Extracting parameters from galaxy redshift-distance surveys

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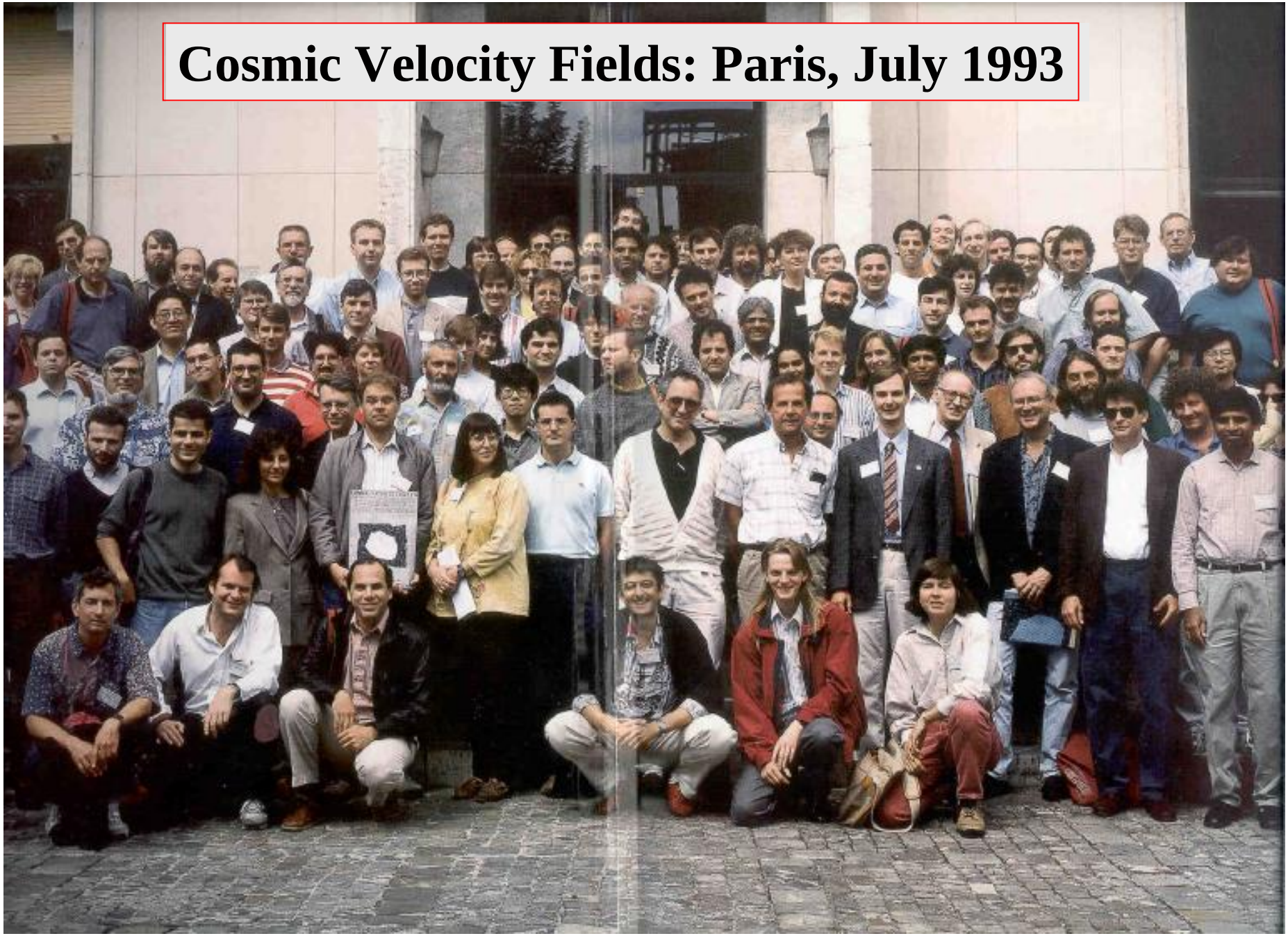
Cosmic Velocity Fields: Paris, July 1993



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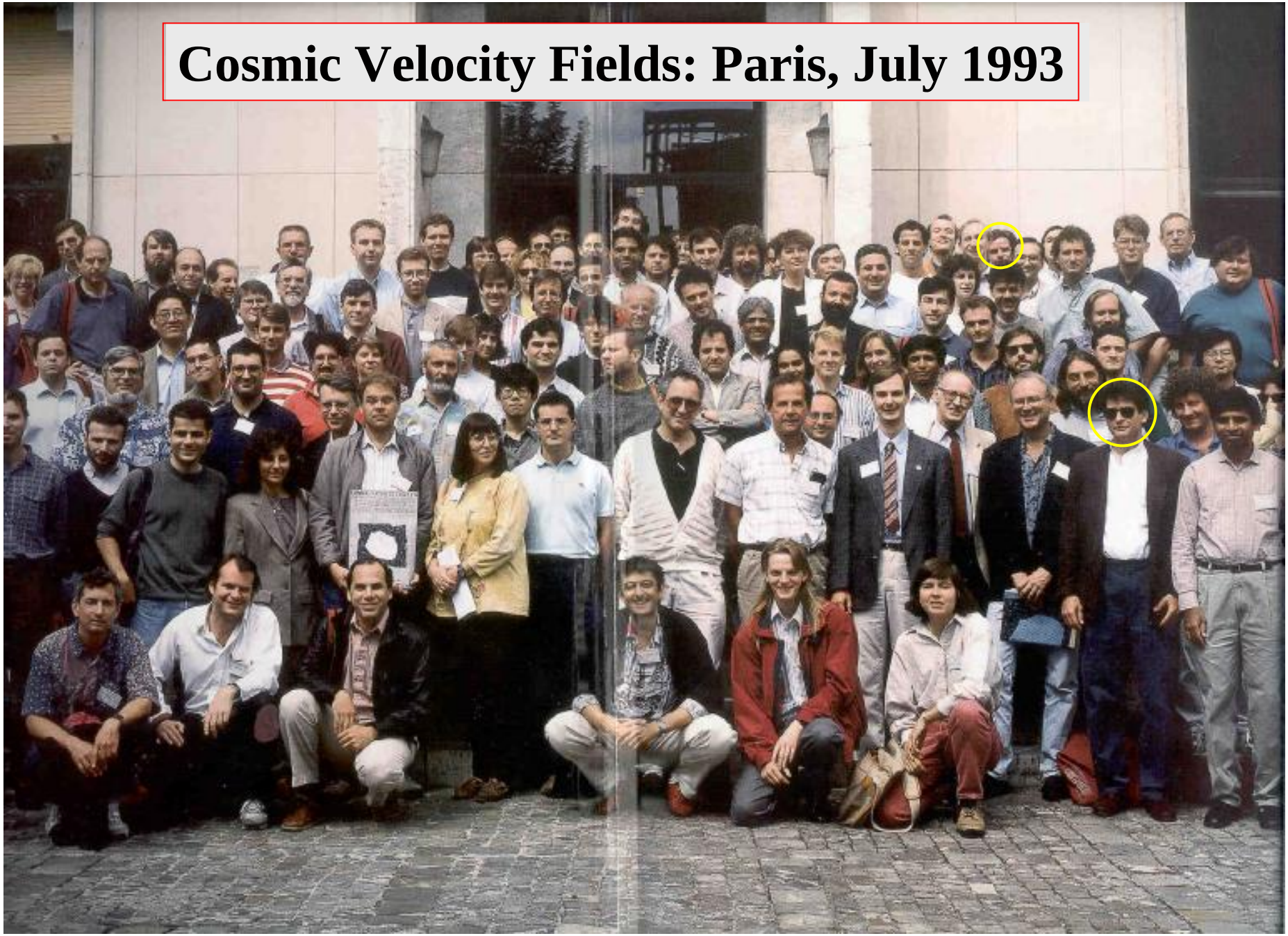

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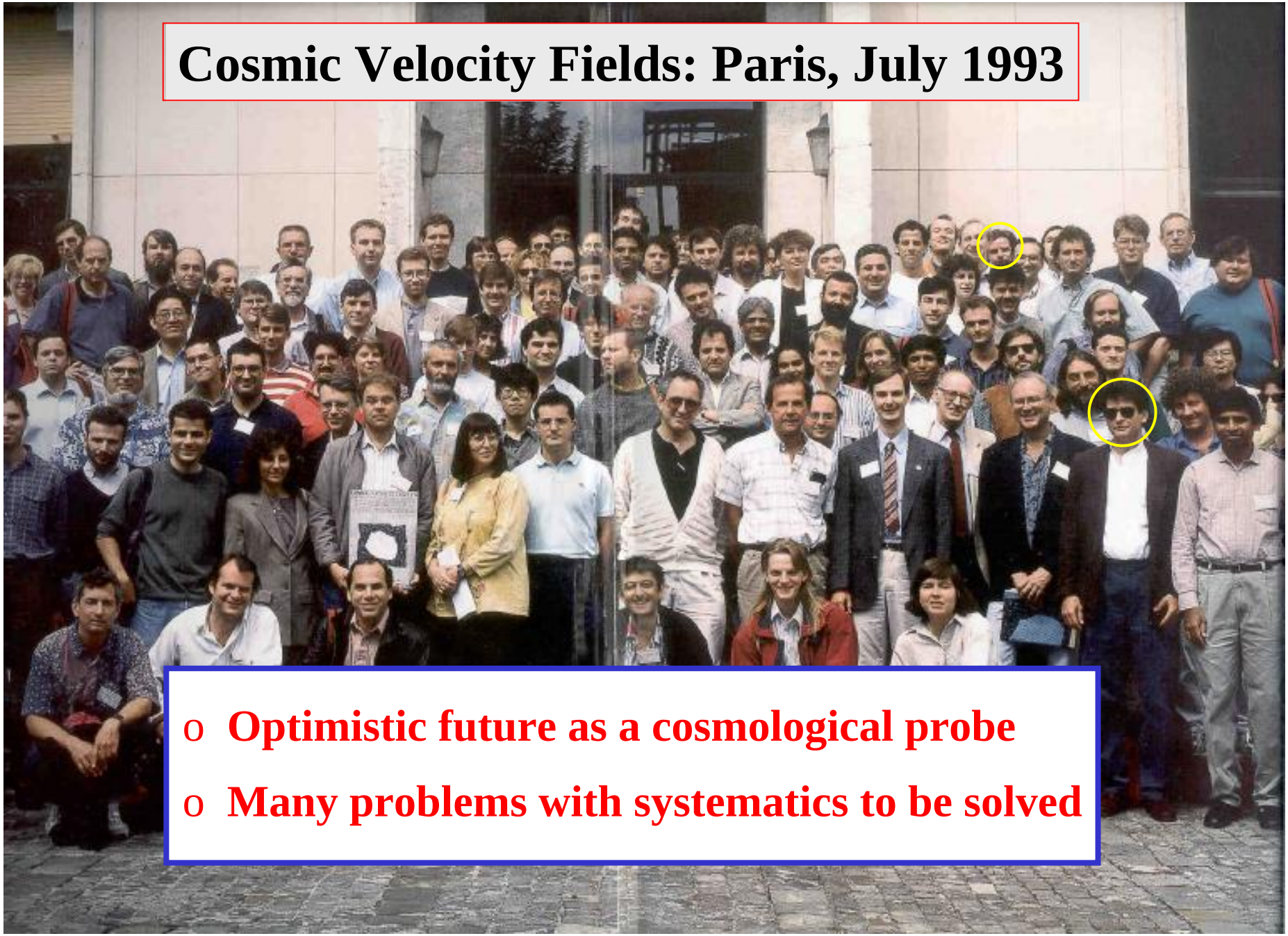

- Optimistic future as a cosmological probe
- Many problems with systematics to be solved

Peculiar Velocities as Cosmological Probes

We can constrain cosmological parameters with peculiar velocities via:-

- **Bulk flow statistics**
- **Pairwise velocities**
- **Velocity-density reconstructions**
  - **POTENT**
  - **VELMOD / ITF**

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Distance Indicators and Datasets

- Tully-Fisher
  Mark III, SFI, SCI, SCII, Shellflow
- Fundamental Plane
  Mark III, SMAC, ENEAR, EFAR

Individual distance errors of 20 – 30%;

$O(10^3)$ galaxies

- SBF
  Tonry et al.
- Type Ia SNe
  Riess et al.

