Cosmology from the Deeply Nonlinear Regime: the SDSS Two-Point Correlation Function

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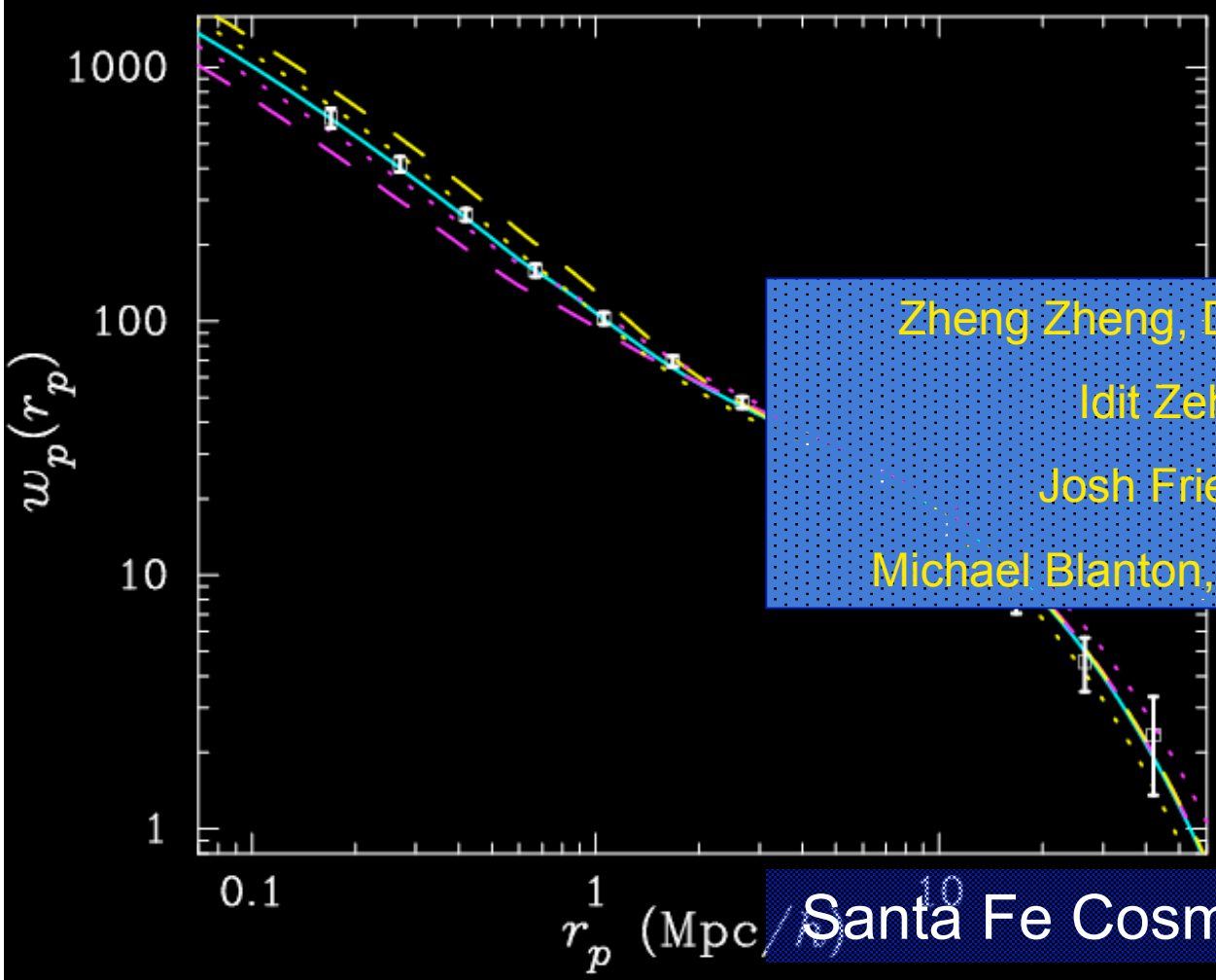
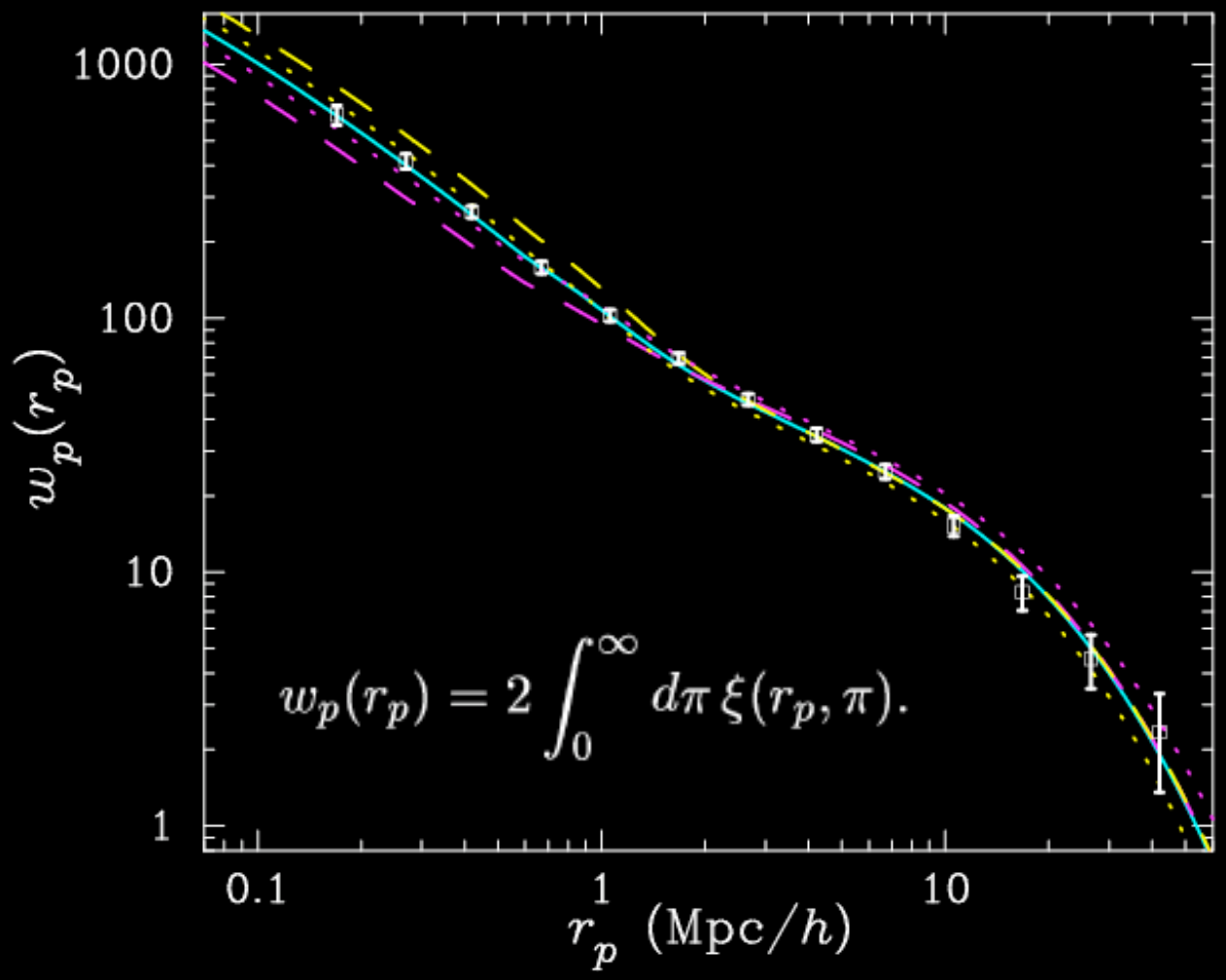


Santa Fe Cosmology Workshop 2004

July 15, 2004

Theory & measurement of the **Galaxy Correlation Function**

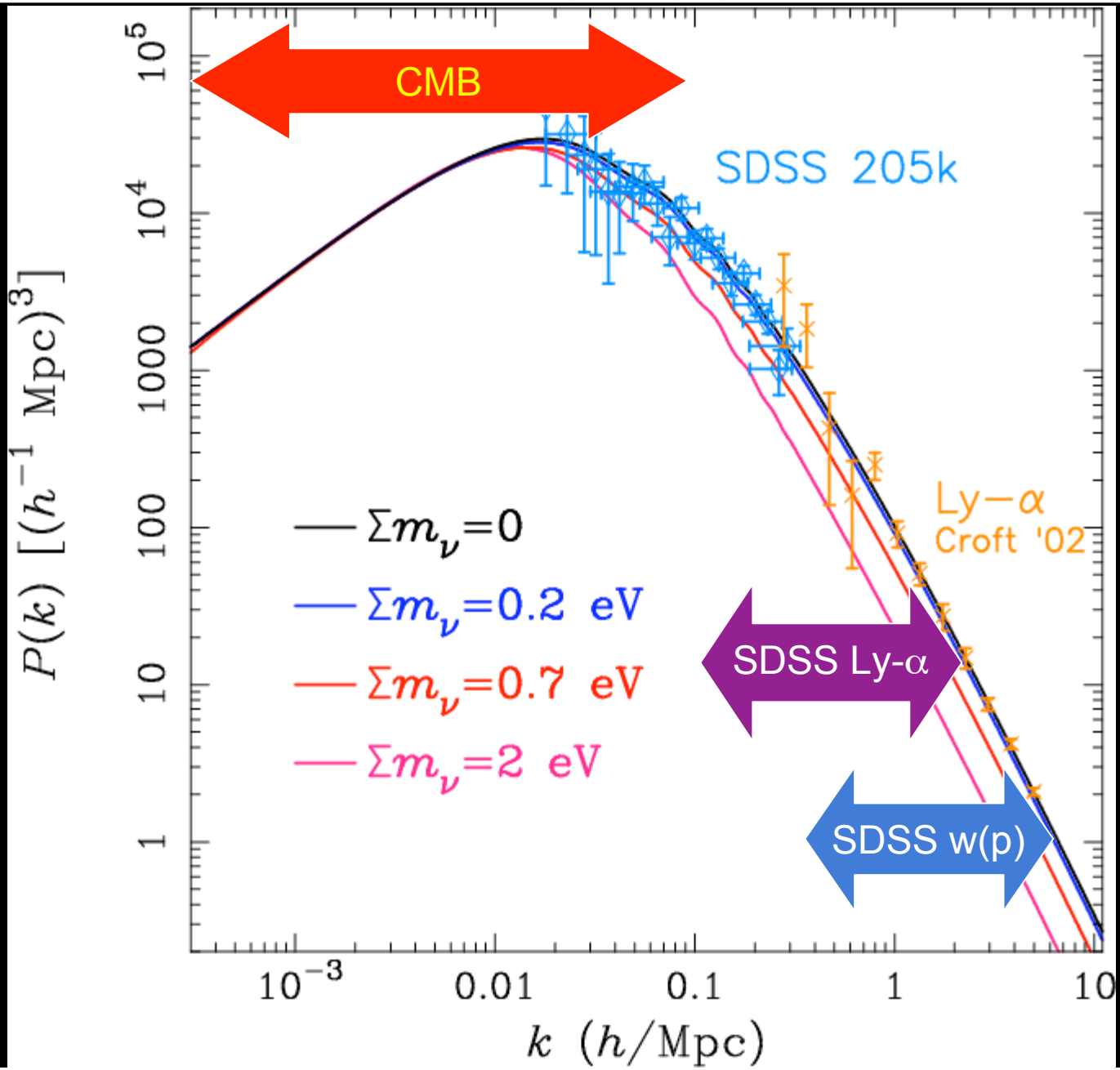


Zehavi et al., 2004:

