS	10^{-35} s	10^{32}	10^{14} GeV (10^{26} K)	
EW UNIFICATION	10^{-12} s	10^{21}	10^{3} GeV (10^{15} K)	UNCLEAR
QUARK-HADRON TRANSITION	10^{-6} s	10^{12}	1 GeV (10^{12} K)	
ν DECOUPLING	1 s	10^{15}	1 MeV (10^{9} K)	
$e^{\pm}$ ANNIHILATION	1 s	10^{15}	1 MeV (10^{9} K)	
NUCLEOSYNTHESIS	1–100 s	10^{14}	1 – 0.1 MeV (10^{9-8} K)	
MATTER-RADIATION EQUALITY	$1400\,(\Omega_0 h^2)^{-2}$ yrs	$2.3\times10^{4}\,\Omega_0 h^2$	5.5 eV (6.6×10^{3} K)	
RECOMBINATION	$1.4\times10^{-5}(\Omega_0 h^2)^{-1/2}$ yrs		0.4 eV	CMB
PHOTON DECOUPLING	$1.8\times10^{5}(\Omega_0 h^2)^{-1/2}$ yrs	1100	0.3 eV	

CONCEPT OF THERMAL EQLBM. ONLY APPROXIMATE.

- CONFORMAL INVARIANCE
- REACTION RATES (INTERACTION STRENGTH)
- $T \sim \frac{1}{a}$

EQUILIBRIUM DISTRIBUTIONS

$$f(\vec{p}) = \frac{1}{e^{(E-\mu)/T} \pm 1} \qquad \begin{matrix} + & \text{FERMIONS} \\ - & \text{BOSONS} \end{matrix}$$

$$E^2 = |\vec{p}|^2 + m^2$$

$$g = 2s+1$$

$$n = \frac{g}{2\pi^2} \int_m^\infty \frac{\sqrt{E^2-m^2}\, E\, dE}{e^{(E-\mu)/T} \pm 1}$$

$$\rho = \frac{g}{2\pi^2} \int_m^\infty \frac{\sqrt{E^2-m^2}\, E^2\, dE}{e^{(E-\mu)/T} \pm 1}$$

$$p = \frac{g}{6\pi^2} \int_m^\infty \frac{(E^2-m^2)^{3/2}\, dE}{e^{(E-\mu)/T} \pm 1}$$

$$dp = \frac{E\, dE}{\sqrt{E^2-m^2}}$$

$$d^3p \to 4\pi p^2\, dp$$

USEFUL INTEGRALS

$$y(n) = \frac{1}{\Gamma(n)} \int_0^\infty \frac{x^{n-1}\, dx}{e^x - 1} \quad \leftarrow n \text{ odd}$$

n even

$$\int_0^\infty \frac{x^{n-1}}{e^x - 1} = \Gamma(n)\left(1 + \frac{1}{2^n} + \frac{1}{3^n} + \cdots\right)$$

$$B_n = \frac{(2n)!}{2^{2n-1}\pi^{2n}}\left(1 + \frac{1}{2^{2n}} + \frac{1}{3^{2n}} + \cdots\right)$$

SPECIAL CASES OF INTEREST

(1) $T \gg m$, $T \gg \mu$

$$\rho_B \longrightarrow \frac{\pi^2}{30} g T^4, \quad \langle E \rangle \equiv \frac{\rho}{n} = \frac{\pi^4 T}{30\, \zeta(3)} \simeq 2.701\, T$$

$$\rho_F \longrightarrow \frac{7}{8}\left(\frac{\pi^2}{30}\right) g T^4; \quad \langle E \rangle = \frac{7\pi^4 T}{180\, \zeta(3)} \simeq 3.151\, T$$