Individual distance errors of 5 – 10%;

$O(10^2)$ galaxies

Velocity – Density Reconstructions

We can compare observed peculiar velocities with the reconstructed density and velocity field from all-sky redshift surveys, via linear theory relations:-

$$\mathbf{v}_{\text{pec}}(\mathbf{r}) = \frac{\Omega_m^{0.6}}{4\pi} \int d^3\mathbf{r}' \frac{\delta(\mathbf{r}')(\mathbf{r}' - \mathbf{r})}{|\mathbf{r}' - \mathbf{r}|^3}$$

$$\nabla \cdot \mathbf{v}_{\text{pec}} = -\Omega_m^{0.6}\,\delta$$

- **density-density comparisons**
- **velocity-velocity comparisons**

Velocity – density reconstructions

Methods can be vulnerable to a number of statistical biases:-

- Calibration bias
- Malmquist bias
- Tensor window bias
- Sampling gradient bias

See e.g. Strauss & Willick (1995), Hendry & Simmons (1995), Hendry (2001)

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True distance distribution

Selection effects

True distance modulus

$$P(r,m,\eta) \propto r^2 n(r) S(m,\eta) \times \phi(\eta) \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left[m - \left(M(\eta) + \mu(r)\right)\right]^2}{2\sigma^2} \right)$$

Line width distribution

True distance distribution

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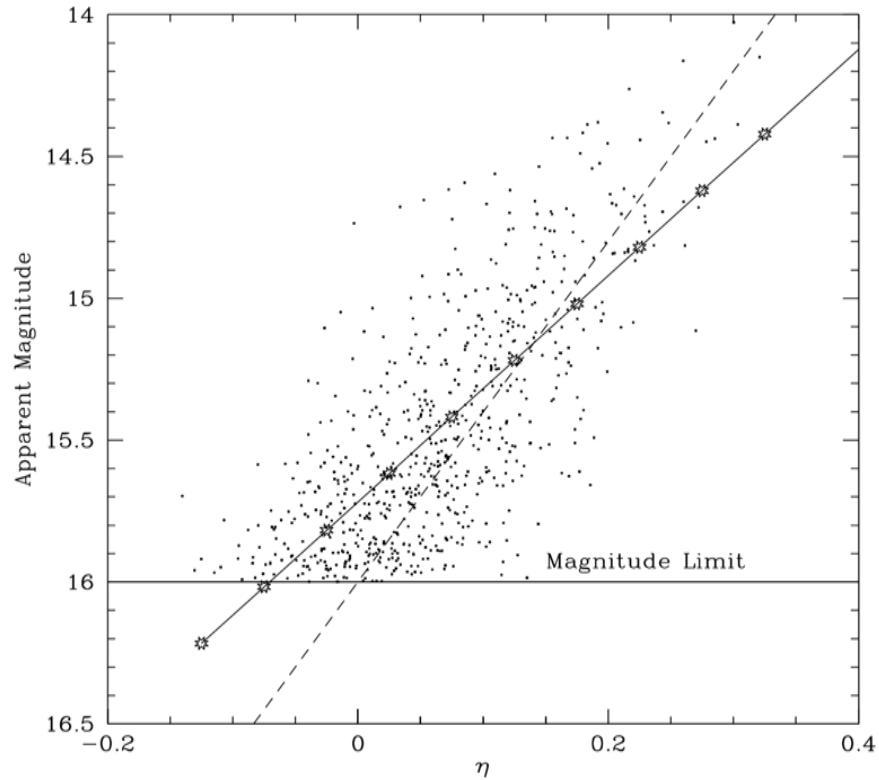
Line width distribution

True line width relation

$$M(\eta) = a\eta + b$$

Calibrate in a *cluster* – assume all galaxies at the same distance:-



From Strauss & Willick

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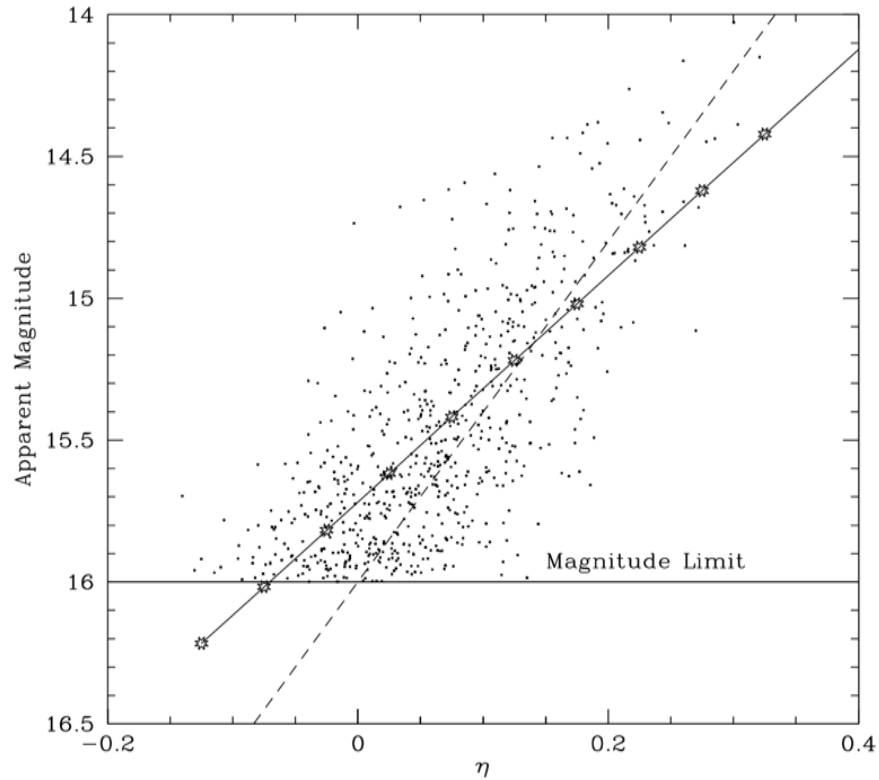

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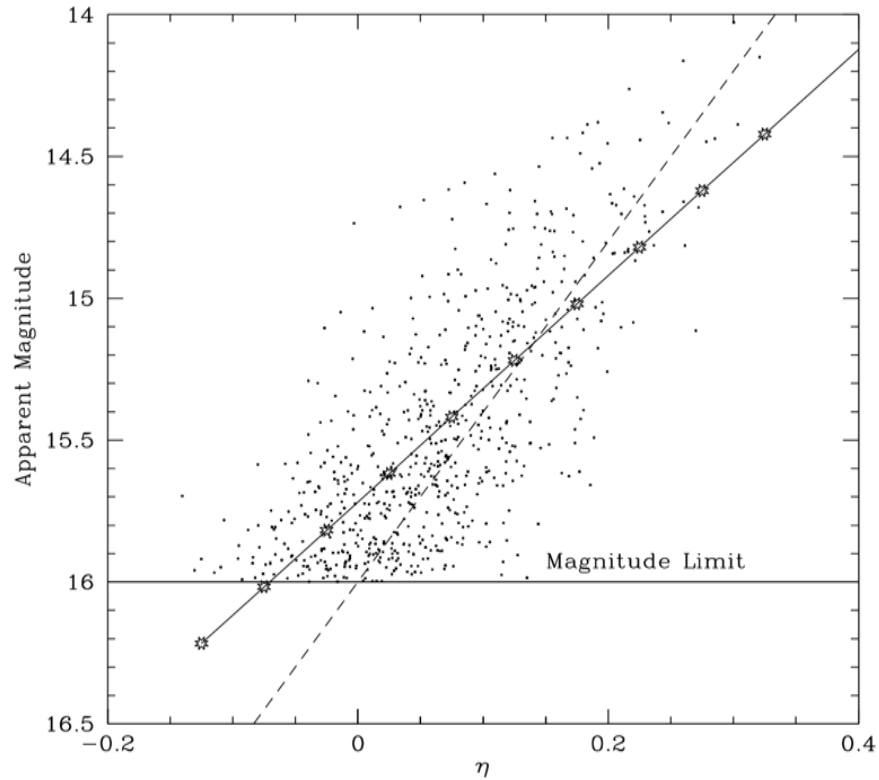
Biases fitted parameters, but can be corrected iteratively



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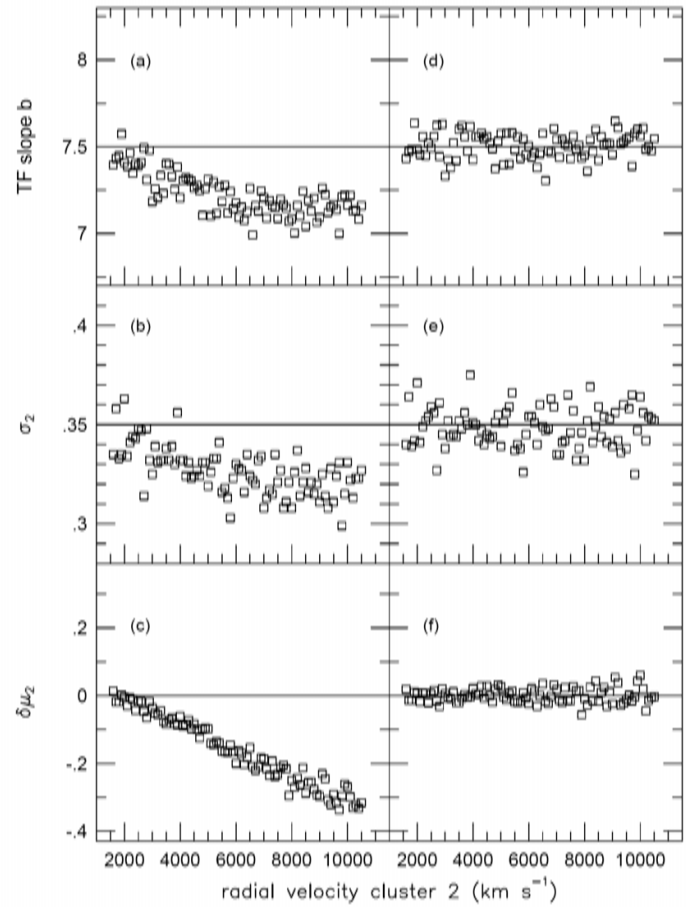

From Strauss & Willick

Could also use *inverse relation* $\boxed{\eta^0(M) = a'M + b'}$

True distance distribution

Selection effects

True distance modulus

$$P(r,m,\eta) \propto r^2 n(r) S(m,\eta)$$

$$\times \Phi\left(m - \mu(r)\right) \frac{1}{\sqrt{2\pi}\sigma_\eta} \exp\left(- \frac{\left[\eta - \eta^0\left(m - \mu(r)\right)\right]^2}{2\sigma_\eta^{\ 2}} \right)$$

Luminosity function

Could also use *inverse relation*

$$\boxed{\eta^0(M) = a'M + b'}$$

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Luminosity function

$$P(\eta \mid m,r) = \frac{P(r,m,\eta)}{\int_{-\infty}^{\infty} P(r,m,\eta)\,d\eta}$$

$$= \frac{S(m,\eta)\exp\left(-\frac{\left[\eta-\eta^0(m-\mu(r))\right]^2}{2\sigma^2}\right)}{\int_{-\infty}^{\infty} S(m,\eta)\exp\left(-\frac{\left[\eta-\eta^0(m-\mu(r))\right]^2}{2\sigma^2}\right)d\eta}$$

No calibration bias provided selection effects **only** on apparent magnitude

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$$\boxed{\eta^0(M) = a'M + b'}$$

True distance distribution

Selection effects

True distance modulus

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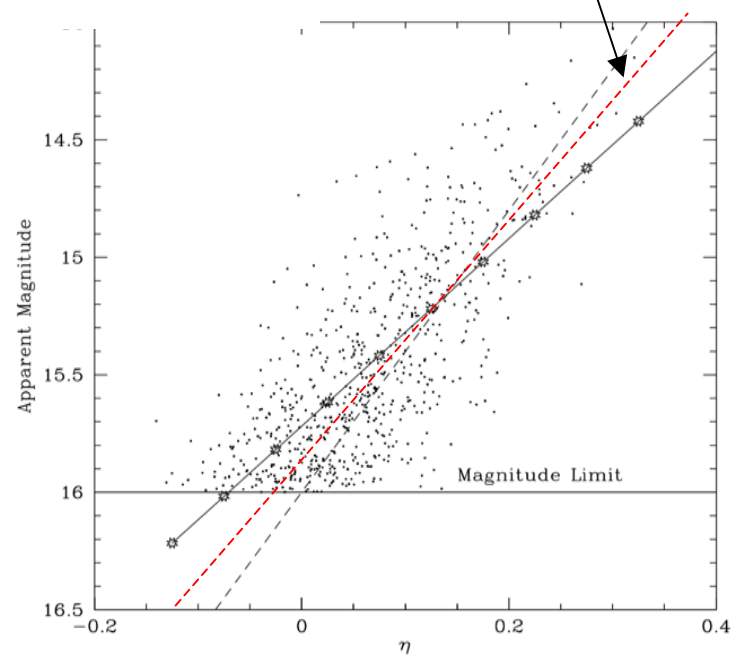
$E(\eta \mid M)$



Velocity – density reconstructions

Methods can be vulnerable to a number of statistical biases:-

- Calibration bias
- Malmquist bias
- Tensor window bias
- Sampling gradient bias

See e.g. Strauss & Willick (1995), Hendry & Simmons (1995), Hendry (2001)

Malmquist bias

Given a properly calibrated distance indicator relation, we construct a distance estimator for more remote galaxies in the obvious way:-

Direct relation:-

$$\hat{M} = a\eta_{\text{obs}} + b$$

Inverse relation:-

$$\hat{M} \quad \text{satisfying} \quad \eta_{\text{obs}} = a'\hat{M} + b'$$

$$d = 10^{0.2(m_{\text{obs}} - \hat{M})}$$

How does d *compare with* r *?*

Malmquist bias

Since $d = d(m,\eta)$ we can compute $p(r,d)$ from $p(r,m,\eta)$

Direct relation:-

$$P(r\,|\,d) = \frac{P(r,d)}{\int_0^\infty P(r,d)\,dr} = \frac{r^2 n(r)\exp\left(-\frac{[\ln r/d]^2}{2\Delta^2}\right)}{\int_0^\infty r^2 n(r)\exp\left(-\frac{[\ln r/d]^2}{2\Delta^2}\right)dr}$$

where

$$\Delta \equiv \left(\frac{\ln 10}{5}\right)\sigma \cong 0.46\sigma$$

True distance distribution

Selection effects

True distance modulus

$$P(r,m,\eta) \propto r^2 n(r) S(m,\eta) \times \phi(\eta)\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{[m-(M(\eta)+\mu(r))]^2}{2\sigma^2}\right)$$

Line width distribution

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Hence $E(r\,|\,d) \neq d$

independent of selection effects, but dependent on **true distance distribution**