$M_{0.1r}$ < -21

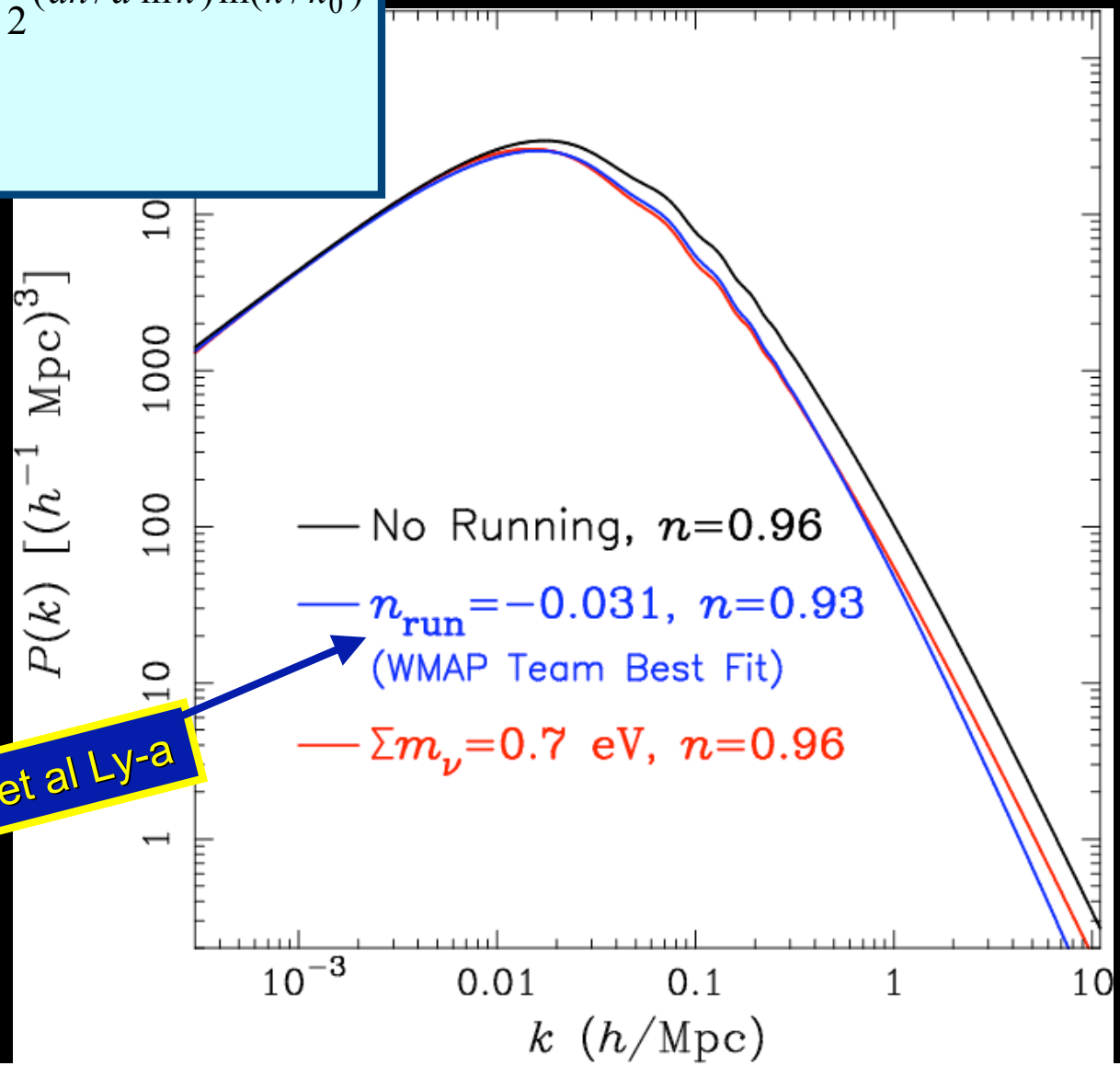
A high-statistics measurement of the galaxy correlation function over a large range of scale

Measuring P(k)



Uncovering the inflationary epoch:
Perturbations away from a power-law spectrum?

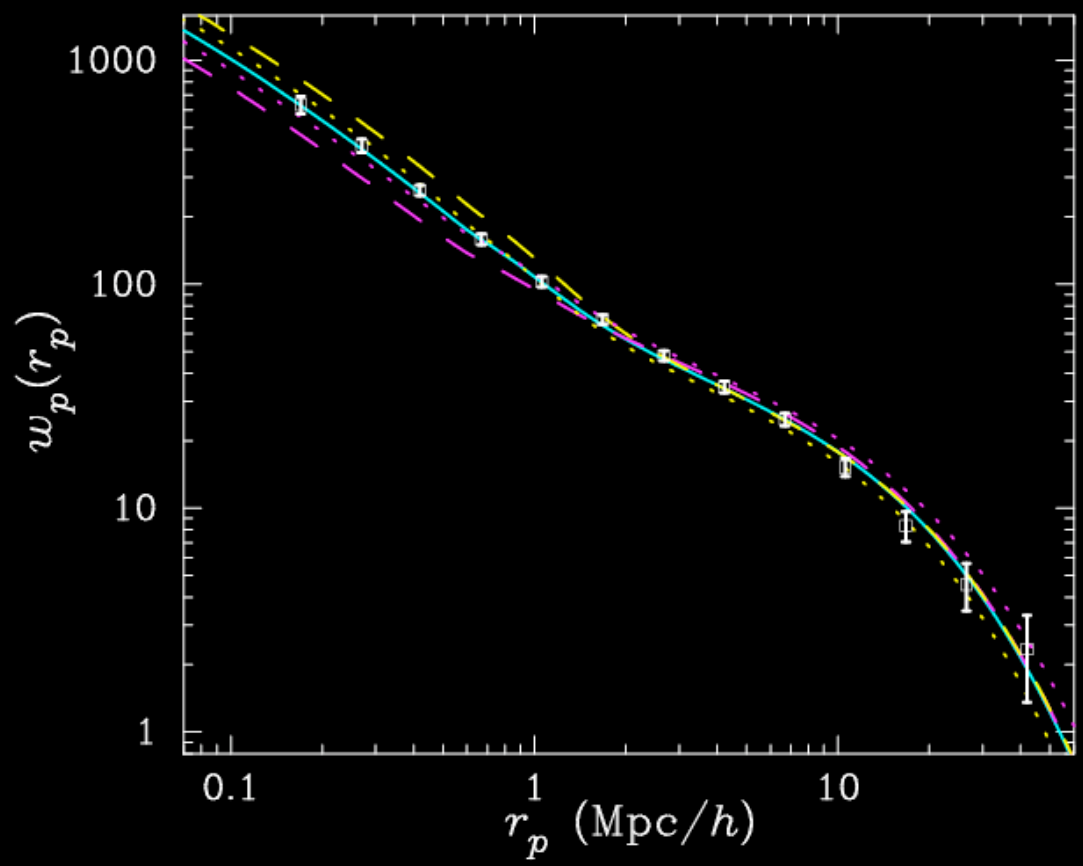
$$P(k) = A(k_0)\left(\frac{k}{k_0}\right)^{n+\frac{1}{2}(dn/d\ln k)\ln(k/k_0)}$$



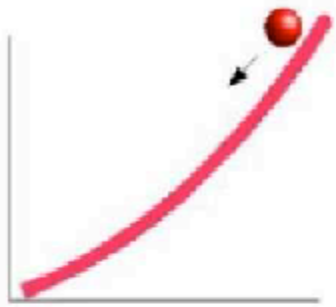
Using WMAPext+2dF+Croft et al Ly-a

Motivation

- Galaxy formation and distribution (Halo Occupation Dist.)
- Inflationary physics (dn/dlnk, n_{run}, α)
- Fundamental particle mass (Neutrino)



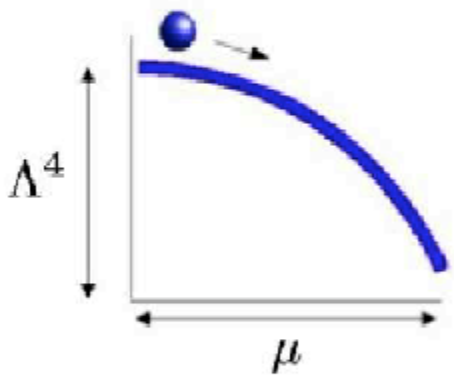
Inflationary Zoology



Large_field

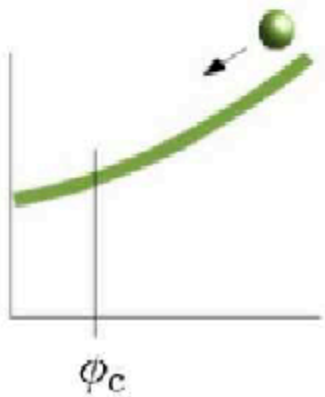
$$V(\phi) = \Lambda^4 (\phi/\mu)^p$$

$$V(\phi) = \Lambda^4 e^{\phi/\mu}$$



Small_field

$$V(\phi) = \Lambda^4 [1 - (\phi/\mu)^p]$$



Hybrid

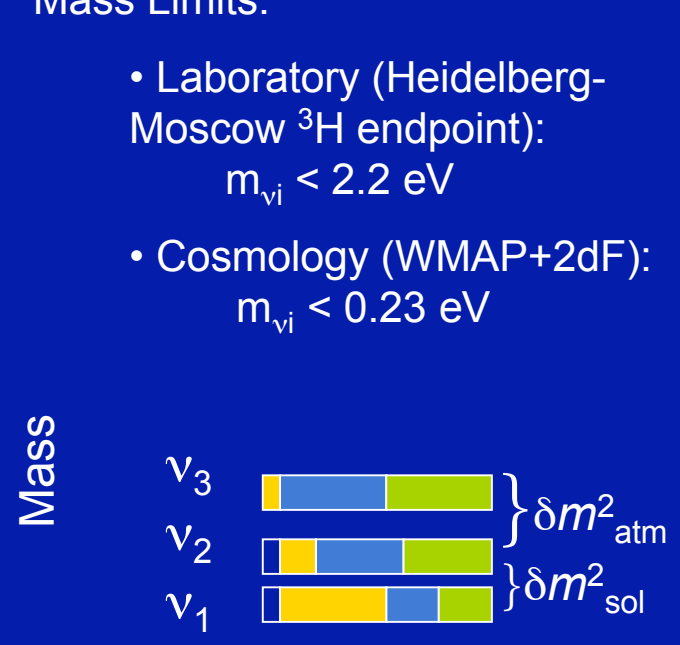
$$V(\phi) = \Lambda^4 [1 + (\phi/\mu)^p]$$

W. Kinney '03

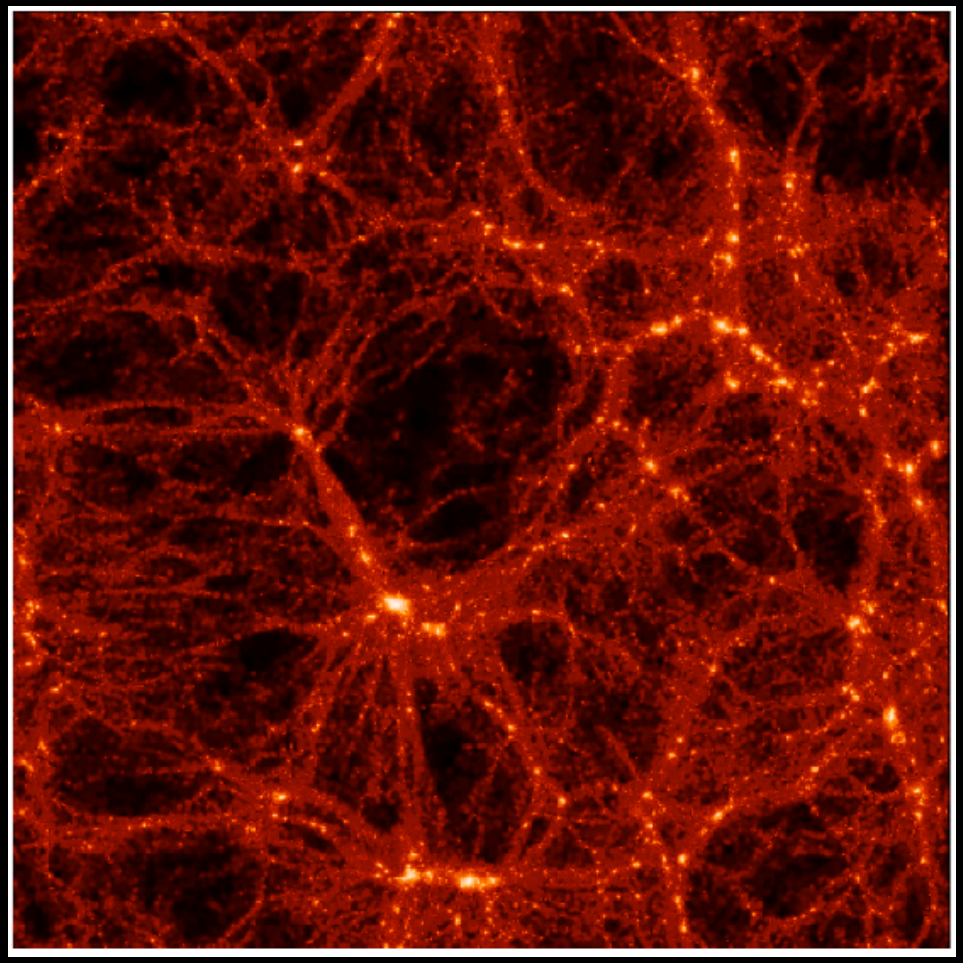
Neutrino Mass differences, but, no mass…