$$n_B \longrightarrow \frac{\zeta(3)}{\pi^2} g T^3$$

$$n_F \longrightarrow \frac{3}{4}\frac{\zeta(3)}{\pi^2} g T^3$$

$$p = \rho/3$$

AVERAGE ENERGY PER PARTICLE

(2) $m \gg T$

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T}$$

$$\rho = mn \qquad \langle E \rangle = m + \frac{3}{2}T$$

$$p = nT \ll \rho$$

(3) WITH $\mu_+ = -\mu_-$, NET FERMION # DENSITY

$$n_+ - n_- = \frac{g}{2\pi^2} \int_m^\infty \frac{E\sqrt{E^2 - m^2}\, dE}{\left(e^{(E-\mu)/T} + 1\right)\left(e^{(E+\mu)/T} + 1\right)}$$

$$\longrightarrow \frac{gT^3}{6\pi^2}\left[\pi^2 \frac{\mu}{T} + \left(\frac{\mu}{T}\right)^3\right] \quad \underline{T \gg m}$$

$$\longrightarrow 2g\left(\frac{mT}{2\pi}\right)^{3/2} \sinh\left(\frac{\mu}{T}\right) e^{-m/T} \quad \underline{T \ll m}$$

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THE CHEMICAL POTENTIAL

- $dE = TdS - PdV + \mu dN$

$$\mu = \left.\frac{\partial E}{\partial N}\right|_{S,V} = \left.\frac{\partial G}{\partial N}\right|_{P,T} = \left.\frac{\partial F}{\partial N}\right|_{T,V}$$

FOR A REACTION BETWEEN i SPECIES OF THE TYPE

$$\sum_i \nu_i A_i = 0$$

TAKING PLACE AT CONST. P & T, G NEEDS TO BE MINIMIZED WRT ONE OF THE N_i, COMBINING THIS WITH THE REACTION EQN. GIVES,

$$\sum_i \nu_i \mu_i = 0$$

IN REACTIONS OF THE TYPE

$$e^- + p \rightarrow \nu_e + n$$

THE μs ARE <u>ADDITIVELY</u> CONSERVED

- FOR A PHOTON GAS, PARTICLE # IS VARIABLE (ABSORPTION/EMISSION BY MATTER). MINIMIZATION OF THE FREE ENERGY ONE HAS $\left.\frac{\partial F}{\partial N}\right|_{T,V} = 0 \Rightarrow \underline{\mu = 0}$

PAIR PRODUCTION

$$e^+ + e^- \longrightarrow \gamma \qquad T > mc^2 \ (6\times10^9 K)$$

$$\mu_+ + \mu_- = \mu_\gamma = 0$$

$$\Rightarrow \mu_+ = -\mu_-$$

SIMPLE CALCULATIONS FOR PHOTONS

$n_\nu \, d\nu = \#(\nu, \nu + d\nu)$

$$n_\nu = \underbrace{\frac{8\pi\nu^2}{c^3}}_{\text{DENSITY OF STATES}} \underbrace{\frac{1}{e^{h\nu/kT} - 1}}_{\text{OCC. \# / MODE}}$$

$$n = \int_0^\infty n_\nu \, d\nu = 16\pi\,\zeta(3)\left(\frac{kT}{hc}\right)^3$$

$$\rho = \int_0^\infty h\nu \, n_\nu \, d\nu = \frac{8\pi^5}{15}\frac{(kT)^4}{(hc)^3} \equiv \sigma T^4$$

EXPANDING UNIVERSE →

PHOTONS IN BOX OF VOLUME V, WITHIN $\nu \to \nu + d\nu$, $N_\nu = n_\nu \, d\nu \, V$