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Assuming $n(r) = n_0 \quad \Rightarrow \quad E(r\,|\,d) = d\exp\left(\tfrac{7}{2}\Delta^2\right)$

Homogeneous Malmquist correction

Malmquist bias

Since $d = d(m,\eta)$ we can compute $p(r,d)$ from $p(r,m,\eta)$

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where

$$s(r) \equiv \int_{-\infty}^{\infty} \mathrm{d}m\, \Phi(m-\mu(r))\, S(m, \eta^0[m-\mu(r)])$$

True distance distribution

Selection effects

True distance modulus

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Luminosity function

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Can estimate inhomogenous corrections using observed $P(d)$ - (Landy & Szalay)

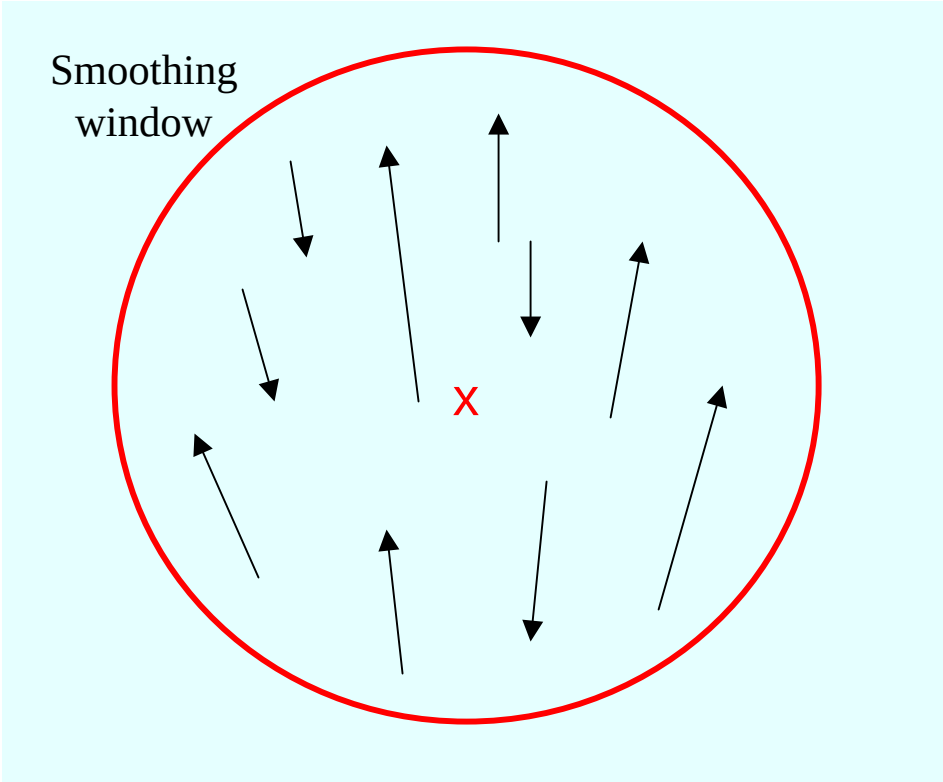
Density – density comparisons

Archetype is POTENT (Bertschinger & Dekel 1988; Dekel et al 1999)

$$\mathbf{v}_{\mathbf{pec}} = -\nabla\Phi_V$$

$$\Phi_V(\mathbf{r}) = -\int_0^{\mathbf{r}} u(r',\theta,\phi)\,dr'$$

Need *only* radial components, but everywhere! Interpolate $u(\mathbf{r})$ on a regular grid



Density – density comparisons

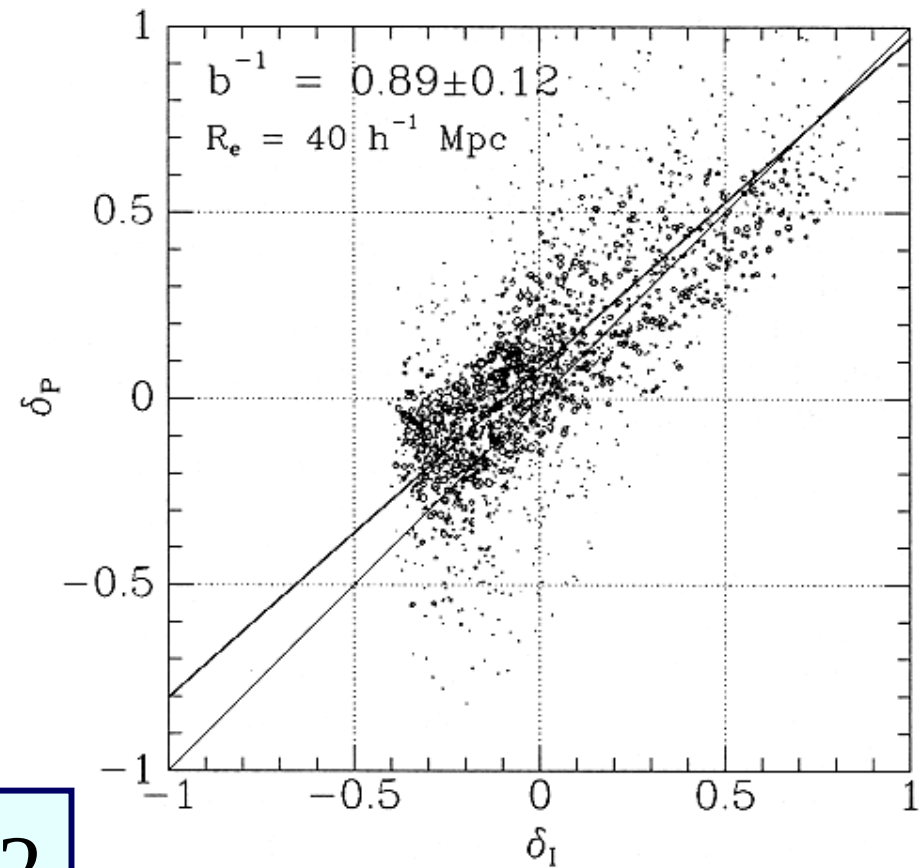
Archetype is POTENT (Bertschinger & Dekel 1988; Dekel et al 1999)

Compare $\mathbf{v}_{\mathbf{pec}}$ with e.g. IRAS δ-field. Assume *linear biasing:* $\delta_{\mathrm{gal}} = b\,\delta$

$\nabla \cdot \mathbf{v}_{\mathbf{pec}}$ versus δ

has slope $\beta = \Omega_m^{0.6} / b$

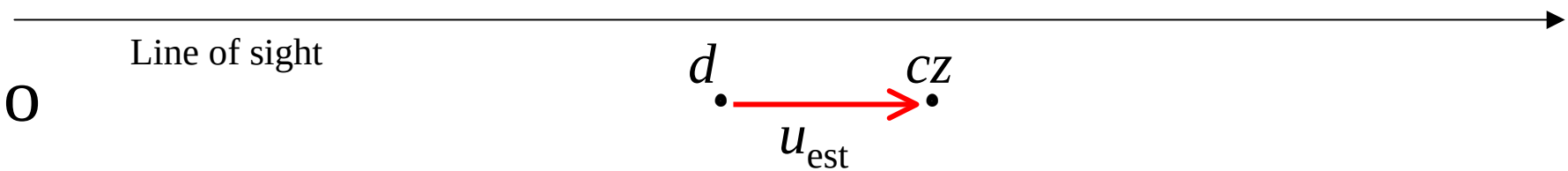
$$\beta_{\mathrm{I}} = 0.89 \pm 0.12$$



Sigad et al. (1998)

Inhomogeneous Malmquist bias

Interpolate $u(\mathbf{r})$ on a *real space* grid



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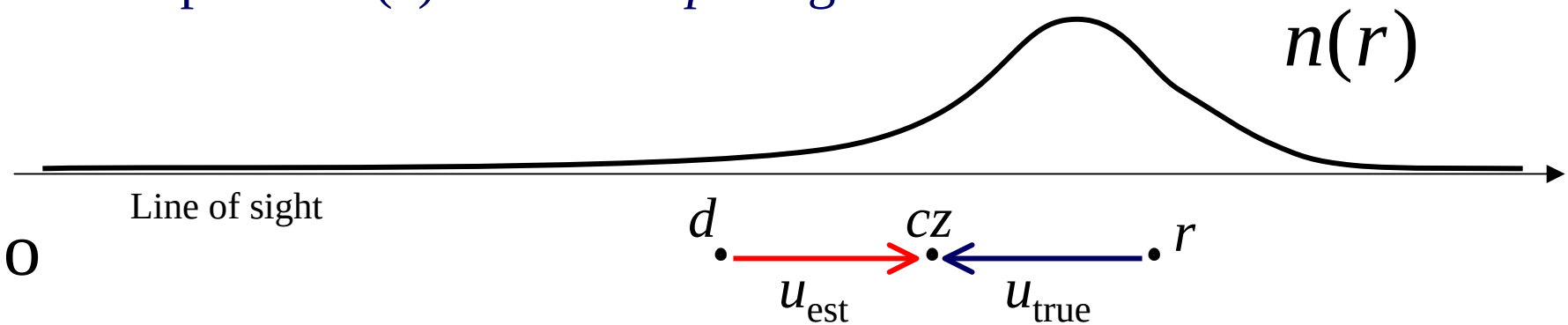

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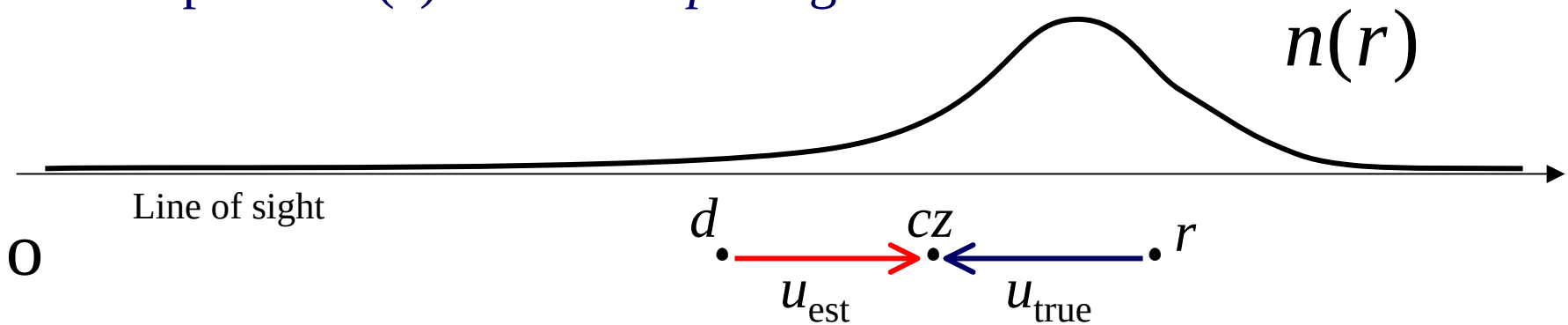


In general $$E(r \mid d) \neq d$$

Bias correction depends on $n(r)$

Velocity – velocity comparisons

Archetype is VELMOD (Willick & Strauss 1997, Willick et al 1998)

Maximise likelihood of observing Tully-Fisher data, given a velocity field and TF model

'Forward' VELMOD

$$L = \prod p(m_i \mid \eta_i, cz_i; \Theta)$$

'Inverse' VELMOD

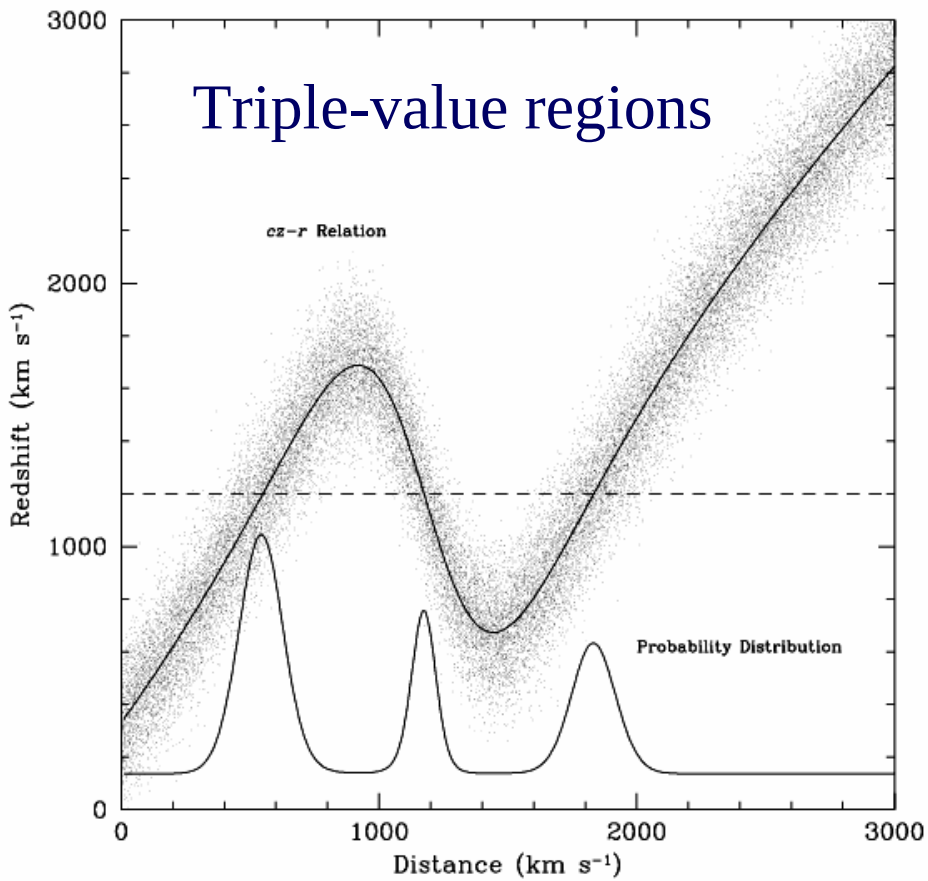
$$L = \prod p(\eta_i \mid m_i, cz_i; \Theta)$$