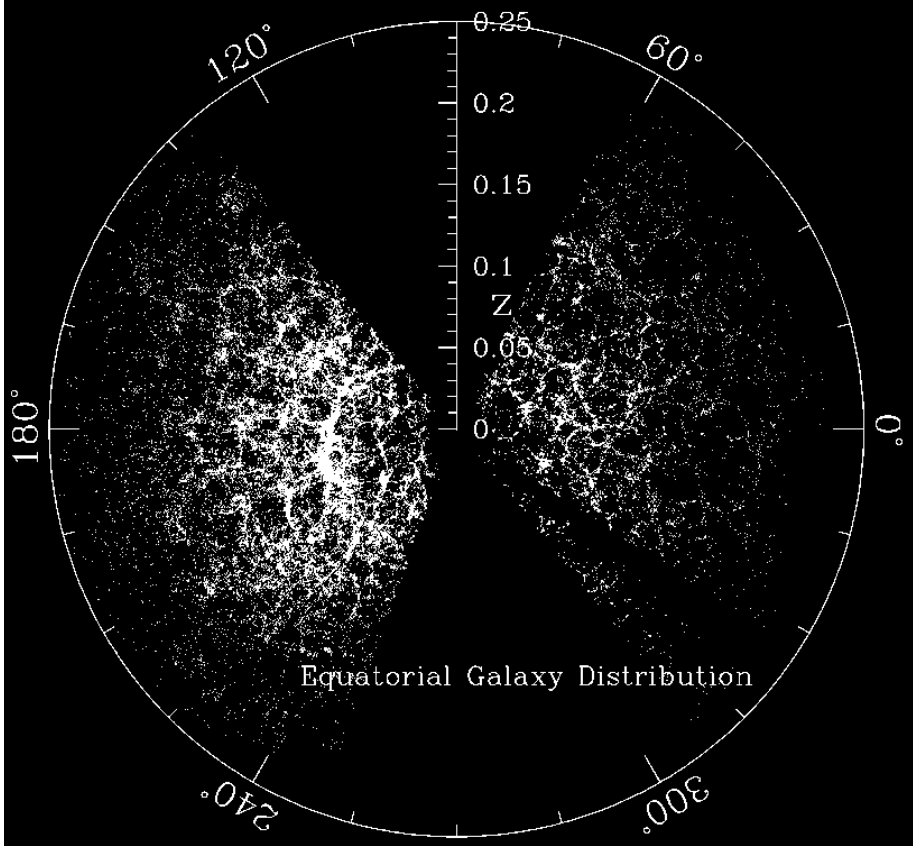
Mass Limits:

- Laboratory (Heidelberg-Moscow ^{3}H endpoint):
  $m_{\nu i} < 2.2$ eV
- Cosmology (WMAP+2dF):
  $m_{\nu i} < 0.23$ eV



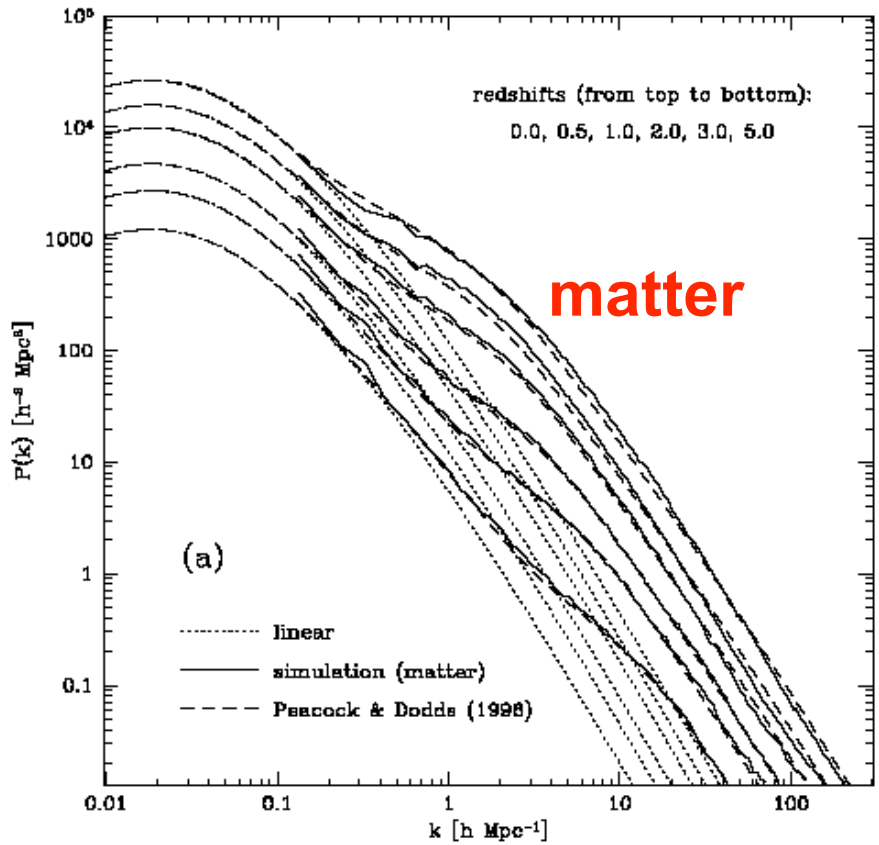
How are galaxies distributed relative to the Dark Matter?

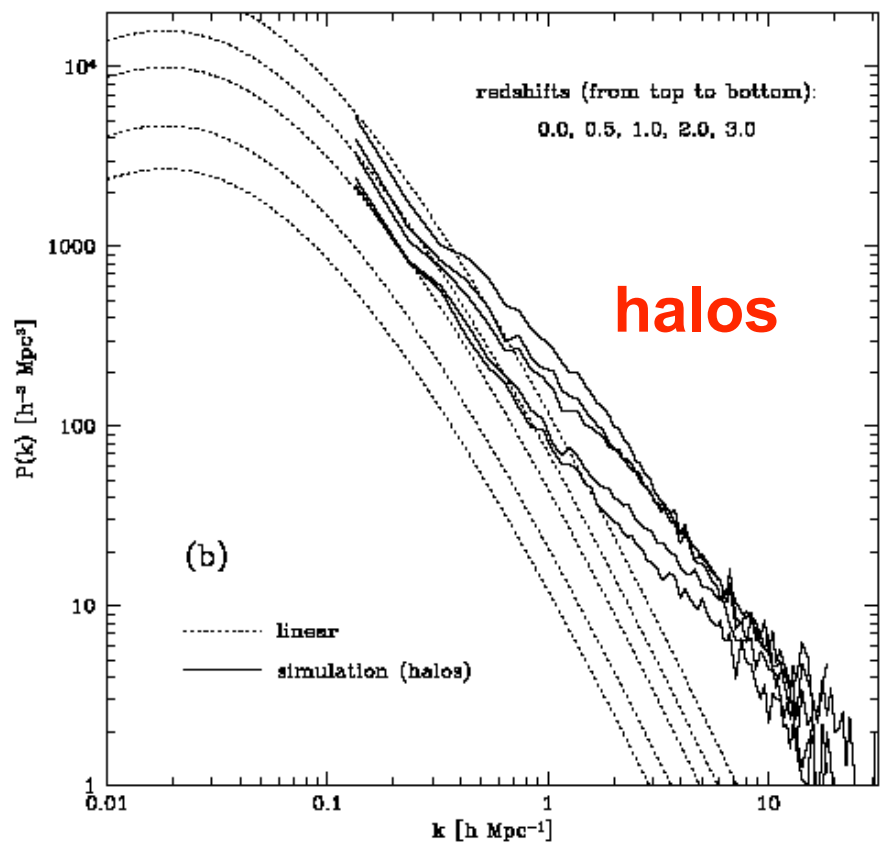




Dark Matter vs. Halos

- b(M):halo bias (from extended Press-Schecter, Mo & White 1996)
- scale dependence of halo bias (calibrated to simulations)





Kravtsov & Klypin (1999)

Galaxies vs. Halos

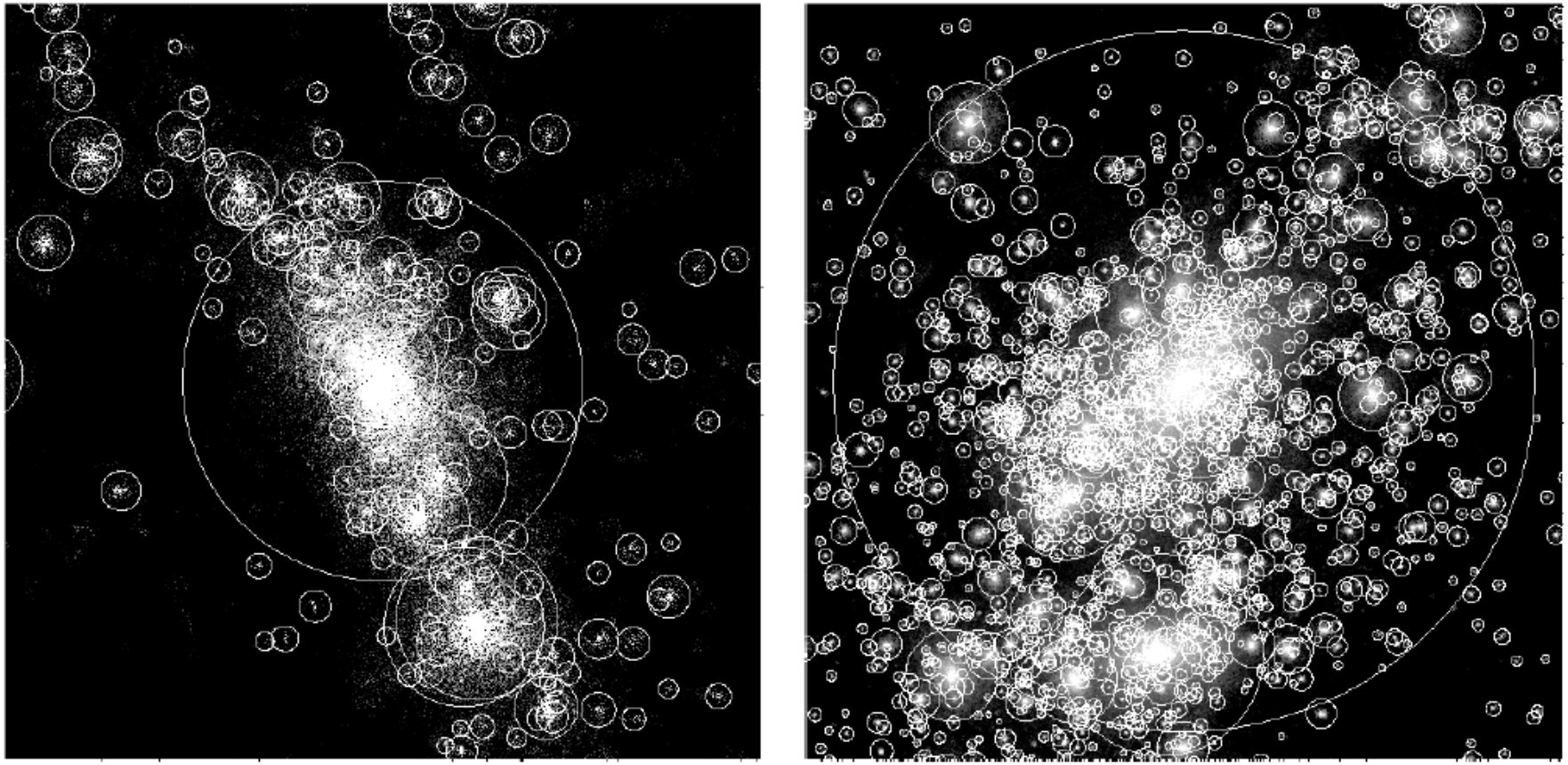


FIG. 1.— Distribution of dark matter particles (points) and dark matter halos (circles) identified by our halo finding algorithm centered on the most massive halo in the $\Lambda\mathrm{CDM}_{80}$ simulation at $z = 3$ (left) and $z = 0$ (right). The radius of the largest circle indicates the actual virial radius, R_{180}, of the most massive halo ($R_{180} = 0.67h^{-1}$ comoving Mpc at $z = 3$ and $R_{180} = 2.1h^{-1}$ Mpc at $z = 0$); the radii of all other halos are scaled using the halo' maximum circular velocities ($r_{\mathrm{h}} = 0.65V_{\mathrm{max}}$ kpc with V_{max} in km s^{-1}).

What is the PDF of Galaxies in Halos?