AFTER EXPANSION,

$$V' = V(a'/a)^3 = \frac{V}{(1+z)^3} \; ; \; d\nu' = (1+z)d$$

OF PHOTONS CONSERVED

$$\Rightarrow n_\nu \, d\nu \, V = n'_{\nu'} \, V' \, d\nu'$$

(7)

$$\Rightarrow \quad \frac{8\pi\nu^2}{c^3}\,\frac{1}{e^{h\nu/kT}-1}\,V\,d\nu$$

$$= \frac{8\pi}{c^3}\,\frac{(\nu')^2}{(1+z)^2}\,\frac{1}{e^{h\nu'/(1+z)kT}-1}\,\frac{(1+z)^3 V'\,d\nu'}{(1+z)}$$

$$= \frac{8\pi}{c^3}(\nu')^2\,\frac{1}{e^{h\nu'/(1+z)kT}-1}$$

$$\Rightarrow \quad \boxed{T' = \frac{T}{1+z} \quad \text{OR} \quad T_0(1+z) = T(z)}$$

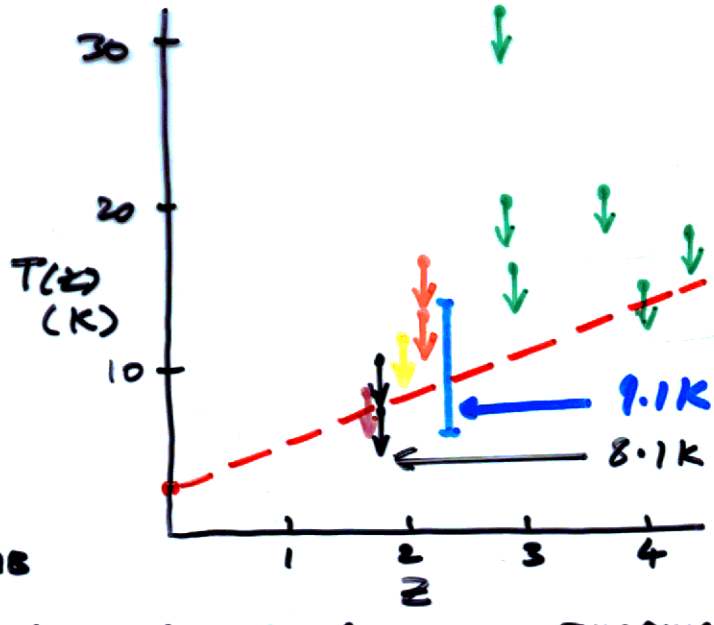
ABSORPTION LINES IN QUASAR SPECTRA

NEUTRAL C GROUND TERM SPLIT INTO 3 LEVELS

0–1 → 23.6 K

1–2 → 38.9 K

EXCITED STATE POPULATIONS → T_{CMB}



SRIANAND ET AL 2000

9.1 K vs 6–14 K

8.1 K vs 7.9 ± 1 K

GE ET AL 9

SYSTEMATICS : COLLISIONAL EXCITATIONS & UV PUMPING / CASCADES

STATUS OF 'EQUILIBRIUM'

WHEN IN LOCAL THERMAL EQUILIBRIUM AND WHEN NOT ?

$$\Gamma \gtrsim H$$

[IGNORING g_* WHICH COMES LATER]

$\Gamma = n \sigma |v|$ → INTERACTION RATE PER PARTICLE

- $|v|$ → AVERAGE RELATIVE VELOCITY
- σ → CROSS-SECTION
- n → # DENSITY OF TARGET PARTICLES

AS LONG AS $\Gamma \gtrsim H$ LTE IS OK, OTHERWISE NEED TO SOLVE BOLTZMANN EQUATION...

- SUPPOSE:
  - (i) $\Gamma \propto T^n$
  - (ii) $a \sim t^{1/2}$ (RADIATION DOMINATED)

THEN $T \sim t^{-1/2}$ $\left[\frac{\dot{T}}{T} = -H = -\frac{1}{2t} \right]$

$$\Rightarrow N_{int} = \int_t^\infty \Gamma(t')\, dt' = \frac{(\Gamma/H)_t}{n-2} \quad \{ \text{for } n > 2 \}$$

SUBSEQUENT TO $\Gamma \sim H$ A PARTICLE REACTS LESS THAN ONCE FOR THE REST OF THE TIME.