Θ = parameters of TF relation and velocity model

VELMOD also requires a parametric model for

$S(m, \eta, r)$, LF, $p(cz \mid r)$

Velocity – velocity comparisons



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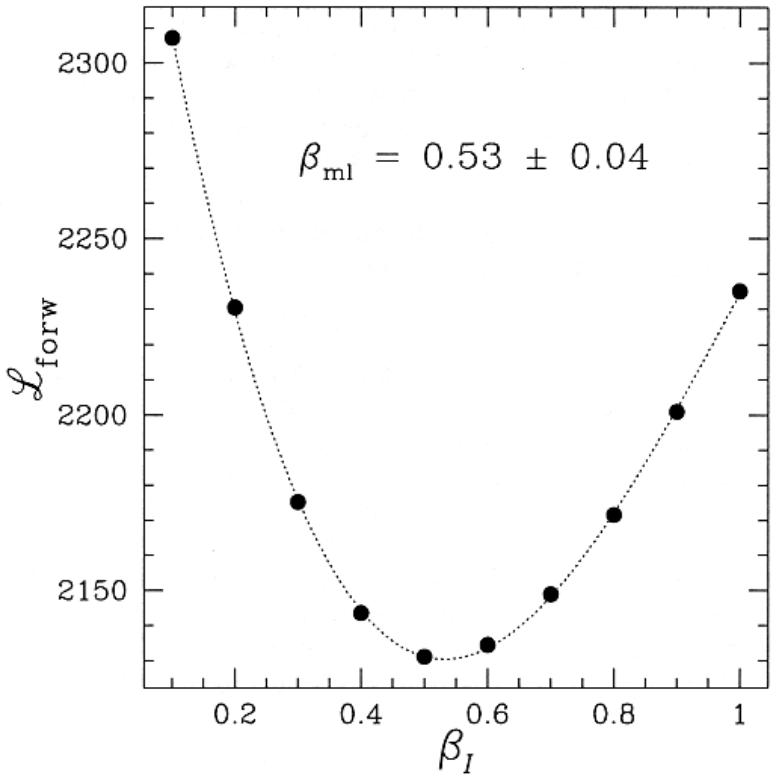
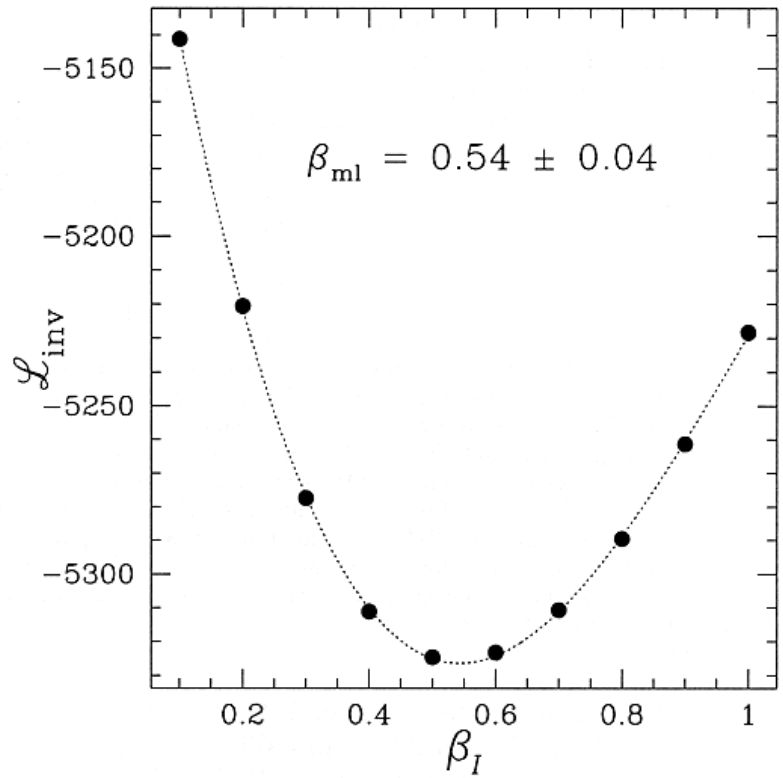
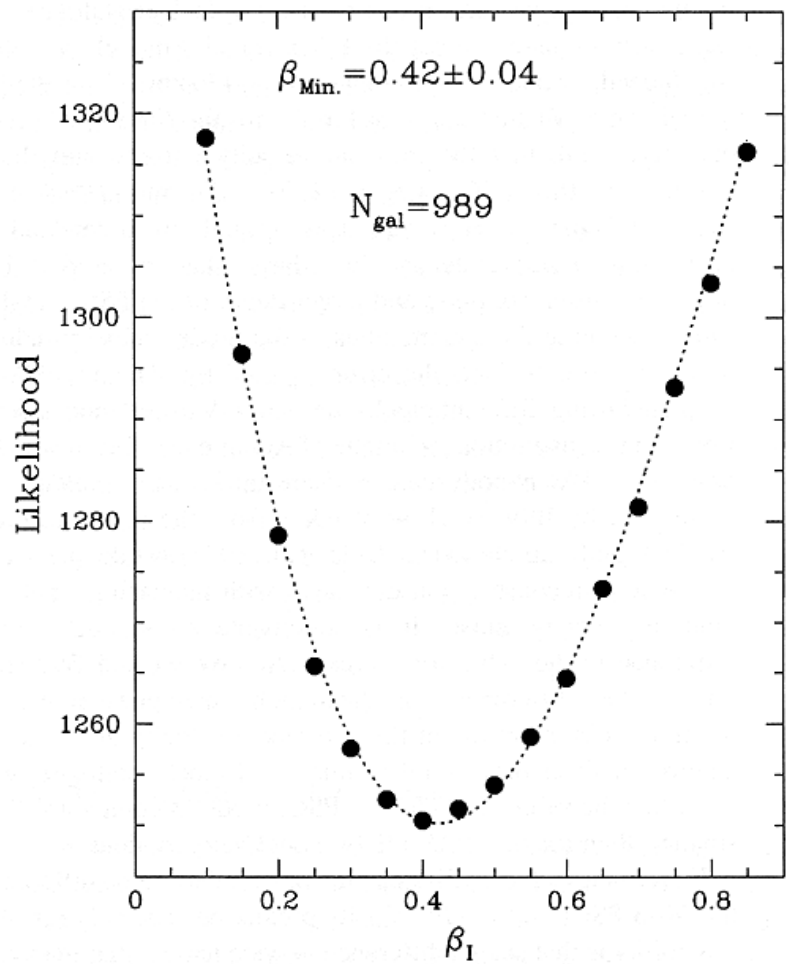
VELMOD Results





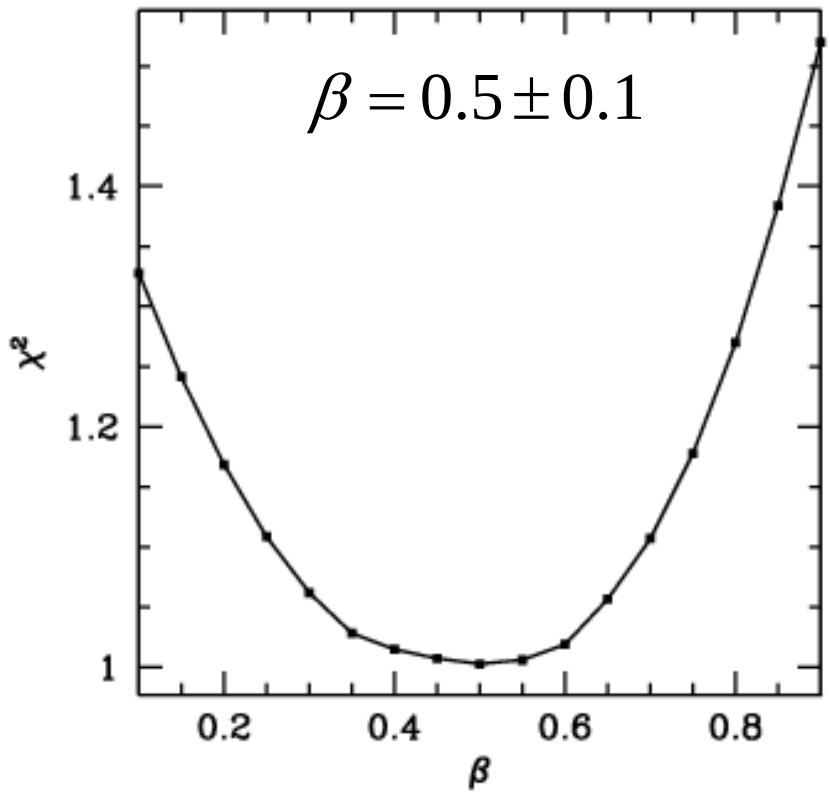
Willick et al 1998: Mark III + IRAS 1.2Jy predicted v-field

VELMOD Results



Branchini et al. 2002

SFI + PSCz v-field



Nusser et al. 2001

ENEAR + PSCz v-field

VELMOD Results

- Consistent picture of $\beta_I \sim 0.4 - 0.6$;
- Good agreement with results of ITF method, but significantly discrepant with POTENT results

What is the origin of this discrepancy?…

- c.f. Berlind et al. 2001, β estimation still OK in non-linear local biasing schemes
- Rauzy & Hendry (2000) – robust approach

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Robust Method

Assumption: luminosity function is **Universal**

$$dP = \frac{1}{A} S(m,r,l,b)\rho(r,l,b)\Phi(M)dldbdrdM$$

Selection effects

Spatial distribution

Luminosity function

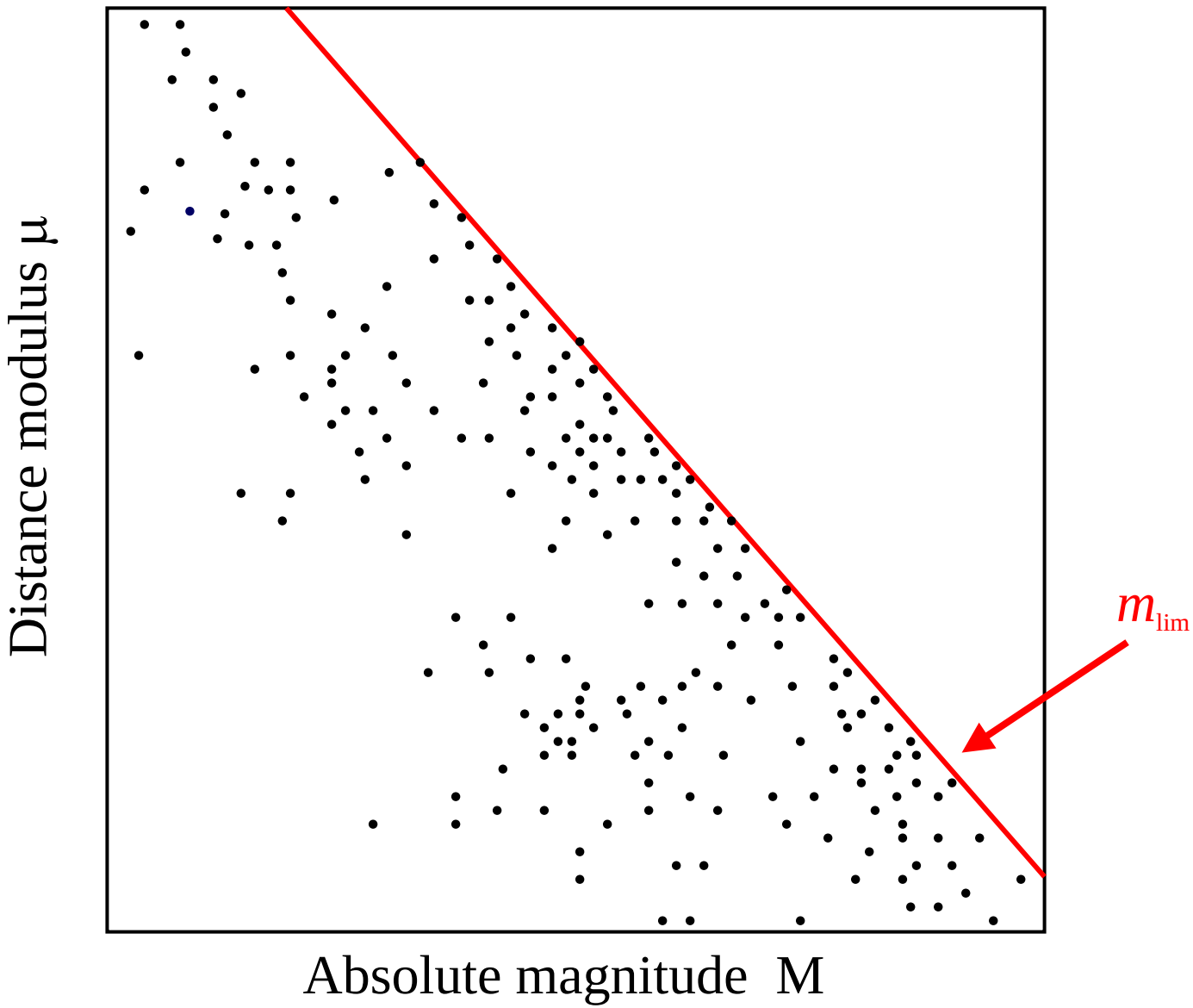
Null hypothesis (Rauzy 2001; Hendry & Rauzy 2004.)