- Binomial?

$$P(N=n|M) = \frac{\mathcal{N}_M}{n!(\mathcal{N}_M - n)!}\, p_M^n (1-p_M)^{\mathcal{N}_M - n},$$

$$\langle N \rangle_M = \mathcal{N}_M p_M$$

$$\langle N(N-1)...(N-j)\rangle = \alpha^2(2\alpha^2-1)...(j\alpha^2-j+1)\langle N\rangle^{j+1}$$

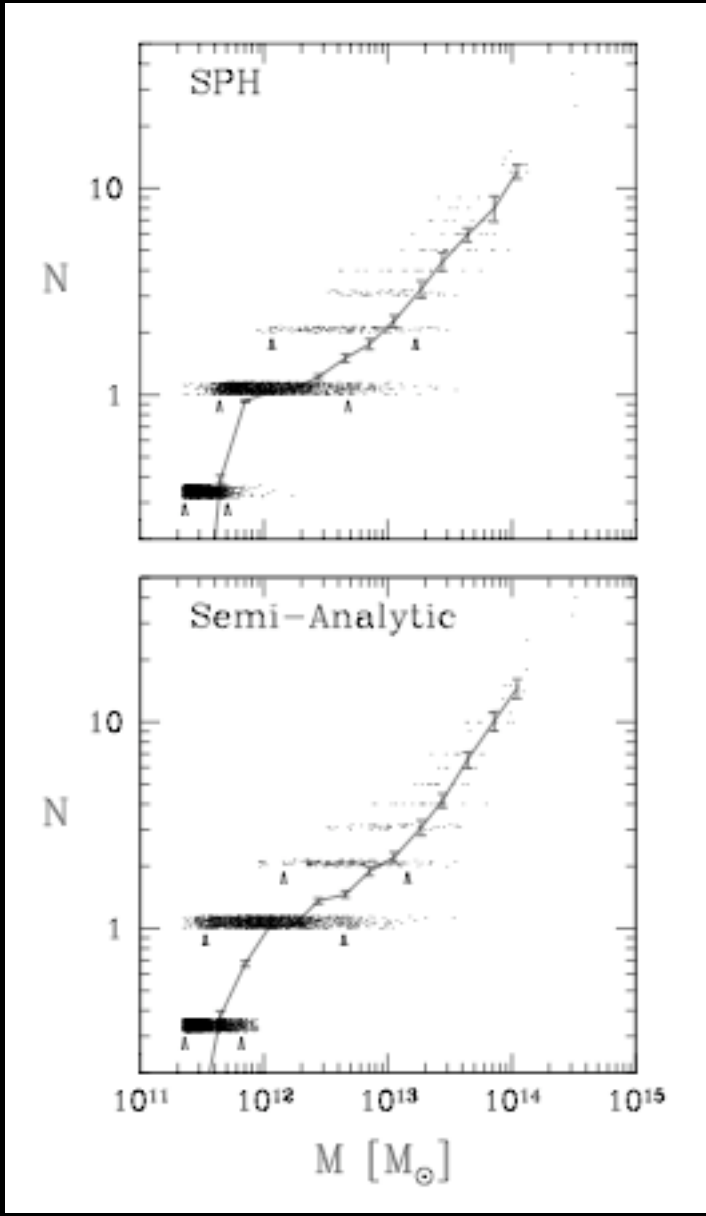
- Nearest Integer?

$$N_l \lesssim \langle N \rangle < N_l + 1$$

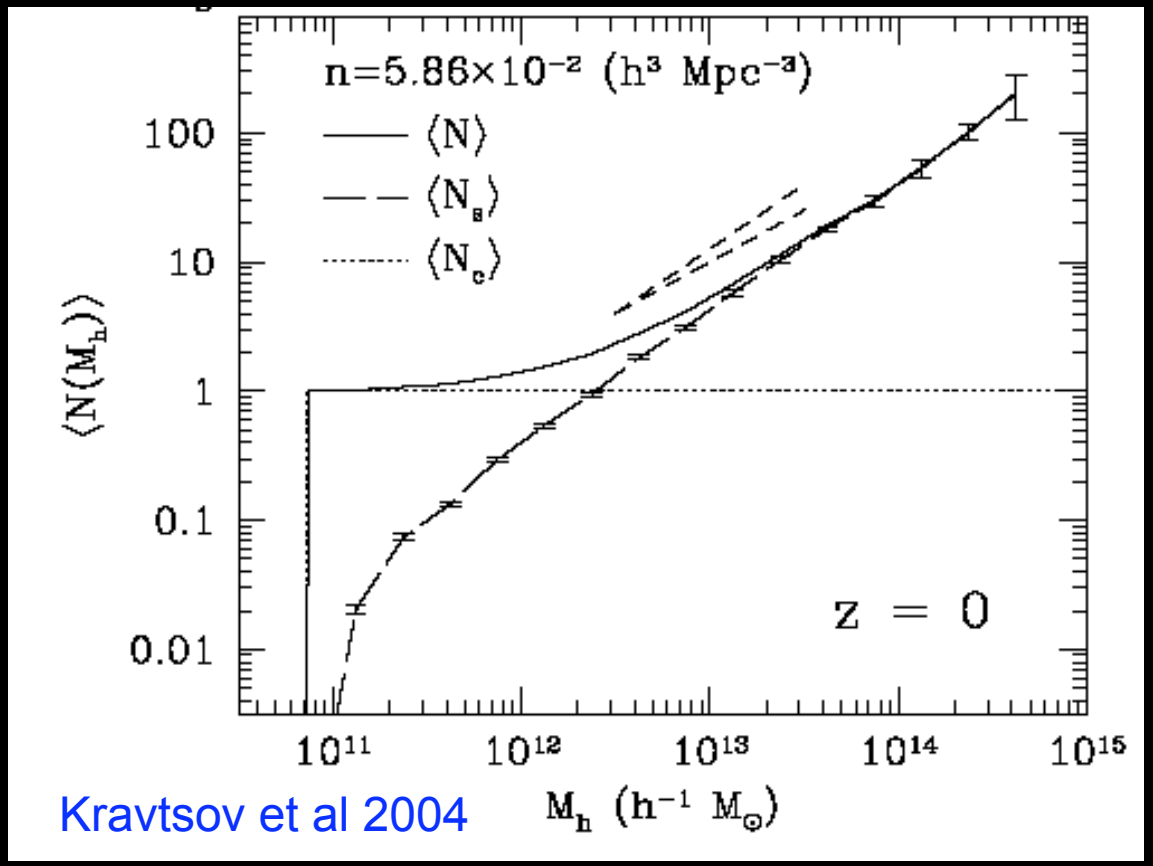
$$\langle N(N-1)\rangle = \langle N\rangle^2 \left(1+\bar{\xi}_2\right)$$
$$\langle N(N-1)(N-2)\rangle = \langle N\rangle^3 \left(1+3\bar{\xi}_2+\bar{\xi}_3\right),$$

- Poisson?

$$\langle N(N-1)...(N-j)\rangle = \langle N\rangle^{j+1}$$



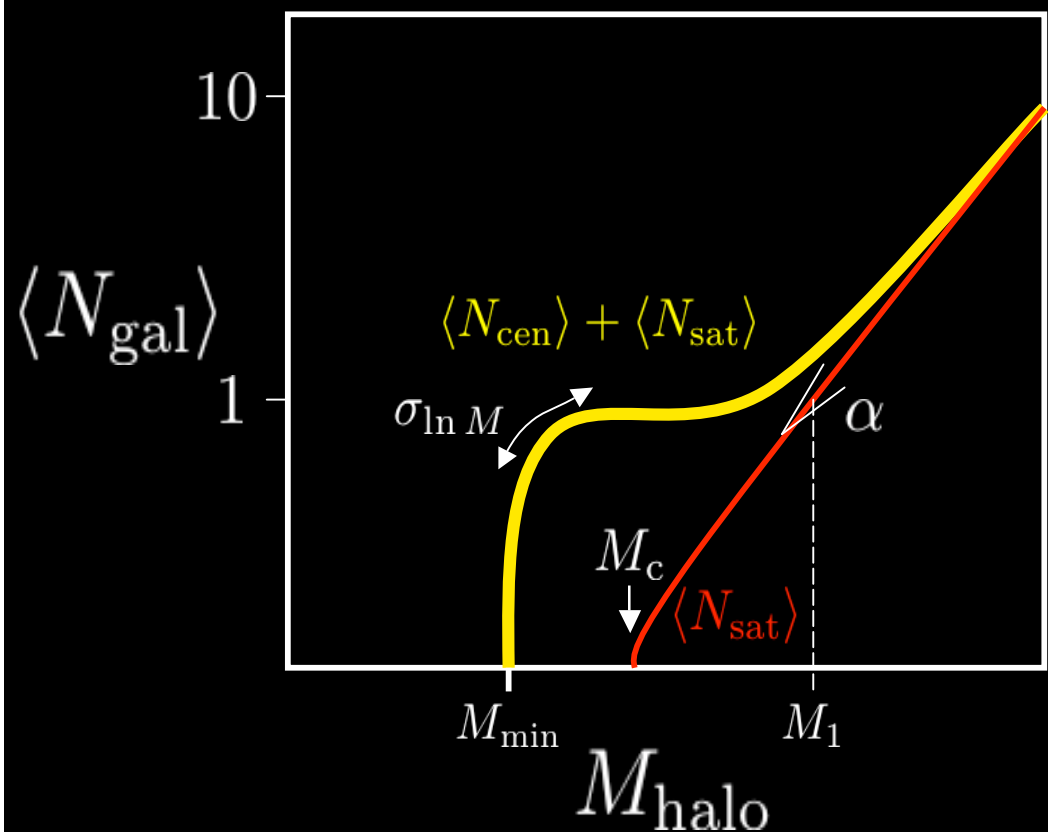
First Moment of P(N|M)



Kravtsov et al 2004

$$\langle N_{\rm sat}|M\rangle = \begin{cases} 0 & M < M_{\rm min} \\ \left(\frac{M+\kappa M_{\rm min}}{M_1}\right)^{\alpha} & M \geq M_{\rm min} \end{cases}$$

Significant Features of the "True" First Moment



$$\langle N_{\rm gal}\rangle = \langle N_{\rm cen}\rangle + \langle N_{\rm sat}\rangle$$

$$\langle N_{\rm cen}\rangle = \frac{1}{2}\mathrm{Erfc}\left(\frac{\ln M_{\rm min}/M}{\sqrt{2}\sigma_{\ln M}}\right)$$

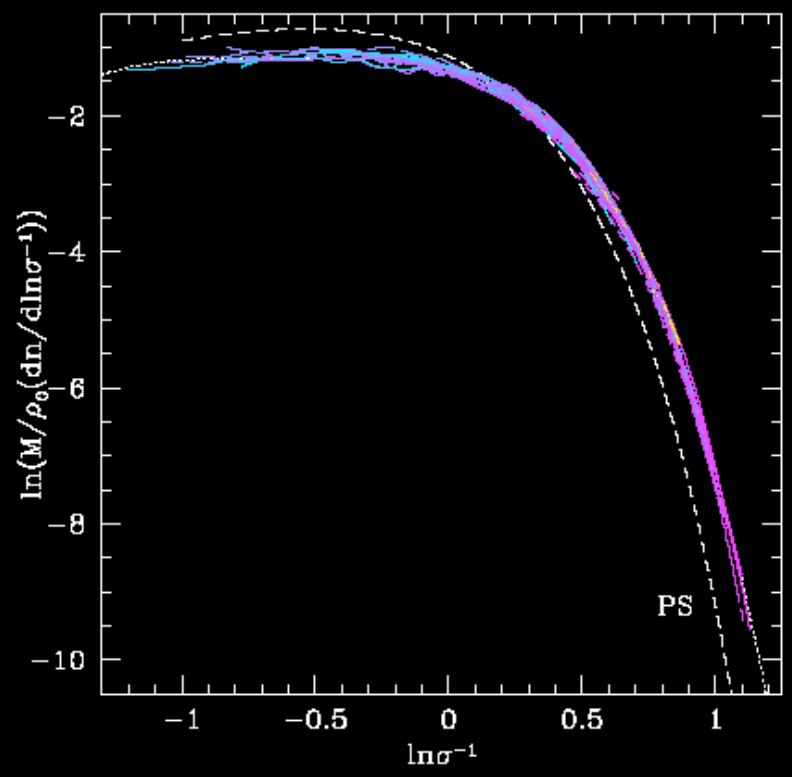
$$\langle N_{\rm sat}|M\rangle = \begin{cases} 0 & M < M_{\rm min} \\ \left(\frac{M+\kappa M_{\rm min}}{M_1}\right)^{\alpha} & M \geq M_{\rm min} \end{cases}$$