CLEARLY STATUS OF LTE DEPENDS ON HOW Γ BEHAVES

SUPPOSE INTERACTIONS MEDIATED BY MASSLESS GAUGE BOSONS

$$\Gamma \sim \alpha^2 T$$

$$H \sim \frac{T^2}{M_{Pl}}$$

$$\Rightarrow \frac{\Gamma}{H} \sim \frac{\alpha^2 M_{Pl}}{T}$$

$\Rightarrow$ FOR $T \lesssim \alpha^2 M_{Pl} \sim 10^{16}$ GeV LTE HOLDS

FOR $T \gtrsim 10^{16}$ GeV NOT CLEAR IF LTE IS A REASONABLE APPROXIMATION OR NOT. WE DON'T REALLY KNOW WHAT'S GOING ON THEN (10^{-38} s)

- NOTE: REACTIONS MEDIATED BY MASSIVE PARTICLES WORK THE OTHER WAY

FOR $T \lesssim \left(\frac{M_X}{100\,\text{GeV}}\right)^{4/3}$ MeV THESE REACTIONS FREEZE OUT. [$M_X \gtrsim T \gtrsim \frac{G_X^{-3/2}}{M_{Pl}^{1/2}}$ TO PROCEED RAPIDLY ...]

FROM NOW ON THE DISCUSSION WILL ASSUME EITHER LTE HOLDS OR PARTICLES ARE FREE-STREAMING

THERMAL HISTORY (BRIEF PLOT)

- $T > 10^{12}$ K. STRONGLY INTERACTING SOUP OF γs, LEPTONS, MESONS, NUCLEONS. HARD TO CALCULATE..
- $T \sim 10^{12}$ K. γs, MUONS, $e^{\pm}$, ν & $\bar{\nu}$, LOW DENSITY OF NEUTRONS & PROTONS ($n_p \simeq n_n$)
- T DROPS BELOW 10^{12} K

  $\mu^+\mu^-$ ANNIHILATE, AFTER ALMOST ALL μs GONE (1.3×10^{11} K) THE $\nu\bar{\nu}$ DECOUPLE. PARTICLES IN LTE ARE $e^{\pm}$, γs & A FEW NUCLEONS ($T \sim 1/a$)
- T DROPS BELOW 10^{11} K

  SMALL Δm_{np} SHIFTS $n_p > n_n$
- T DROPS BELOW 5×10^{9} K

  e^+e^- ANNIHILATE, γs $\nu\bar{\nu}$ IN FREE EXPANSION WITH $T_\gamma \sim 1.4\, T_\nu$ n/p RATIO FREEZES AT ~ 1.5
- $T \sim 10^9$ K — $n + p \rightarrow$ nuclei
- T_γ LOCKED TO T_{GAS} TILL ~ 4000 K
- $T \sim 10^3 - 10^5$ K — MATTER DOMINATION BEGINS

EFFECTIVE DEGREES OF FREEDOM

- WHAT ARE THE μ_i ?

$$e^- + \mu^+ \longrightarrow \nu_e + \bar{\nu}_\mu$$
$$e^- + p \longrightarrow \nu_e + n$$
$$\mu^- + p \longrightarrow \nu_\mu + n$$

{ USING $\mu_+ = -\mu_-$ }

RECALL $\sum_i \mu_i = 0 \Rightarrow \mu_{e^-} - \mu_{\nu_e} = \mu_{\mu^-} - \mu_{\nu_\mu} = \mu_n - \mu_p$

FOR GROSS THERMAL HISTORY CONSIDERATIONS CAN CONSIDER THE FOUR CONSERVED INTRINSIC QUANTUM #S

CHARGE	$\longrightarrow$	N_Q
BARYON #	$\longrightarrow$	N_B
ELECTRON-LEPTON #	$\longrightarrow$	N_E $(e^- \nu_e - e^+ \bar{\nu}_e)$
MUON-LEPTON #	$\longrightarrow$	N_M $(\mu^- \nu_\mu - \mu^+ \bar{\nu}_\mu)$