$$S(m,z,l,b) \equiv \theta(m_{\mathrm{lim}} - m) \times \phi(z,l,b)$$

Step function

Angular and radial Selection function

Robust Method: Completeness



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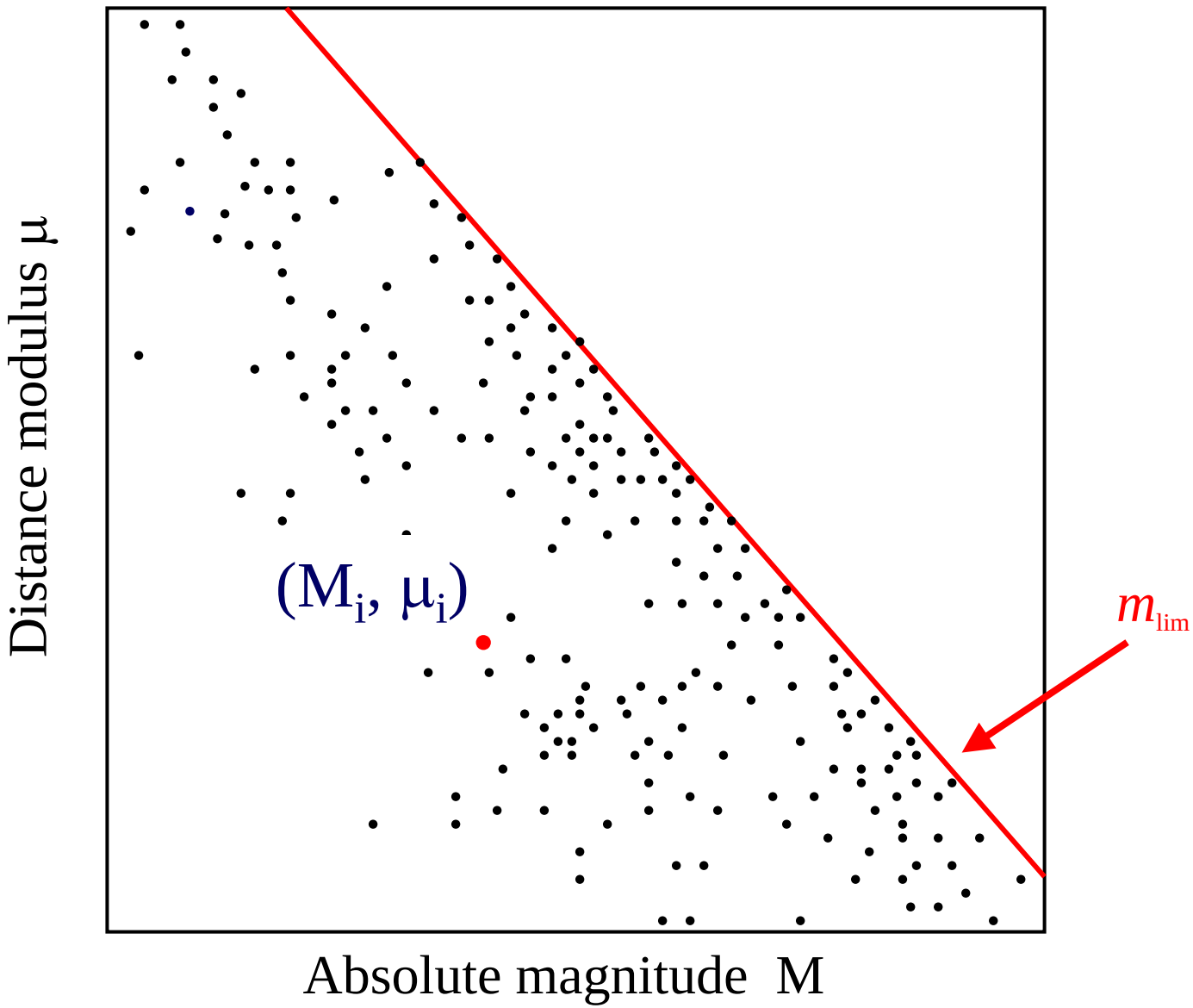

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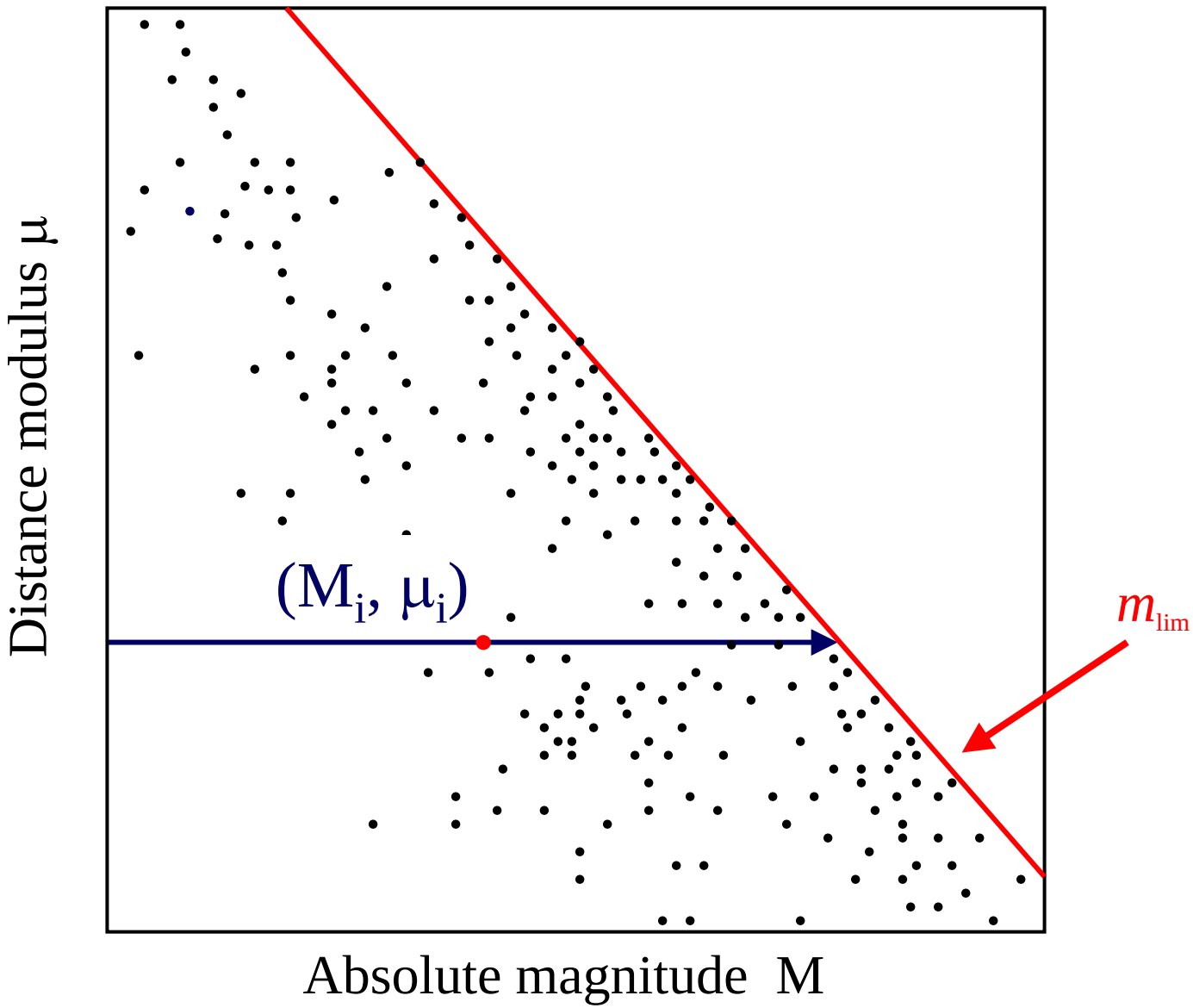

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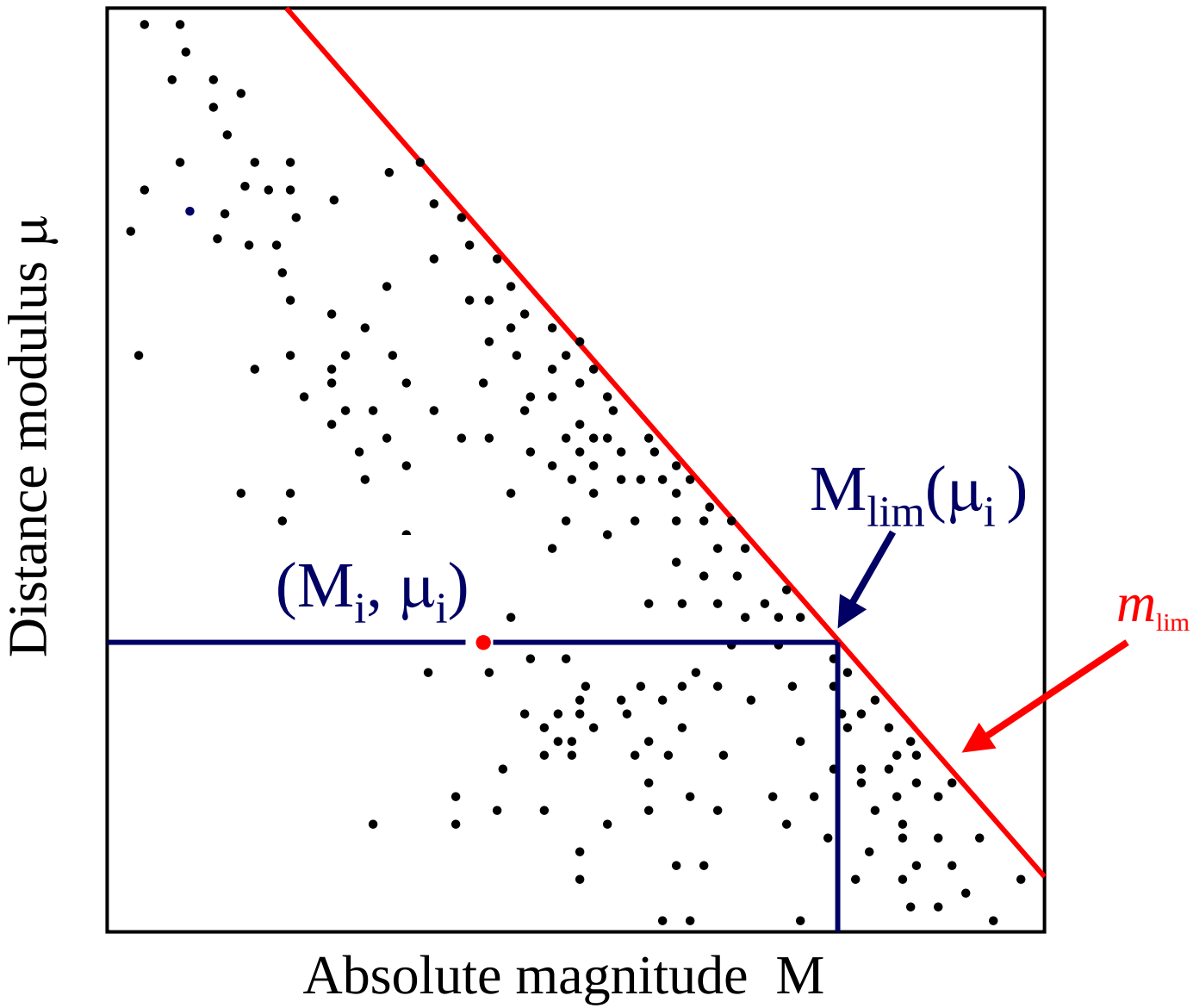