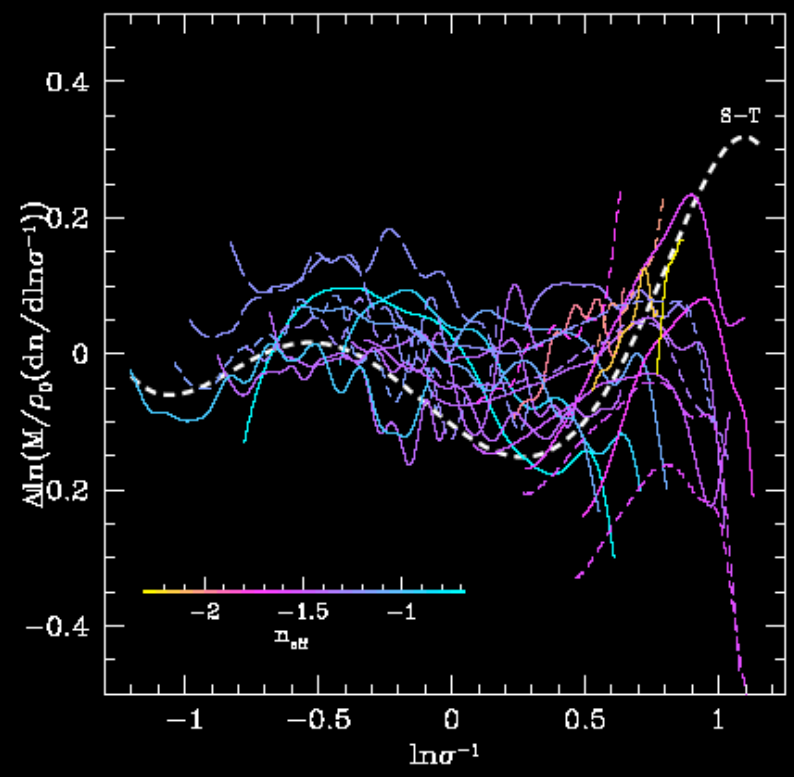
$$\kappa = \frac{M}{M_c}$$

The halo model as a tool

- The halo model will give statistics of the mass and galaxy distribution, given
  - The number density of halos of a given mass n(m), i.e., the mass function
    - Jenkins, et al 2001 with correction for cosmological variation of Δ_{vir} (Evrard et al, 2002; appendix of Hu & Kravtsov 2002)
  - The density distribution within the halos ρ(m): typically an NFW profile
    - Halo concentration dependence on cosmology (Huffenberger & Seljak 2003)
  - The bias of halos of given mass, b(m): analytically from n(m), plus any scale dependence
    - from high-resolution HOT-code simulations (LANL) (Warren & Seljak 2004)

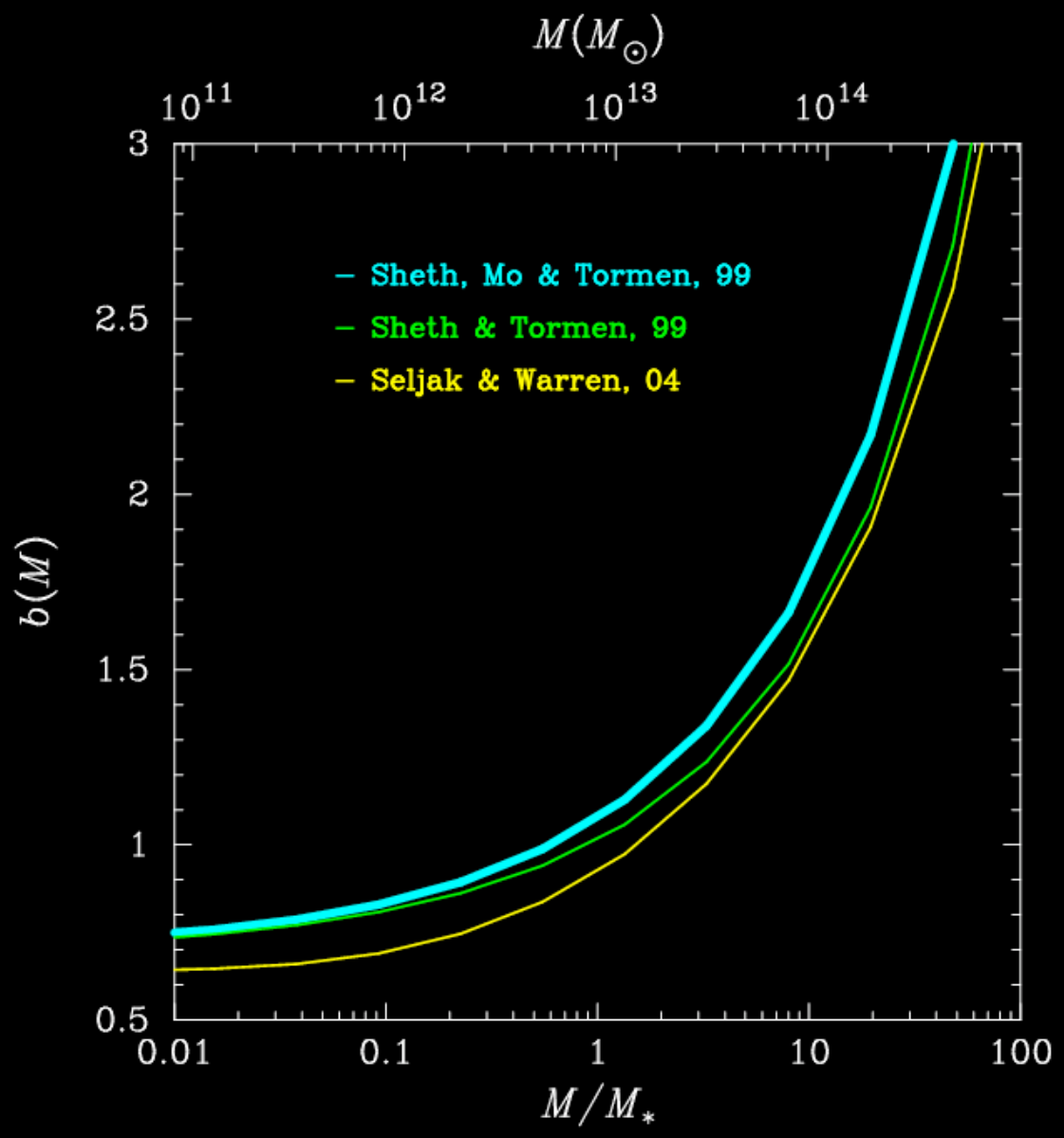
Mass function best estimate





Jenkins et al 2001

Halo Bias Best Estimate



Based on high-resolution dark matter only tree-code simulations by the Los Alamos HOT code

Seljak & Warren (2004)

NFW Concentration best estimate

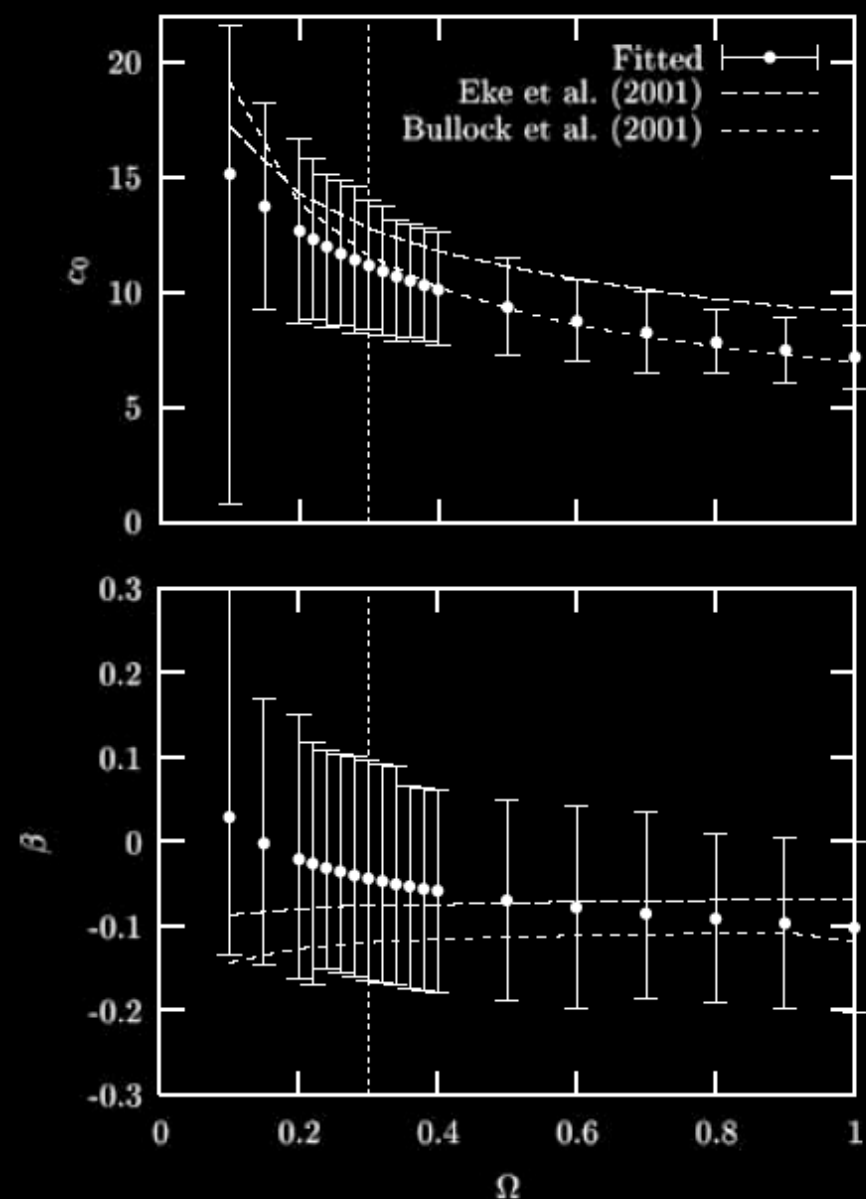
$$c = c_0 \left(\frac{M}{M_*}\right)^{\beta},$$

$$c_0 = 11 \left(\frac{\Omega_m}{0.3}\right)^{-0.35} \left(\frac{n_{\rm eff}}{-1.7}\right)^{-1.6},$$

$$\beta = -0.05,$$

$$n_{\rm eff} \equiv \left.\frac{d \ln P_{\rm lin}(k)}{d \ln k}\right|_{k_*},$$

Huffenberger & Seljak (2003)



Additional physical effects in halo model

- Nonlinear evolution of power spectrum for 2-halo term
  - Smith et al 2001

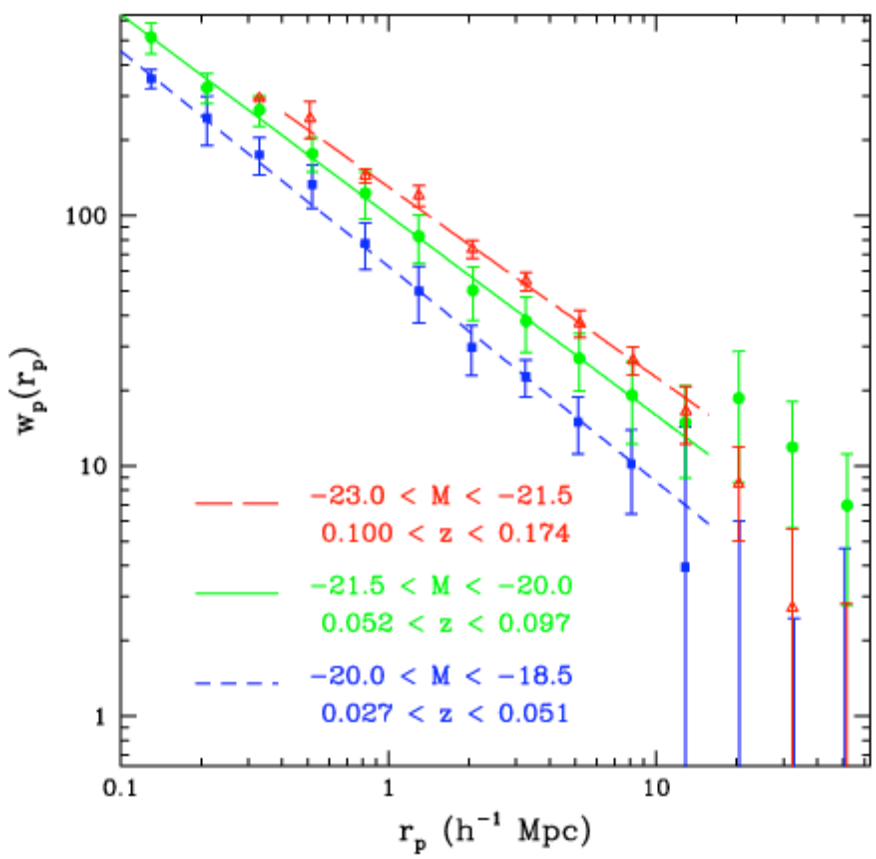
$$P_{\rm gg}^{\rm 2h}(k) = P_{\rm lin}(k)\left[\frac{1}{\bar{n}_g}\int_0^\infty dM\frac{dn}{dM}N_{\rm avg}(M)b_h(M)y_g(k,M)\right]^2,$$