$\Rightarrow \exists$ ONLY 4 INDEPENDENT μ_S DETERMINED BY THE N_S ABOVE (ALL $N_S \sim \frac{1}{a^3}$)

RELATIVE TO n_γ, N_B IS VERY SMALL, $N_Q \sim 0$ AND IT SEEMS REASONABLE TO ALSO TAKE N_B & N_M SMALL. IF

$$N_Q \simeq N_B \simeq N_E \simeq N_M \simeq 0$$

$\Rightarrow \boxed{\mu_i = 0}$ $\because$ THE N_S ARE ODD FUNCTIONS OF THE 4 μ_i (REMEMBER "$\mu_+ = -\mu_-$" & "$n_+ - n_-$")

$\hookrightarrow$ THIS SIMPLIFIES LIFE CONSIDERABLY

RADIATION TEMPERATURE & PRESSURE

$$\rho_R = T^4 \sum_i \left(\frac{T_i}{T}\right)^4 \frac{g_i}{2\pi^2} \int_{x_i}^{\infty} \frac{\sqrt{u^2 - x_i^2}\, u^2\, du}{e^u \pm 1}$$

$$p_R = T^4 \sum_i \left(\frac{T_i}{T}\right)^4 \frac{g_i}{6\pi^2} \int_{x_i}^{\infty} \frac{(u^2 - x_i^2)^{3/2}\, du}{e^u \pm 1}$$

$\mu_i \to 0$

$x_i = \frac{m_i}{T}$, THE SUM IS OVER ALL SPECIES IN THERMAL EQUILIBRIUM

SINCE NONRELATIVISTIC SPECIES CONTRIBUTE VERY LITTLE, WE CAN IGNORE THEM. THIS GIVES

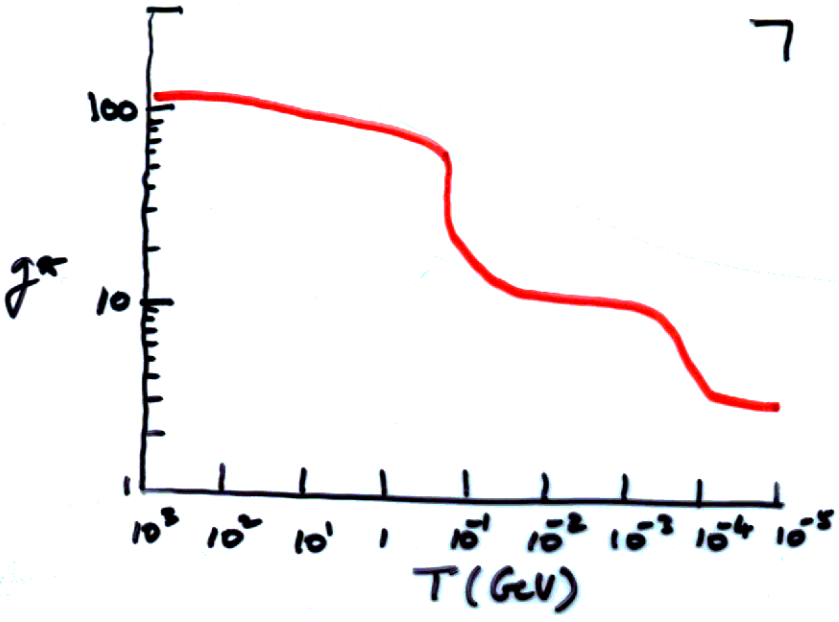
$$\rho_R = \frac{\pi^2}{30} g_* T^4$$

$$p_R = \frac{\rho_R}{3} = \frac{\pi^2}{90} g_* T^4$$

$$g_* = \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i=\text{fermions}} g_i \left(\frac{T_i}{T}\right)^4$$

COUNTS # OF EFFECTIVELY MASSLESS DEGREES OF FREEDOM

SINCE AT DIFFERENT TIMES DIFFERENT SPECIES CONTRIBUTE TO g_* IT IS A STRONG FUNCTION OF TIME.