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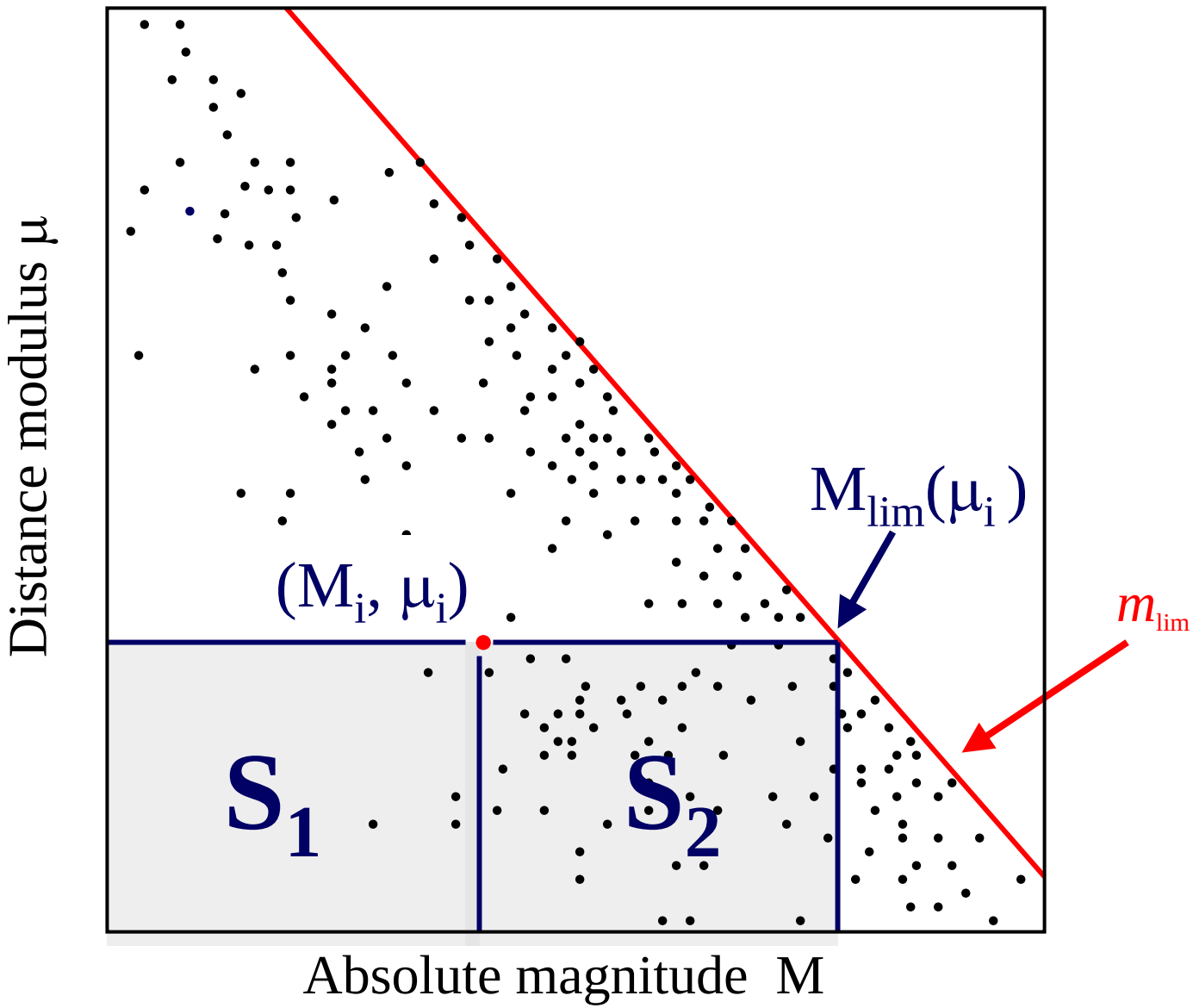
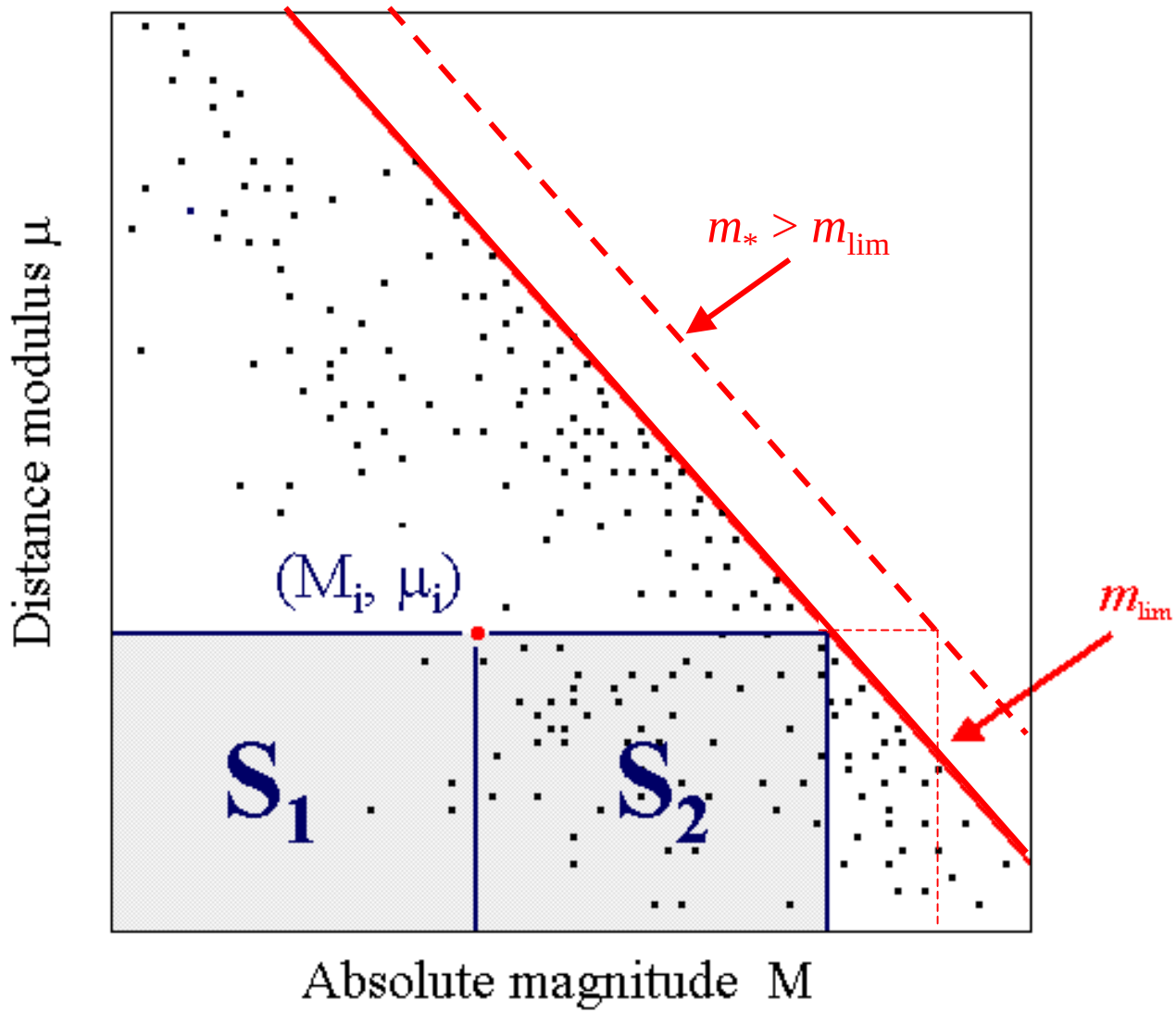