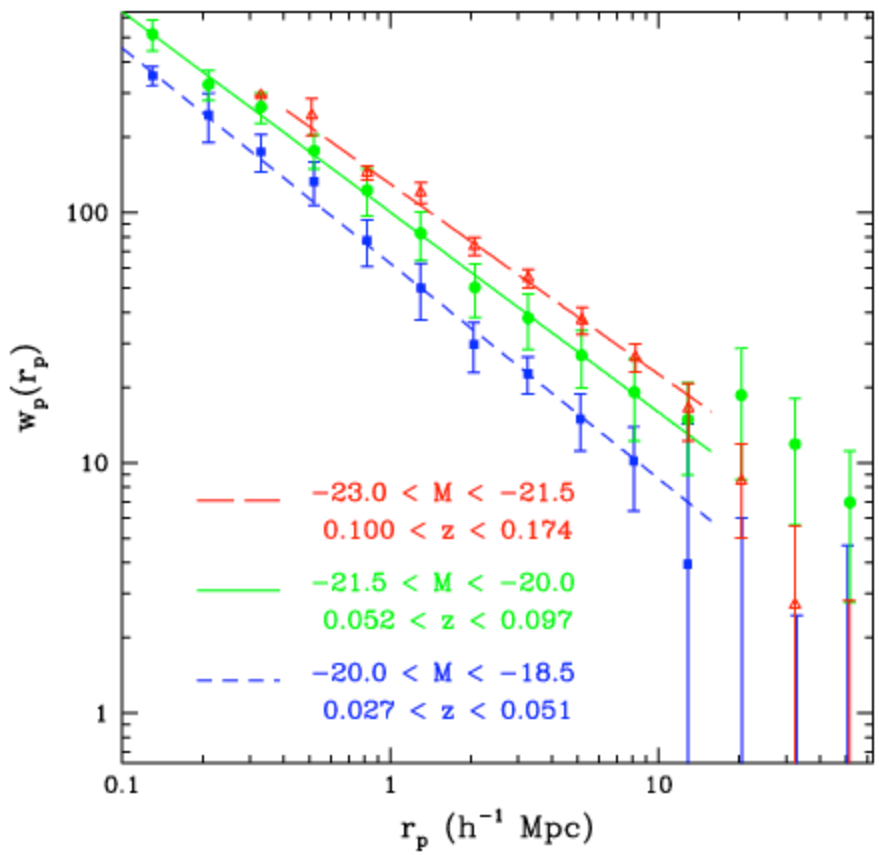
- Halo Exclusion

$$R_{\rm vir}(M) < r/2$$

$$R_{\rm vir}(M_1)+R_{\rm vir}(M_2) < r$$

The 2-point correlation function

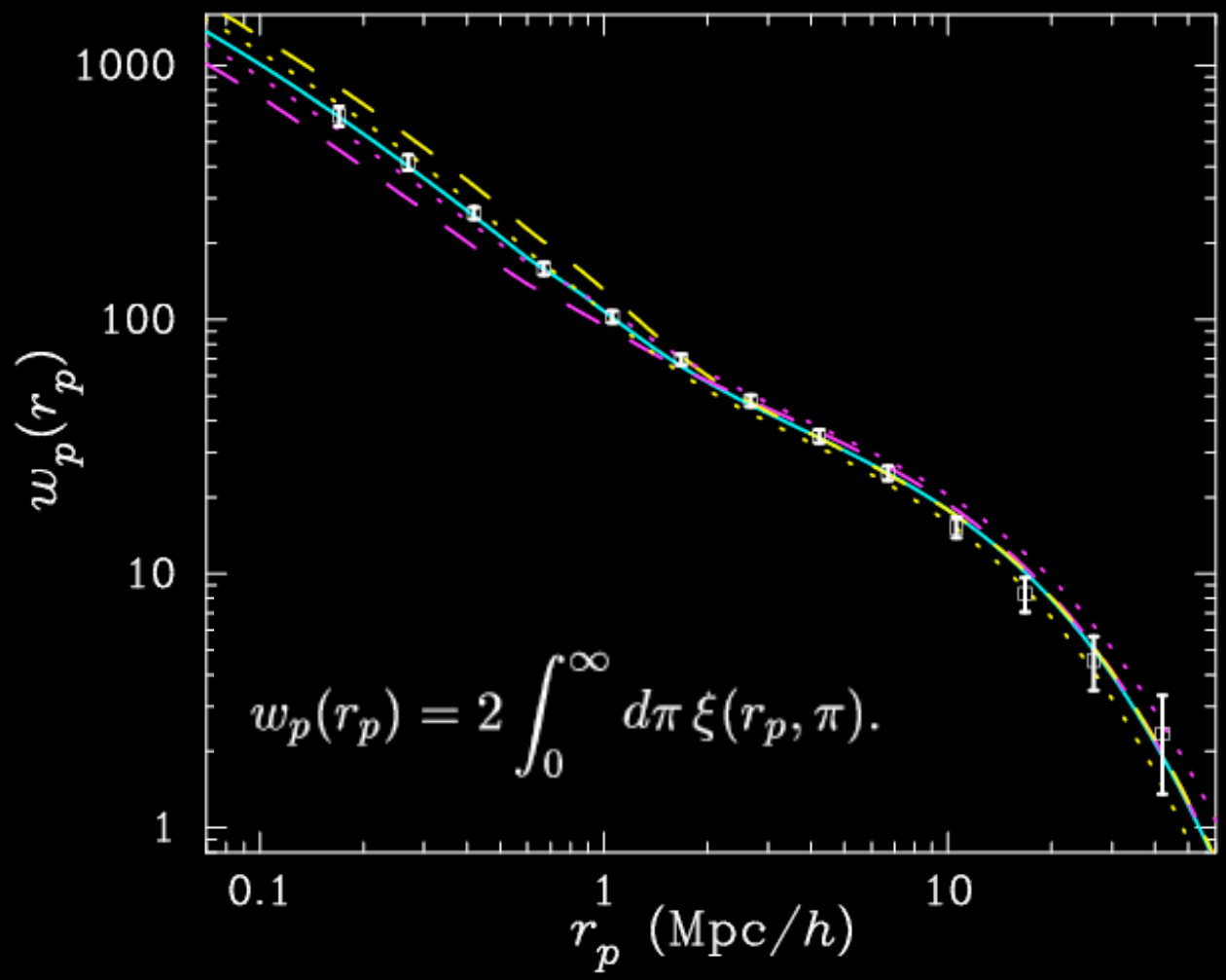




$$w_p(r_p) = 2\int_0^\infty \xi\left[(r_p^2 + y^2)^{1/2}\right] dy,$$

Zehavi et al 2001

Unprecedented precision



Zehavi et al., 2004

$M_{0.1r} < -21$

A high-statistics measurement of the galaxy correlation function over a large range of scale

A complete descriptor of the galaxy correlation function on a wide range of scales:

Galaxy correlation function:

$$\xi_{\rm gg}(r) = \xi_{\rm gg}^{\rm 1h}(r) + \xi_{\rm gg}^{\rm 2h}(r) + 1.$$

$$1 + \xi_{\rm gg}^{\rm 1h}(r) = \frac{1}{2}\,\bar{n}_{\rm g}^{-2} \int n(M)\langle N(N-1)\rangle_M \lambda(r|M)\,dM;$$

$$\xi_{\rm gg}^{\rm 2h}(r) = \xi_{\rm mm}^{\rm lin}(r)\;\bar{n}_{\rm g}^{-2} \int n(M_1) b_h(M_1)\langle N\rangle_{M_1}\,dM_1$$

$$\int n(M_2) b_h(M_2)\langle N\rangle_{M_2} \lambda(r|M_1, M_2)\,dM_2$$

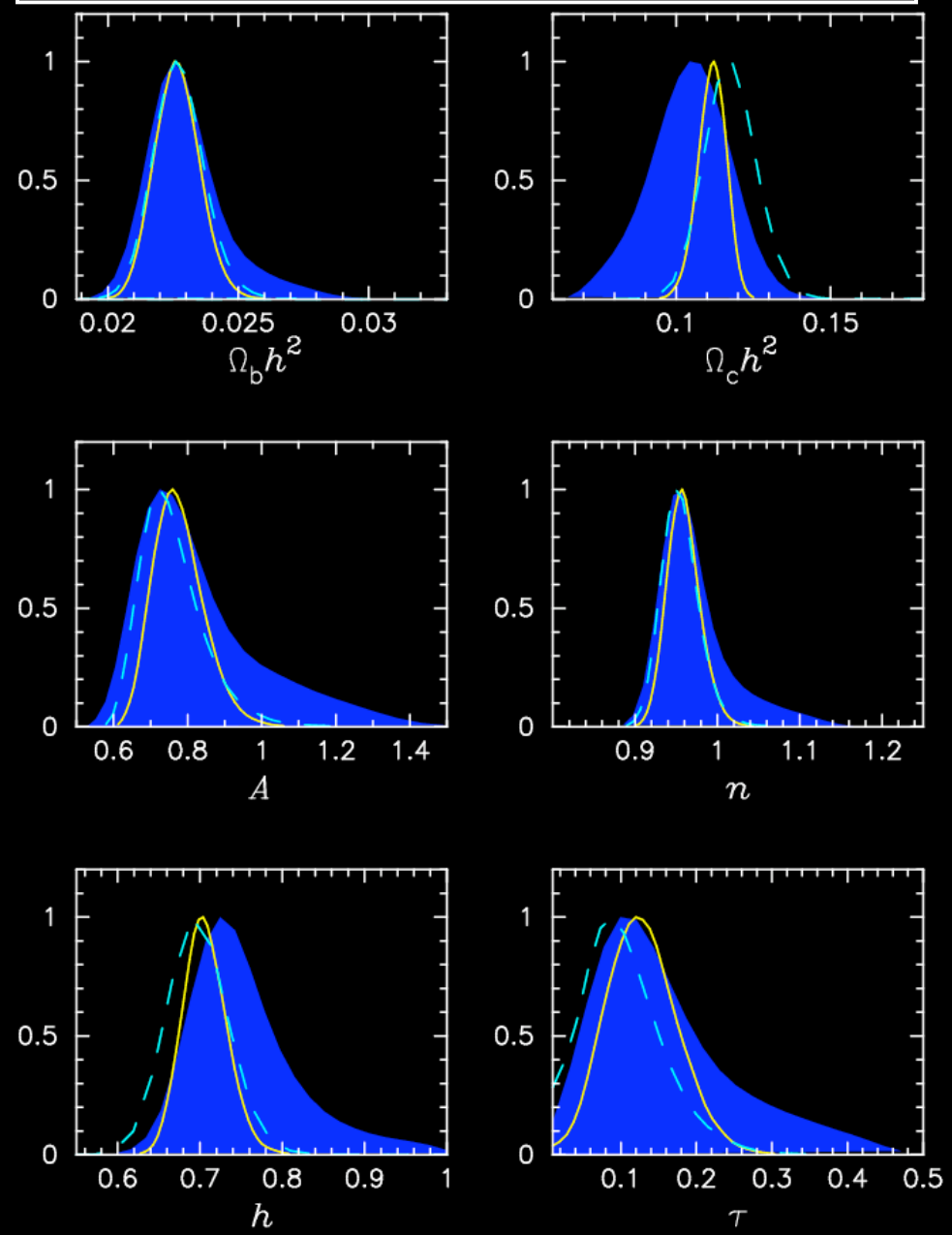
First and higher order moments of HOD:

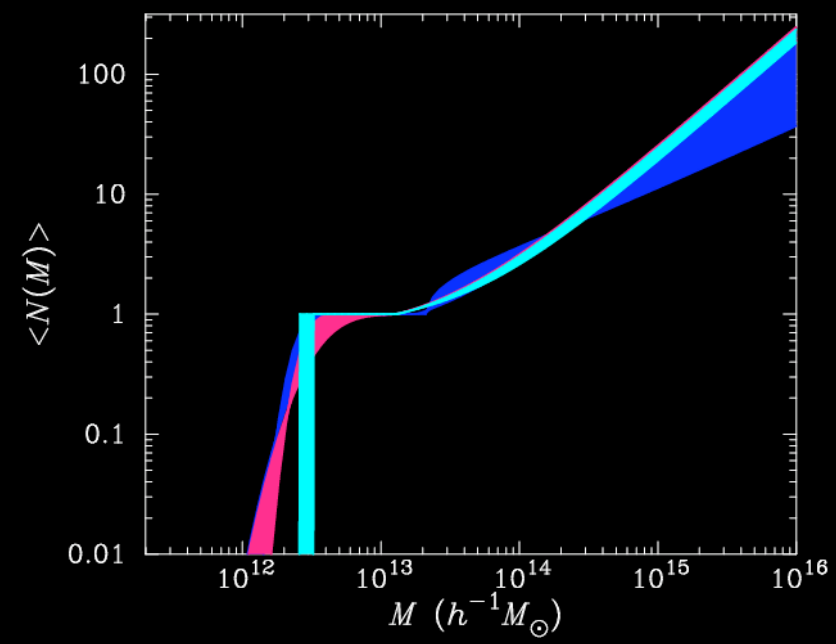
$$\langle N\rangle_M = \sum_N N\,P(N|M),$$

$$\langle N(N-1)...(N-j)\rangle = \langle N\rangle^{j+1}.$$

Cosmology plus Galaxies:

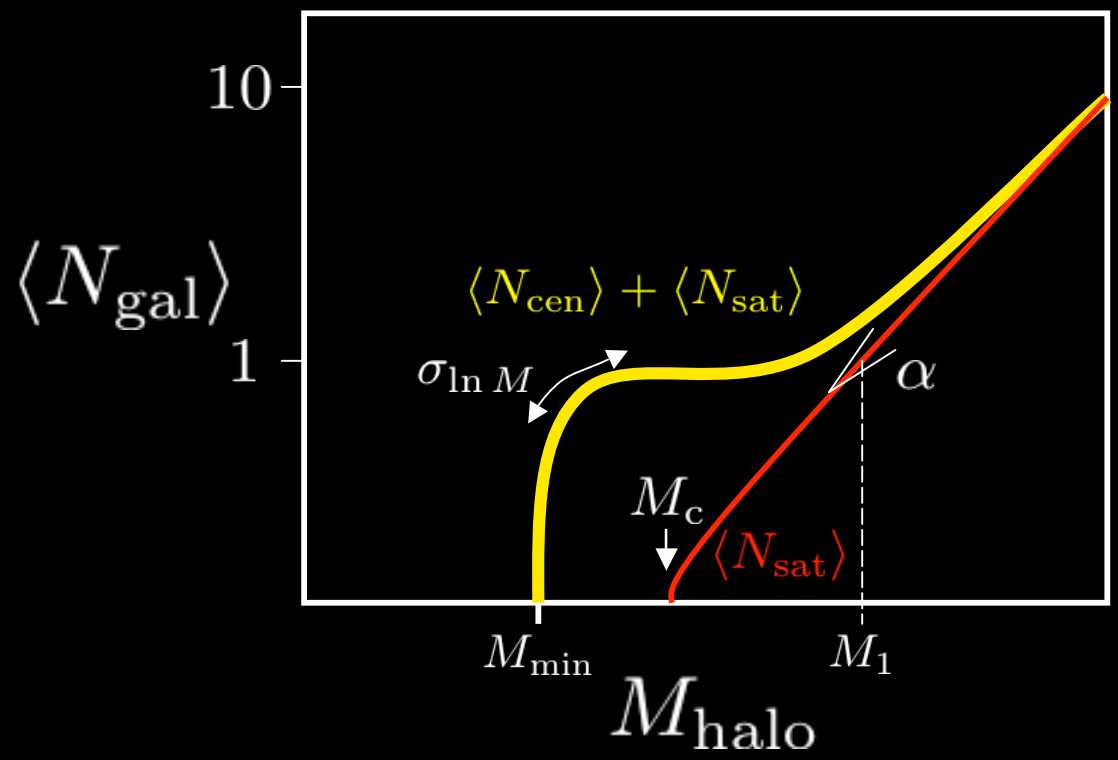
$p=(\theta, \Omega_b h^2, \Omega_c h^2, A_s, n_s, M_1, \alpha)$





Parameter	CMB+$w_p(r_p)$ $2p$	CMB+SDSS 3D $P_g(k)$	CMB
A	$0.768^{+0.068}_{-0.063}$	$0.747^{+0.077}_{-0.071}$	$0.79^{+0.15}_{-0.12}$
n	$0.959^{+0.019}_{-0.020}$	$0.956^{+0.020}_{-0.021}$	$0.974^{+0.037}_{-0.036}$
τ	$0.129^{+0.020}_{-0.024}$	$0.105^{+0.017}_{-0.028}$	$0.158^{+0.093}_{-0.084}$
h	$0.706^{+0.026}_{-0.026}$	$0.697^{+0.028}_{-0.028}$	$0.755^{+0.059}_{-0.058}$
$\Omega_c h^2$	$0.1115^{+0.0045}_{-0.0044}$	$0.1176^{+0.0069}_{-0.0069}$	$0.104^{+0.013}_{-0.013}$
$\Omega_b h^2$	$0.0226^{+0.0008}_{-0.0009}$	$0.0227^{+0.0009}_{-0.0009}$	$0.0230^{+0.0013}_{-0.0013}$
$M_1[10^{13}h^{-1}M_\odot]$	$4.51^{+0.54}_{-0.54}$	–	–
κ	$4.70^{+0.52}_{-0.74}$	–	–
$\sigma_{\rm cen}$	0	–	–
α	1	–	–
$M_{\rm min}[10^{12}h^{-1}M_\odot]$	$2.94^{+0.33}_{-0.33}$	–	–
Ω_m	$0.271^{+0.026}_{-0.027}$	$0.291^{+0.034}_{-0.034}$	$0.231^{+0.054}_{-0.056}$
σ_8	$0.821^{+0.028}_{-0.028}$	$0.834^{+0.049}_{-0.050}$	$0.802^{+0.072}_{-0.073}$
$\chi^2_{\rm eff}$/DOF	1459.6/1376	1477.2/1383	1452.5/1365
AIC	1475.6	1491.2	1464.5
BIC	1525.6	1527.9	1495.8

How many parameters are needed for this sample's HOD?



2p: M_1, κ
3p: M_1, κ, σ_{cen}
4p: M_1, κ, σ_{cen},α

$$\kappa = \frac{M}{M_c}$$

Aikake and Bayesian Information Criteria:

(see Liddle 2004)

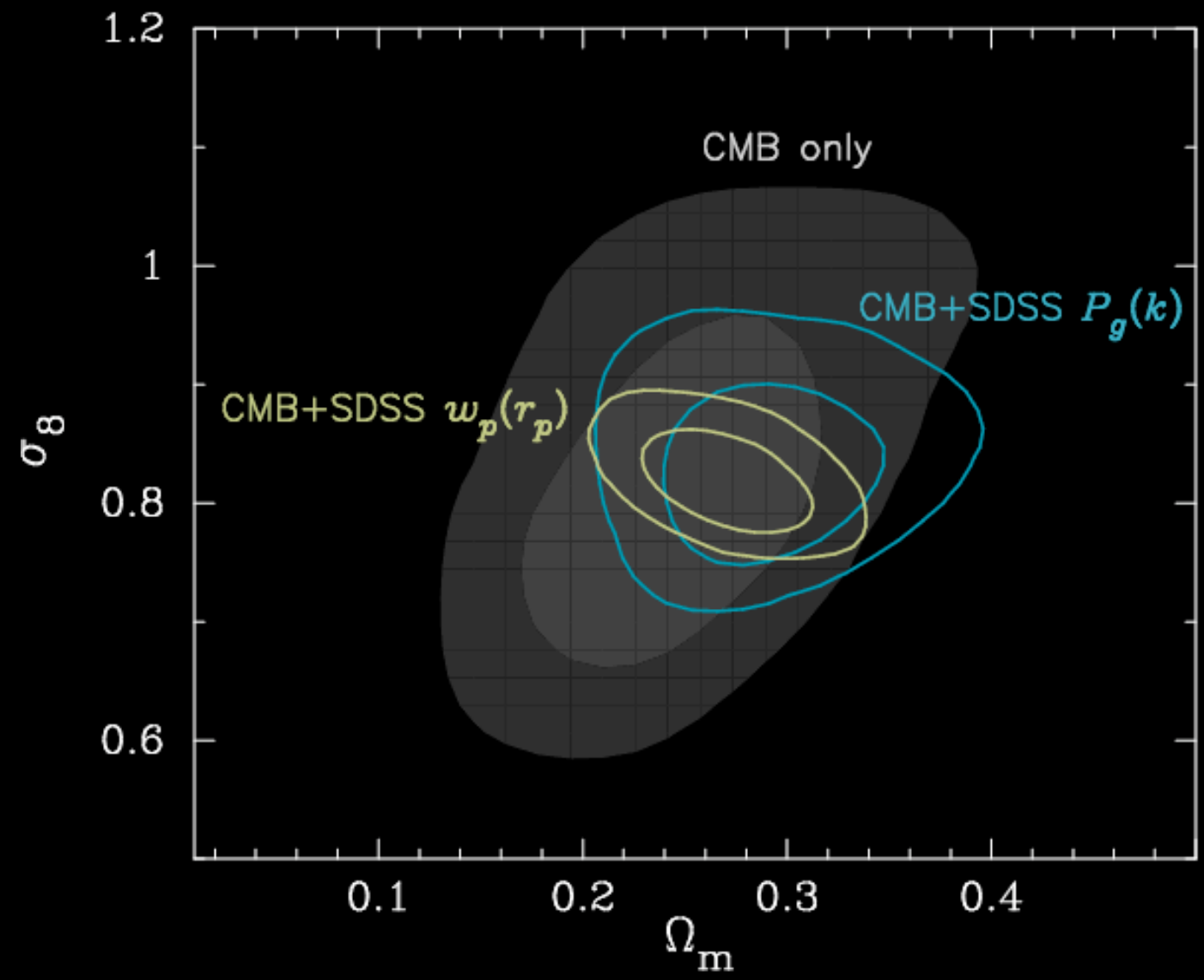
$$\mathrm{AIC} = -2 \ln \mathcal{L} + 2k$$

$$\mathrm{BIC} = -2 \ln \mathcal{L} + k \ln N$$

Information Criteria & Parameters

Parameter	CMB+$w_p(r_p)$ $2p$	CMB+$w_p(r_p)$ $3p$	CMB+$w_p(r_p)$ $4p$
A	$0.768^{+0.068}_{-0.063}$	$0.782^{+0.077}_{-0.072}$	$0.830^{+0.115}_{-0.100}$
n	$0.959^{+0.019}_{-0.020}$	$0.963^{+0.022}_{-0.022}$	$0.977^{+0.031}_{-0.031}$
τ	$0.129^{+0.020}_{-0.024}$	$0.141^{+0.021}_{-0.029}$	$0.173^{+0.029}_{-0.038}$
h	$0.706^{+0.026}_{-0.026}$	$0.715^{+0.030}_{-0.031}$	$0.732^{+0.040}_{-0.040}$
$\Omega_c h^2$	$0.1115^{+0.0045}_{-0.0044}$	$0.1097^{+0.0053}_{-0.0052}$	$0.1072^{+0.0064}_{-0.0064}$
$\Omega_b h^2$	$0.0226^{+0.0008}_{-0.0009}$	$0.0228^{+0.0009}_{-0.0009}$	$0.0232^{+0.0011}_{-0.0011}$
$M_1[10^{13}h^{-1}M_\odot]$	$4.51^{+0.54}_{-0.54}$	$4.38^{+0.56}_{-0.56}$	$2.96^{+0.72}_{-0.88}$
κ	$4.70^{+0.52}_{-0.74}$	$4.22^{+0.53}_{-0.73}$	$7.2^{+1.4}_{-2.0}$
$\sigma_{\rm cen}$	0	$0.43^{+0.11}_{-0.18}$	$0.39^{+0.11}_{-0.17}$
α	1	1	$0.79^{+0.23}_{-0.23}$
$M_{\rm min}[10^{12}h^{-1}M_\odot]$	$2.94^{+0.33}_{-0.33}$	$3.17^{+0.41}_{-0.43}$	$2.92^{+0.19}_{-0.23}$
Ω_m	$0.271^{+0.026}_{-0.027}$	$0.262^{+0.030}_{-0.030}$	$0.246^{+0.036}_{-0.037}$
σ_8	$0.821^{+0.028}_{-0.028}$	$0.821^{+0.030}_{-0.030}$	$0.838^{+0.039}_{-0.039}$
$\chi^2_{\rm eff}$/DOF	1459.6/1376	1459.5/1375	1459.5/1374
AIC	1475.6	1477.5	1479.5
BIC	1525.6	1517.5	1524.6