→ FOR $T \ll$ MeV, ONLY RELATIVISTIC SPECIES ARE THE 3 ν_s AND THE PHOTON

→ FOR 100 MeV $\gtrsim T \gtrsim$ 1 MeV THE e^+e^- CAN BE ADDED TO THIS CATEGORY

→ FOR $T \gtrsim$ 300 GeV ALL OF THE SM PARTICLES ENTER

g_* ($T \ll$ MeV) = 3.36 $(\nu\bar{\nu}\gamma)$

g_* (100 MeV $\gtrsim T \gtrsim$ 1 MeV) = 10.75 $(e^+e^-\nu\bar{\nu}\gamma)$

g_* ($T \gtrsim$ 300 GeV) = 106.75 (GLUONS (8), $W^\pm$ Z^0, ETC.)

DURING THE EARLY RADIATION DOMINATED ERA (MOSTLY THE ERA OF INTEREST)

$$\rho \simeq \rho_R$$

AND IF $g_* \simeq$ CONST., $a \sim t^{1/2}$

$$\Rightarrow \boxed{H = 1.66 \sqrt{g_*} \frac{T^2}{M_{Pl}}}$$

ENTROPY

IF WE ARE IN LTE, THE 2ND LAW APPLIES:

$$dS(V,T) = \frac{1}{T}\left[d\left(\rho_{eq}(T)V\right) + p_{eq}(T)\,dV\right] \quad \text{— (1)}$$

$$\Rightarrow \quad \frac{\partial S}{\partial V} = \frac{1}{T}\left[\rho_{eq}(T) + p_{eq}(T)\right]$$

$$\frac{\partial S}{\partial T} = \frac{V}{T}\frac{d\rho_{eq}(T)}{dT}$$

THE INTEGRABILITY CONDITION

$$\frac{\partial^2 S}{\partial T\,\partial V} = \frac{\partial^2 S}{\partial V\,\partial T}$$

YIELDS

$$\frac{d\rho_{eq}(T)}{dT} = \frac{1}{T}\left[\rho_{eq}(T) + p_{eq}(T)\right] \quad \text{— (2)}$$

$T^{\mu\nu}{}_{;\nu} = 0$ FOR A PERFECT FLUID IN AN FRW UNIVERSE GIVES THE ENERGY CONSERVATION EQN.

$$a^3\frac{d\rho_{eq}}{dt} = \frac{d}{dt}\left[a^3(\rho_{eq} + p_{eq})\right]$$

THE ABOVE TWO EQUATIONS COMBINED:

$$\boxed{\frac{d}{dt}\left[\frac{a^3}{T}\left(\rho_{eq}(T) + p_{eq}(T)\right)\right] = 0}$$

— A CONSERVATION LAW.

[$S(R^3,T)$ CONSE RV

SUBSTITUTION OF (2) IN (1) ABOVE GIVES

$$\boxed{S(V,T) = \frac{V}{T}\left[\rho_{eq}(T) + p_{eq}(T)\right]}$$

WHEN ALL THE PARTICLES ARE HIGHLY RELATIVISTIC $p_{ky} = \frac{1}{3} \rho_{ky}$, AND THE ENTROPY DENSITY

$$s = \frac{S}{a^3} = \frac{\rho + p}{T} \approx \boxed{\frac{2\pi^2}{45} g_{*s} T^3}$$

$$g_{*s} = \sum_{i\,\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i\,\text{fermions}} g_i \left(\frac{T_i}{T}\right)^3$$