Robust Method: Completeness



Robust Method: Completeness

Define:-

$$\zeta = \frac{F(M)}{F(M_{\lim})}$$

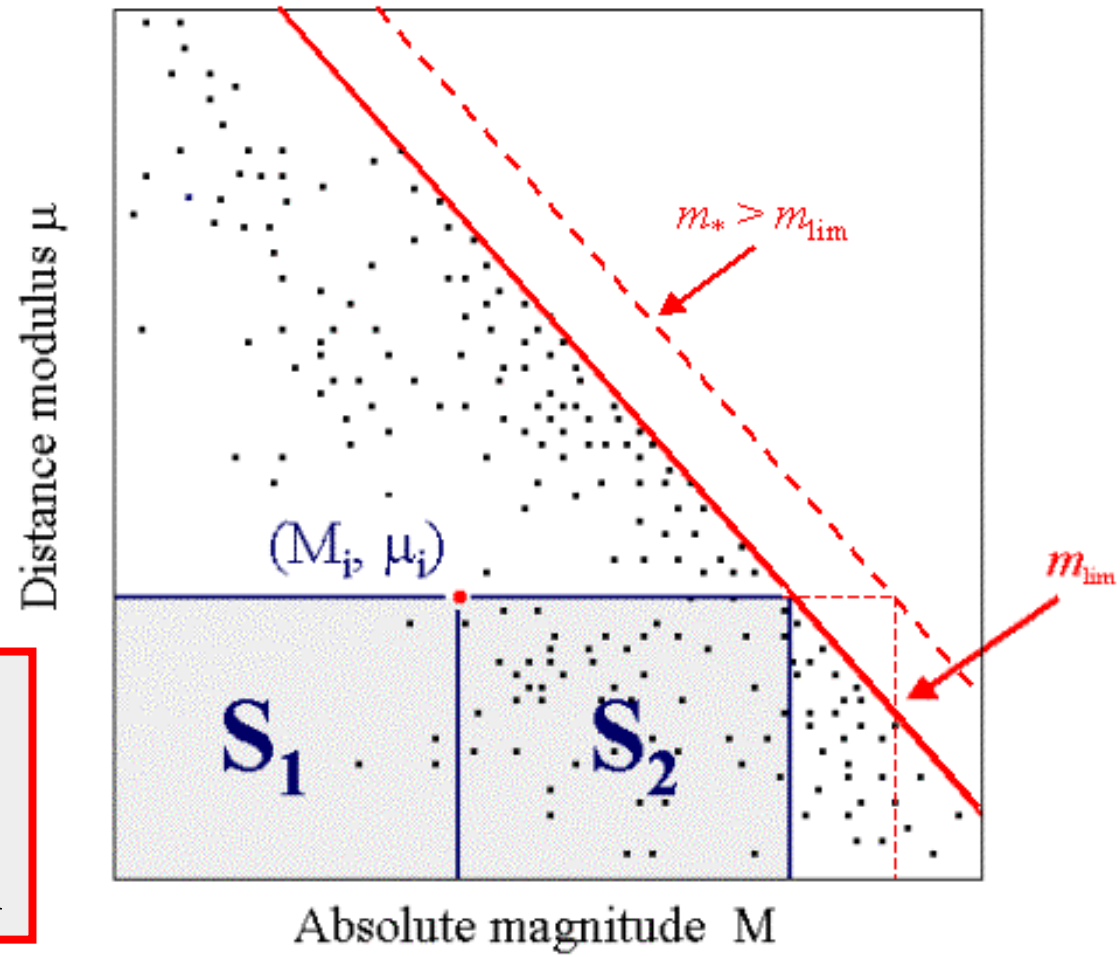
where

$$F(M) = \int_{-\infty}^{M} f(x)dx$$

Can show:-

P1: $\zeta \in \mathrm{U}[0,1]$

P2: ζ, μ uncorrelated



Robust Method: Completeness

Also:-

$$\hat{\zeta}_i = \frac{r_i}{n_i + 1}$$

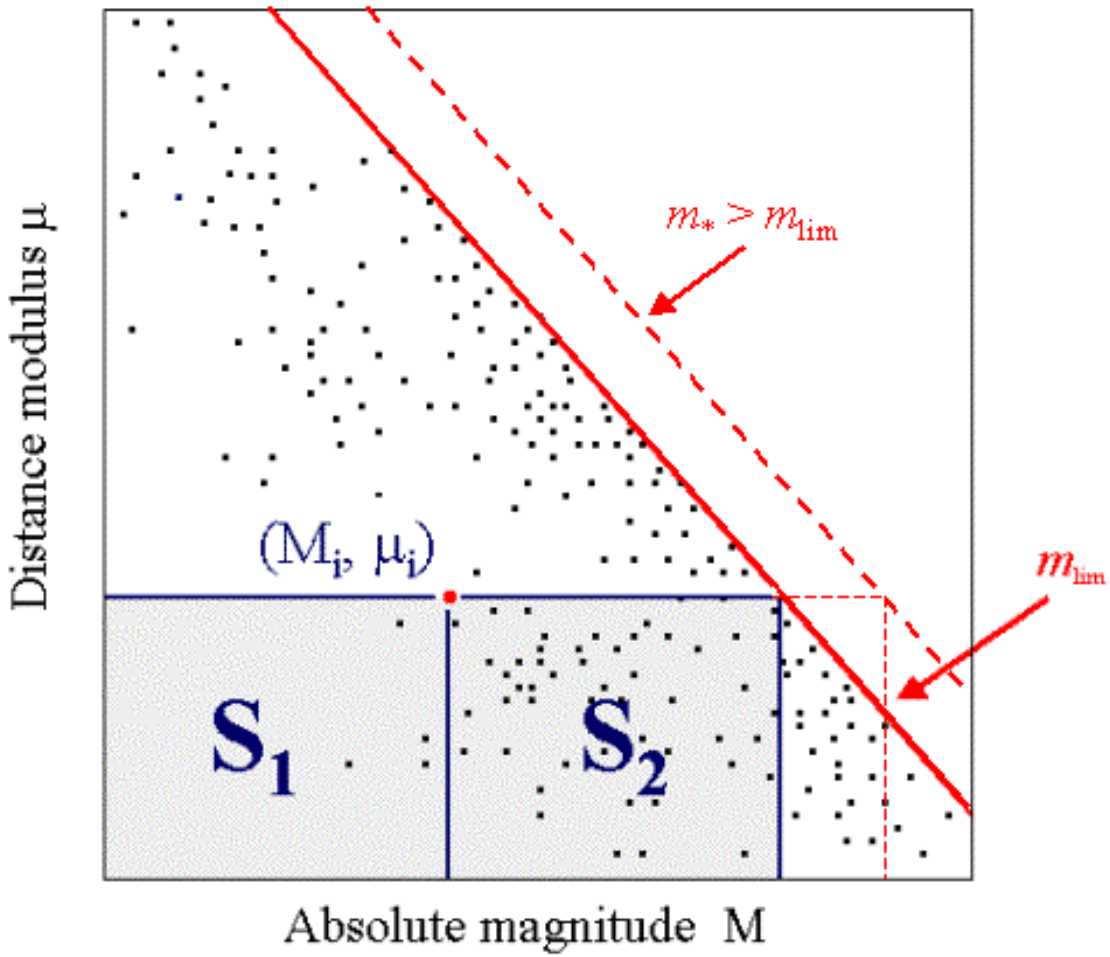
$$r_i = n(S_1)$$

$$n_i = n(S_1 \cup S_2)$$

$$E_i = \frac{1}{2} \qquad V_i = \frac{1}{12}\frac{n_i - 1}{n_i + 1}$$

but *only* for

$$m_* \leq m_{\lim}$$



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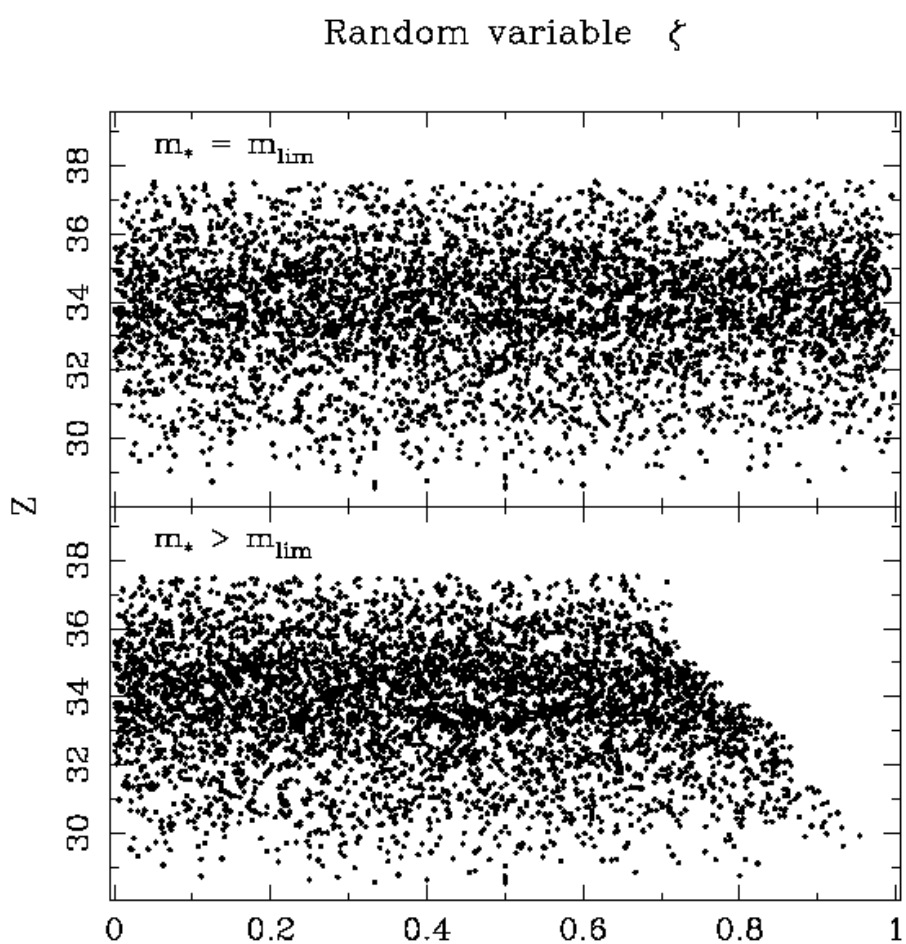

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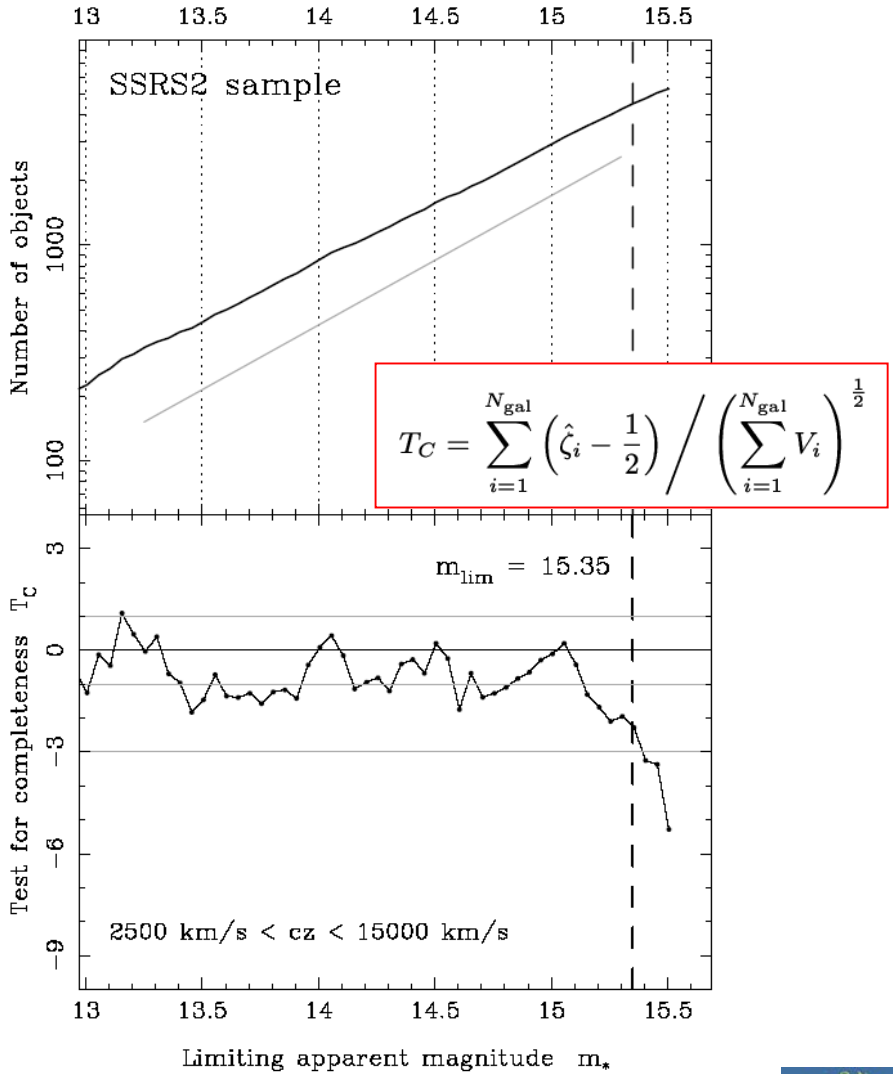


Robust Method: Velocity Field Model

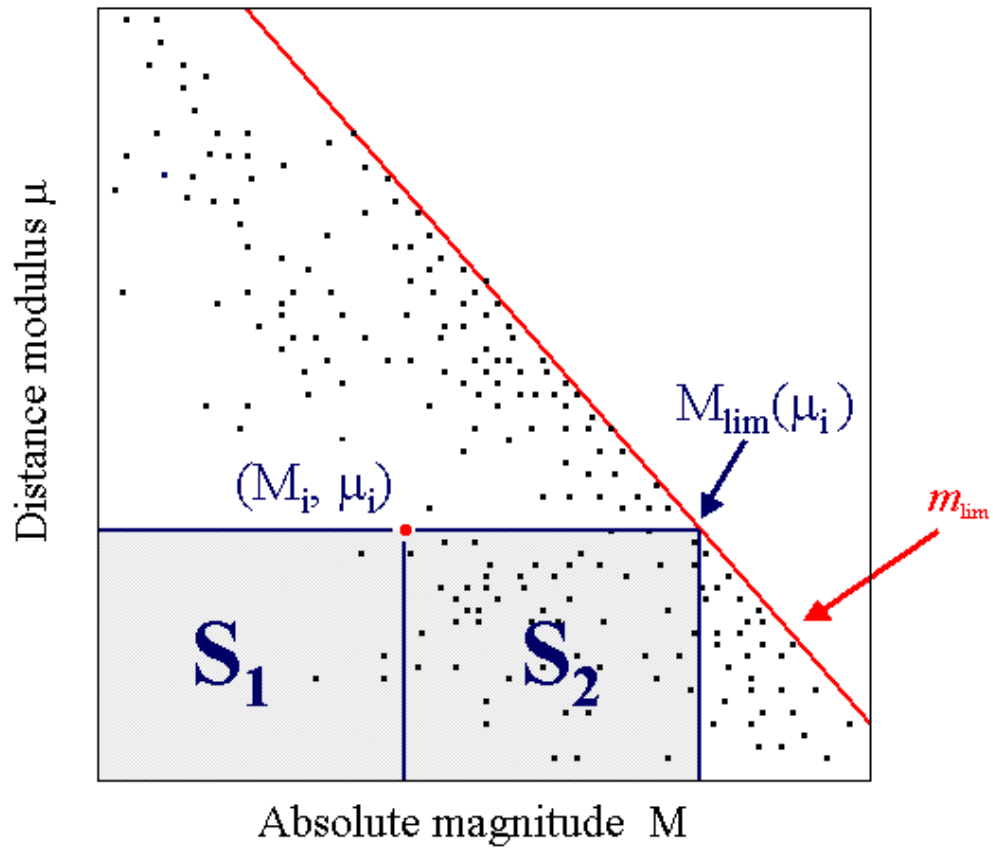
Assuming $\mathrm{v}(\mathbf{r}) \equiv \mathrm{v}_\beta(\mathbf{r})$

define

$$u_\beta = -5\log_{10}\left(1 - \mathrm{v}_\beta / cz\right)$$

Can show:-

P3: ζ_β, u_β uncorrelated



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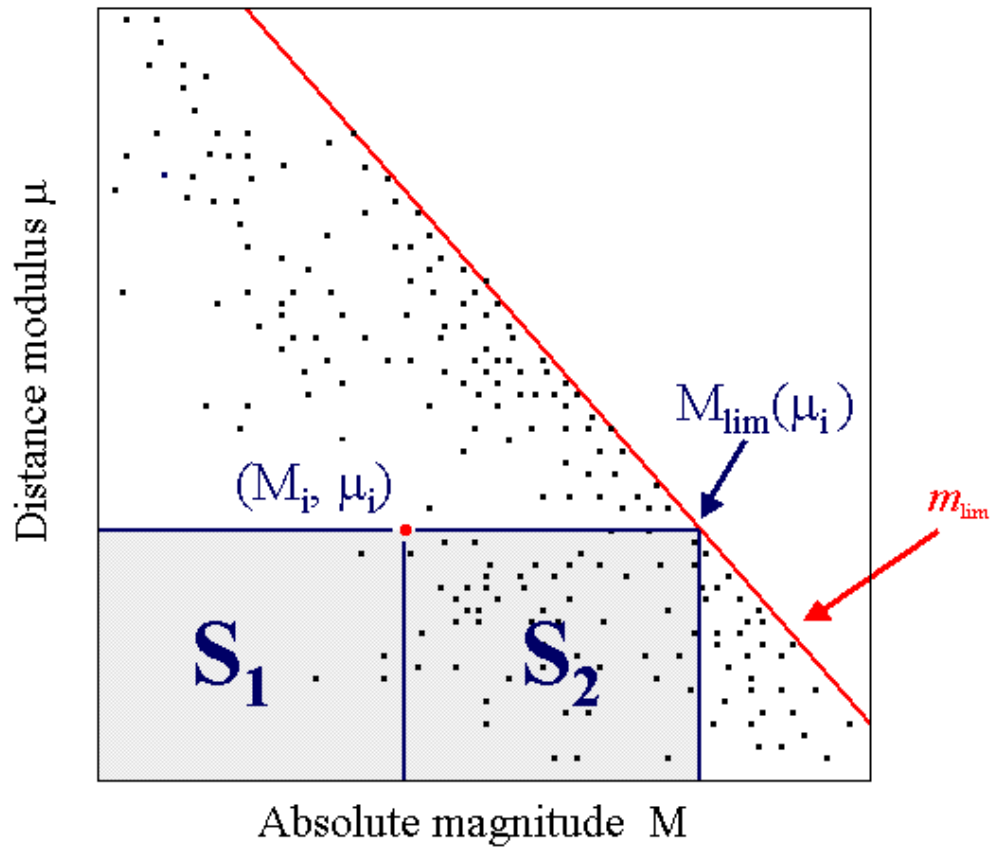
Estimate β via