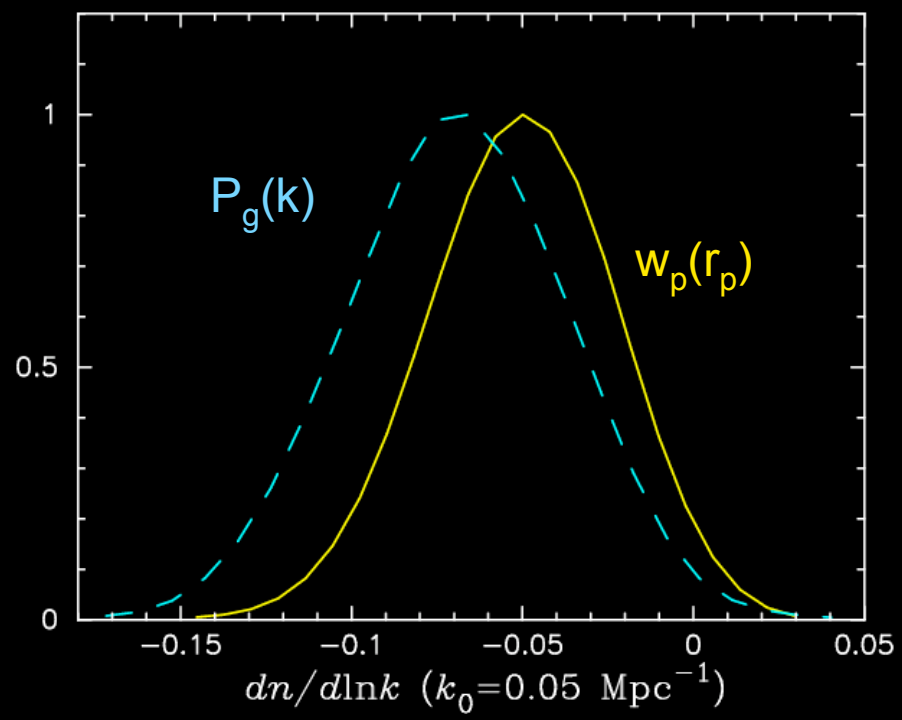
σ_8 vs. Ω_m



	CMB + w_p (M<-21)	CMB + SDSS $P_g(k)$
$\Omega_b h^2$	0.023±0.001	0.023±0.001
$\Omega_c h^2$	0.112±0.005	0.118±0.007
n_s	0.96±0.02	0.96±0.02
A	0.77±0.07	0.75±0.08
τ	0.10±0.04	0.10±0.05
Ω_m	0.27±0.03	0.29±0.04
σ_8	0.82±0.08	0.83±0.05
H_0	71±3 km s^{-1} Mpc^{-1}	70±3 km s^{-1} Mpc^{-1}
M_1	$(4.51\pm0.55)\times10^{13}$ M_o	-
κ	4.7±1.4	-
M_{min}	$(2.9\pm0.3)\times10^{12}$ M_o	-
b_g	1.45±0.09	-

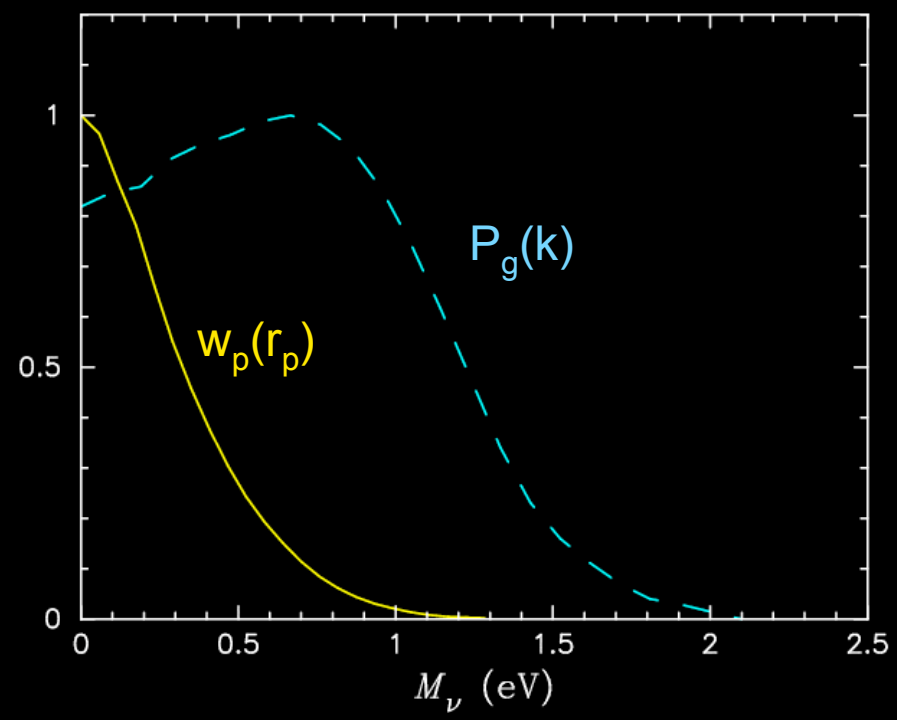
Extensions to ΛCDM: massive neutrinos & scalar spectral index running

6 cosmological parameters + dn/dlnk



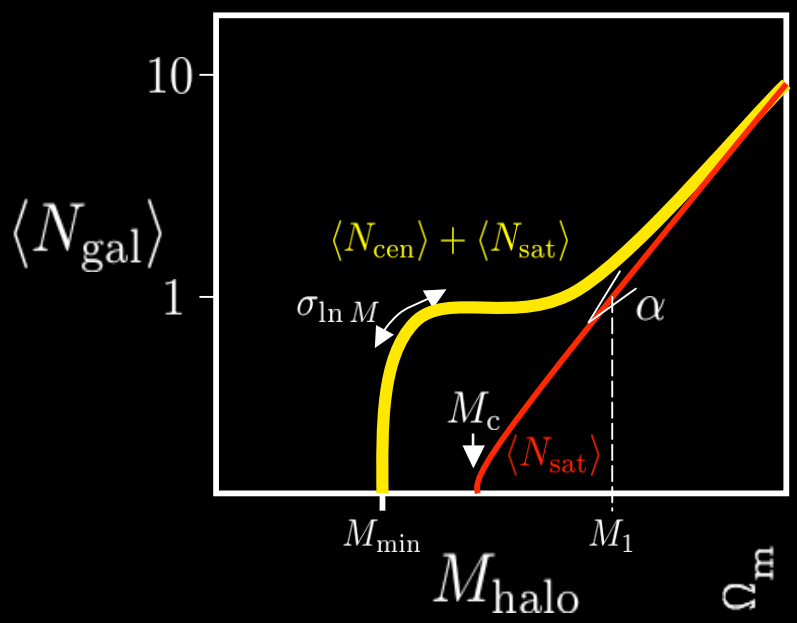
$$dn/d\ln k = -0.051 \pm 0.027$$

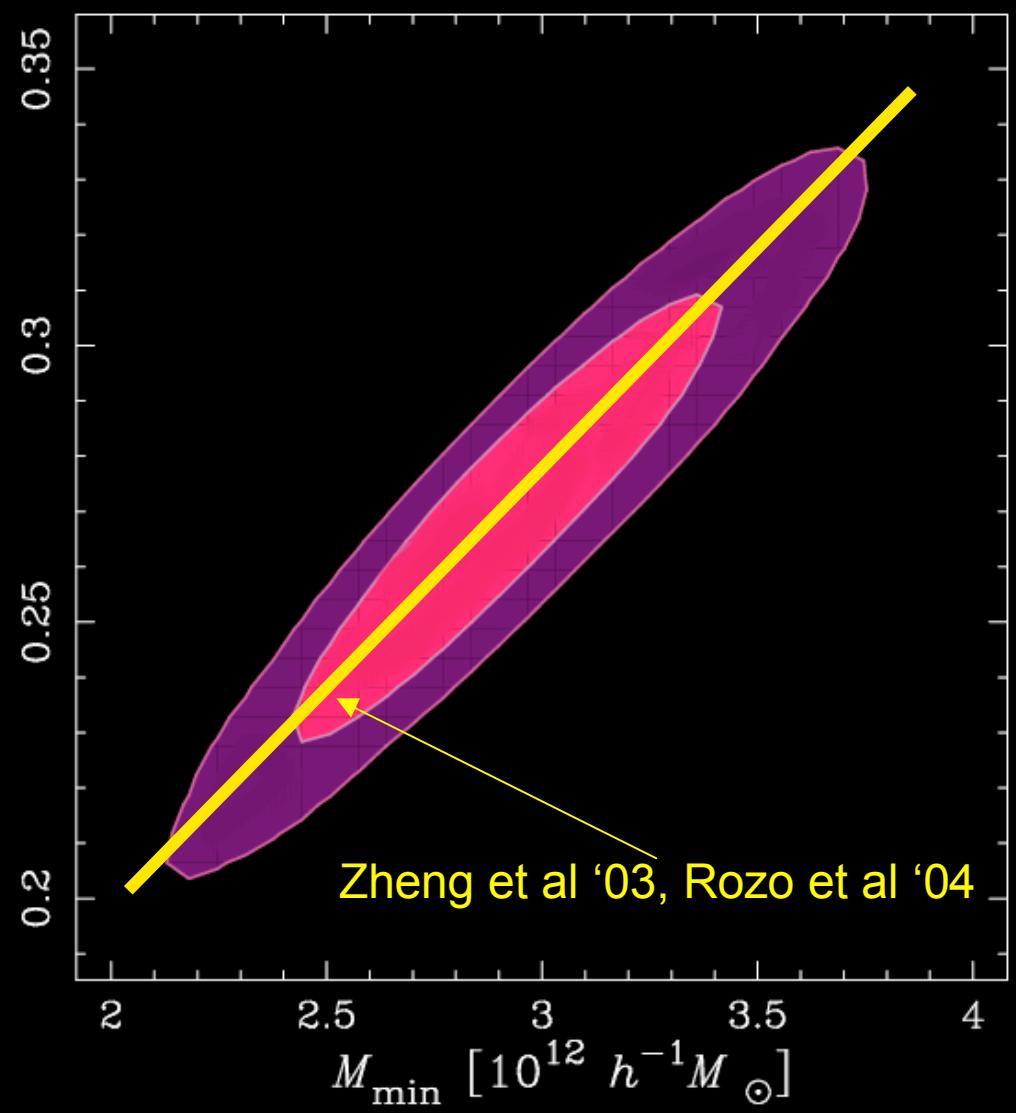
6 cosmological parameters + M_ν



$$m_\nu < 0.23\,\mathrm{eV}\ (95\%\ \mathrm{C.L.})$$

Degeneracy - Ω_m & M_{min}





Power Spectrum and Bias

$$P_{\rm gal}(k) = P^{1h}_{\rm gal}(k) + P^{2h}_{\rm gal}(k)\,, \qquad \text{where}$$

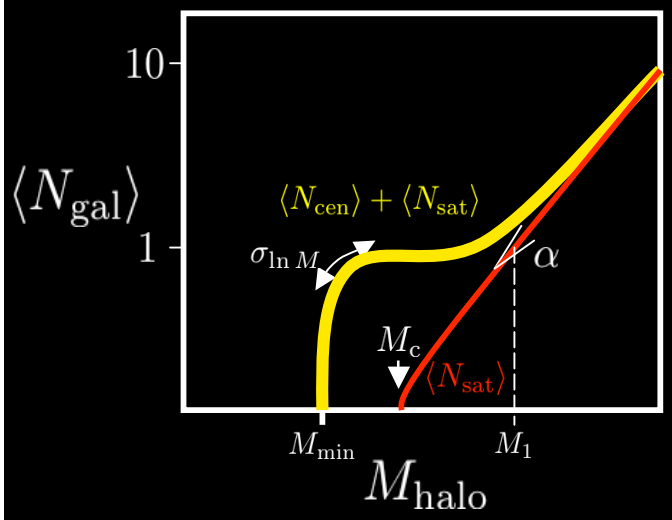
$$P^{1h}_{\rm gal}(k) = \int dm\, n(m)\, \frac{\langle N_{\rm gal}(N_{\rm gal}-1)|m\rangle}{\bar{n}^2_{\rm gal}}\, |u_{\rm gal}(k|m)|^p\,,$$

$$P^{2h}_{\rm gal}(k) \approx P^{\rm lin}(k) \left[\int dm\, n(m)\, b_1(m)\, \frac{\langle N_{\rm gal}|m\rangle}{\bar{n}_{\rm gal}}\, u_{\rm gal}(k|m)\right]^2 .$$

Gives bias for respective galaxy sample: $P_g(k)$

$$b_{\rm gal} = \int dm\, n(m)\, b_1(m)\, \frac{\langle N_{\rm gal}|m\rangle}{\bar{n}_{\rm gal}}$$

Galaxies to dark matter …



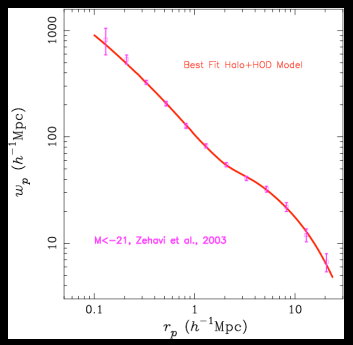
$$w_p(r_p) \Leftrightarrow \xi(r)$$

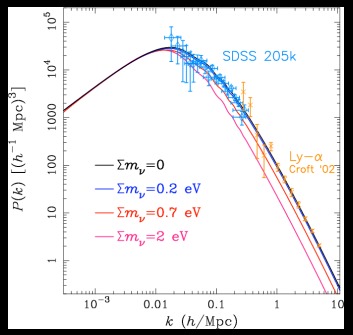
+

$$P_g(k)$$

+

Multiplicity + N-point
+ redshift distortions





To cosmology…

Ω_m Ω_Λ τ/z_{re} H_0

To fundamental physics…

A_s n_s n_{run} m_ν