→ FAPP $g_* = g_{*s}$

ANOTHER WAY TO RESTATE OUR PREVIOUS CONSERVATION LAW IS THAT

$$S = \boxed{g_{*s} T^3 a^3 = \text{CONST.}} \quad (s \propto \bar{a}^3)$$

→ $T \sim (1/a) g_{*s}^{-1/3}$

OF SPECIES IN COMOVING VOLUME $V = a^3$

$$N \equiv a^3 n = \frac{n}{T^3}$$

FOR A MASSLESS SPECIES, AFTER DECOUPLING (T DROPPING BELOW MASS THRESHOLD)

$$T \sim \frac{1}{a} \quad \left(= \frac{T_0 a_0}{a}\right) \quad [\text{NO } g_{*s}]$$

FOR A MASSIVE SPECIES ($m \gg T_0$)

$$T \sim \frac{1}{a^2}$$

IF $m \sim T_0$ THERE IS NO THERMAL DISTBN. IN THE ABSENCE OF INTERACTIONS

DECOUPLING EXAMPLE : NEUTRINOS

ν_i MAINTAINED IN EQLBM. VIA INTERACTIONS OF THE TYPE

$$\nu_e + \bar{\nu}_\mu \longleftrightarrow e^- + \mu^+ \quad \text{ETC.}$$

WEAK INTERACTION CROSS-SECTION

$$\sigma \simeq G_F^2 T^2$$

DENSITY OF PARTICLES

$$n \simeq T^3$$

$$\Rightarrow \quad \Gamma_{int} = n \sigma |v| \overset{v \sim \text{UNITY}}{\simeq} G_F^2 T^5$$

THUS

$$\boxed{\frac{\Gamma_{int}}{H} \propto \frac{G_F^2 T^5}{T^2/M_{Pl}}} \leftarrow \text{SHOWN EARLIER}$$

$$\boxed{\propto \left(\frac{T}{1\,\text{MeV}}\right)^3}$$

FOR $T < 1\,\text{MeV}$ LIGHT ν_i DECOUPLE FROM THE PLASMA & THEIR TEMPERATURE THEN FALLS AS $1/a$

VERY SHORTLY AFTER ν DECOUPLING $T < m_e$ e^+e^- ANNIHILATE OUT & THE RESULTING ENTROPY GOES TO γ_i & NOT ν_i.

AT $T \gtrsim m_e$, $g_* = \frac{11}{2}$ [γ's & e^+e^- PAIRS]

$T \ll m_e$, $g_* = 2$ [ONLY γ's]

RECALL $g_* (aT)^3 = \text{CONST.}$

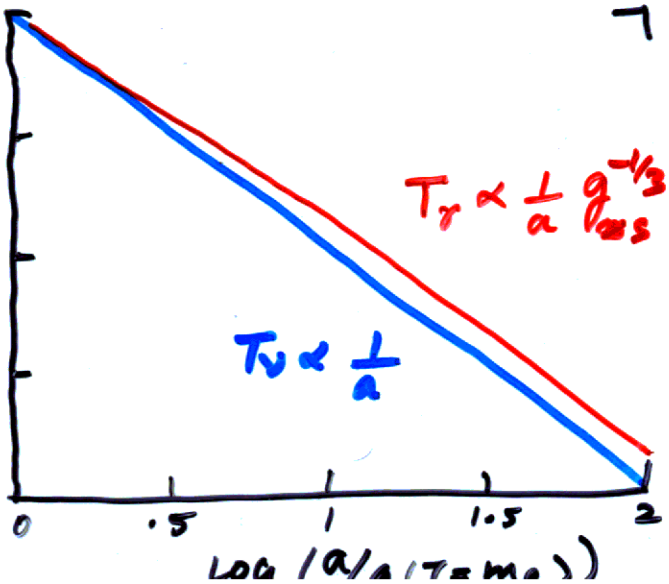
$\Rightarrow$ aT AFTER e^+e^- ANNIHILATION IS GREATER THAN aT BEFORE ANNIHILATION BY A RELATIVE FACTOR OF $g_*^{1/3}$ AT THESE TIMES, I.E. BY A RELATIVE FACTOR

$$\left(\frac{11}{2} \times \frac{1}{2}\right)^{1/3} = \left(\frac{11}{4}\right)^{1/3}$$

WHILE aT_γ GOES UP BY THIS FACTOR aT_ν STAYS (RELATIVELY) CONST.

HENCE, $$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3} = 1.4 \quad \text{TODAY}$$

$$\Rightarrow \boxed{T_\nu \simeq 1.96\ \text{K}}$$



MATTER - RADIATION PHOTON DECOUPLING

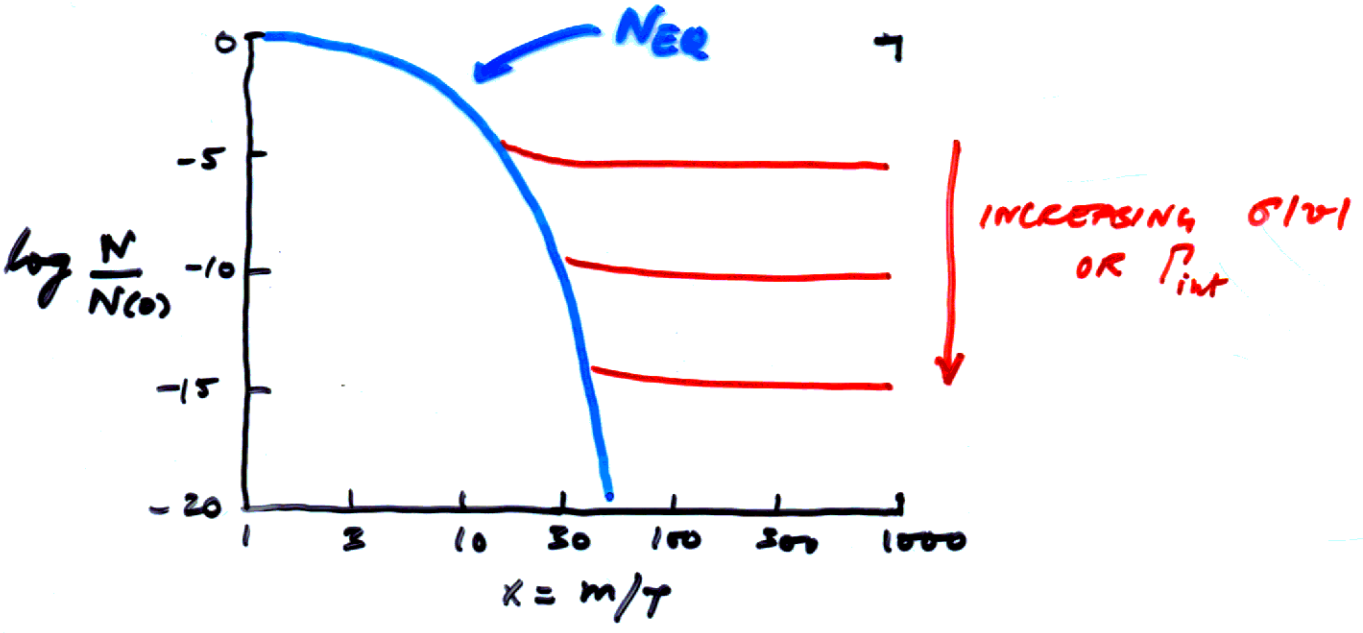
CMB LECTURES

FREEZE OUT: 'RELICS'

MASSIVE PARTICLE ABUNDANCE IN EQLBM.

$$\frac{n}{s} \sim (mT)^{3/2} e^{-m/T}$$

SINCE $m \gg T$ TODAY, ABUNDANCE WOULD BE NEGLIGIBLE

SAVED BY NONEQUILIBRIUM NATURE OF EVOLUTION: IF $\Gamma_{int} < H$ AT A TEMPERATURE S.T. m/T IS NOT $\gg 1$, CAN STILL HAVE SIGNIFICANT COMPONENT TODAY

(NUCLEONS, WIMPS, MASSIVE ν's ETC.)