$$\rho(\zeta_i, u_i) = 0$$



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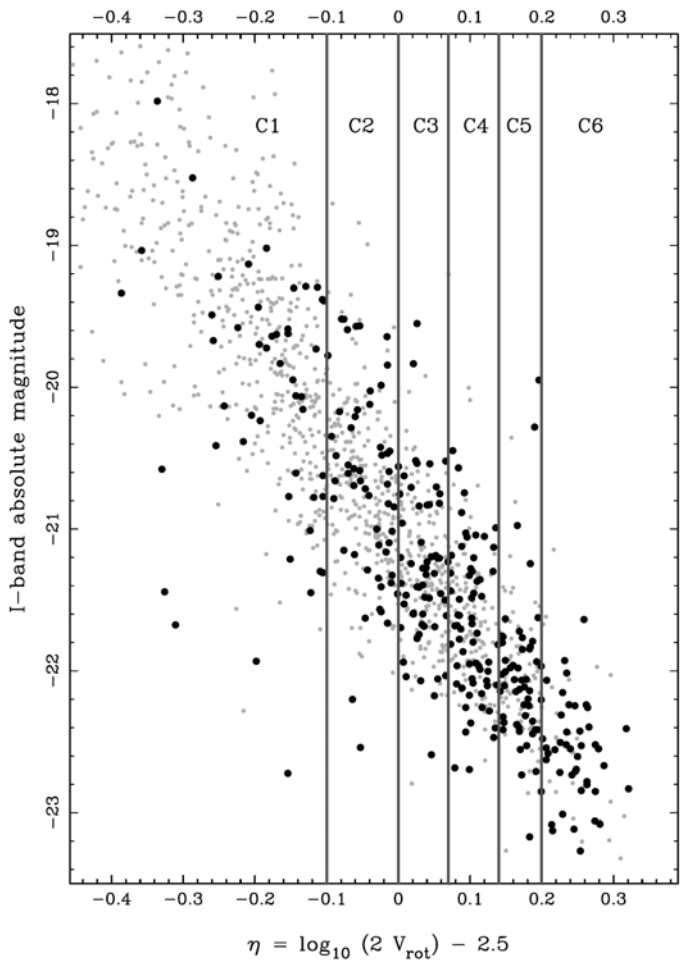
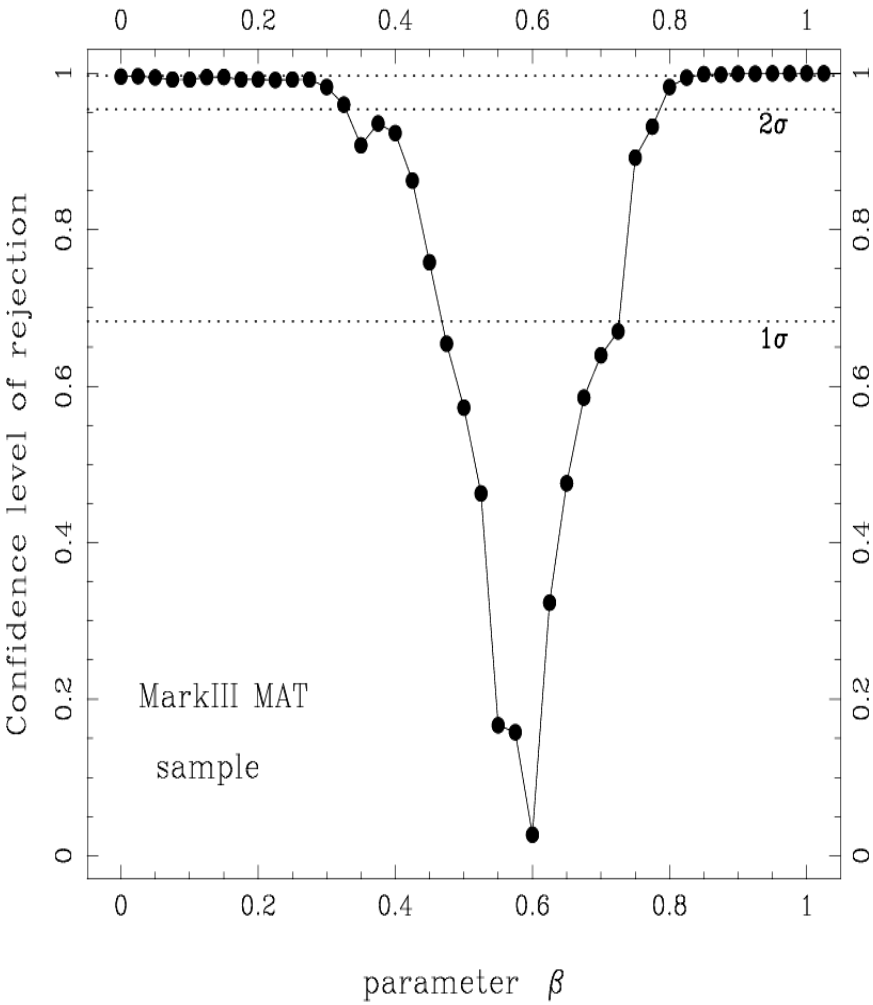


From Rauzy & Hendry 2000

- Non-parametric: independent of LF, spatial distribution.
- Insensitive to Malmquist bias.
- Very conservative use of TF information.
- Monte Carlo error estimates straightforward – permutation test.
- Rejection test on values of β

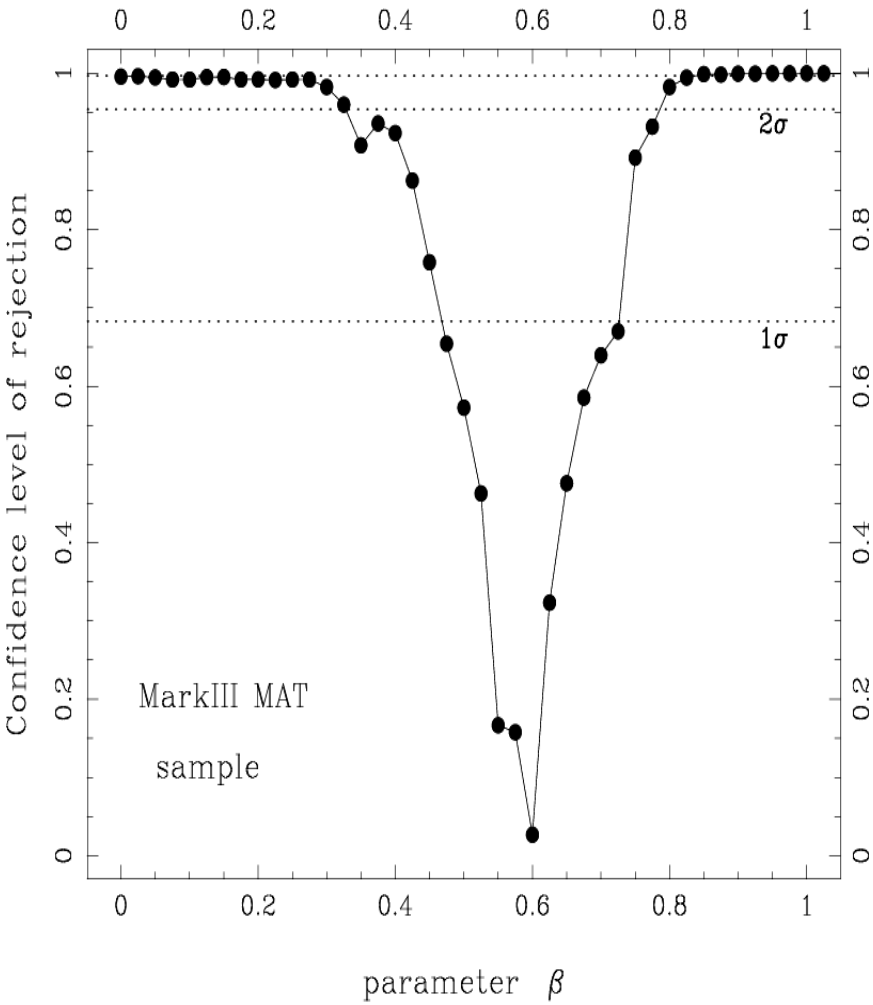
Robust Method



$$\beta_{\mathrm{I}} = 0.6 \pm 0.1$$

From Rauzy & Hendry 2000

Robust Method



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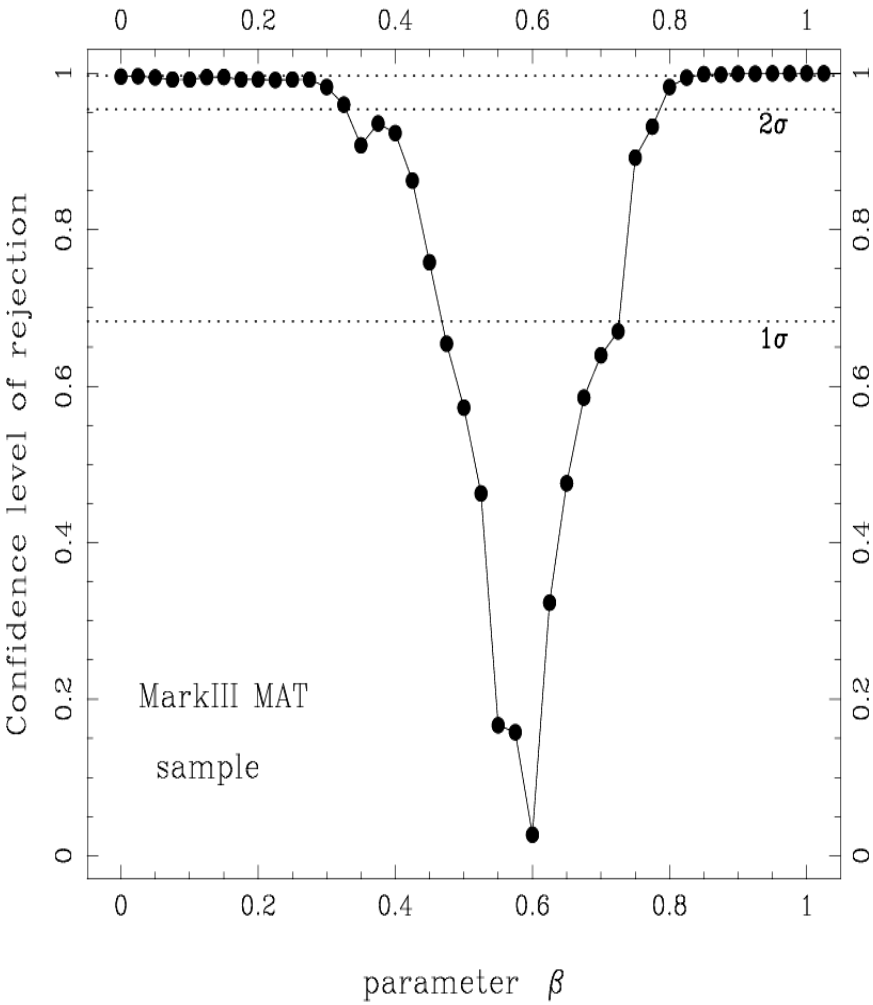
<u>Strength</u>:

Robust support for VELMOD analysis: validity of inhomogeneous Malmquist corrections

Robust Method



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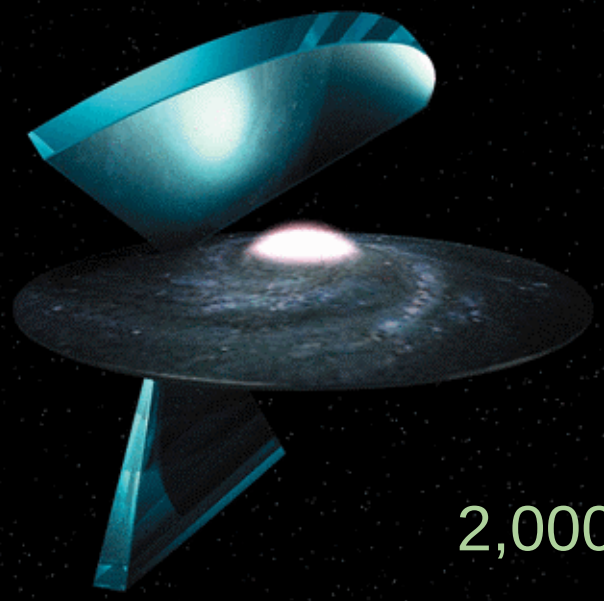
<u>Strength</u>:

Robust support for VELMOD analysis: validity of inhomogeneous Malmquist corrections

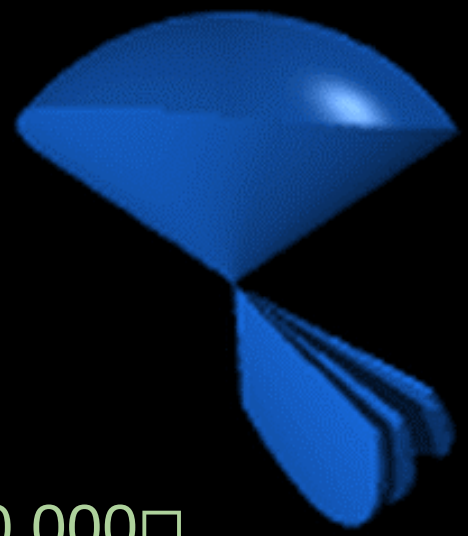
<u>Weakness</u>:

Completeness requirement may restricts sample size and depth

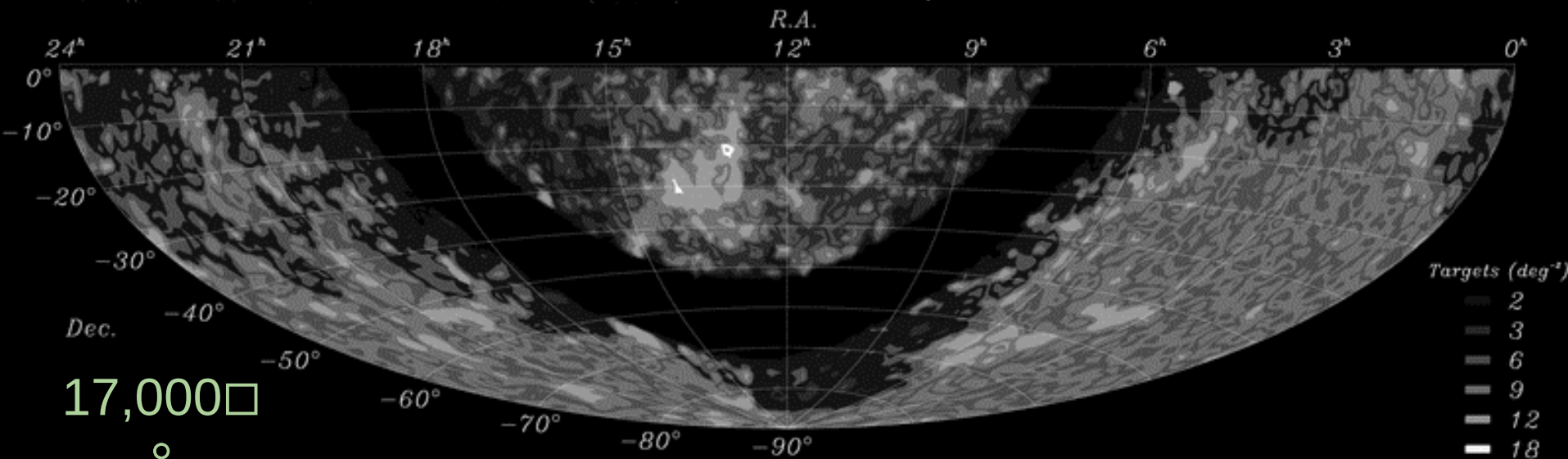
The future: 6DF



2,000□°



10,000□°

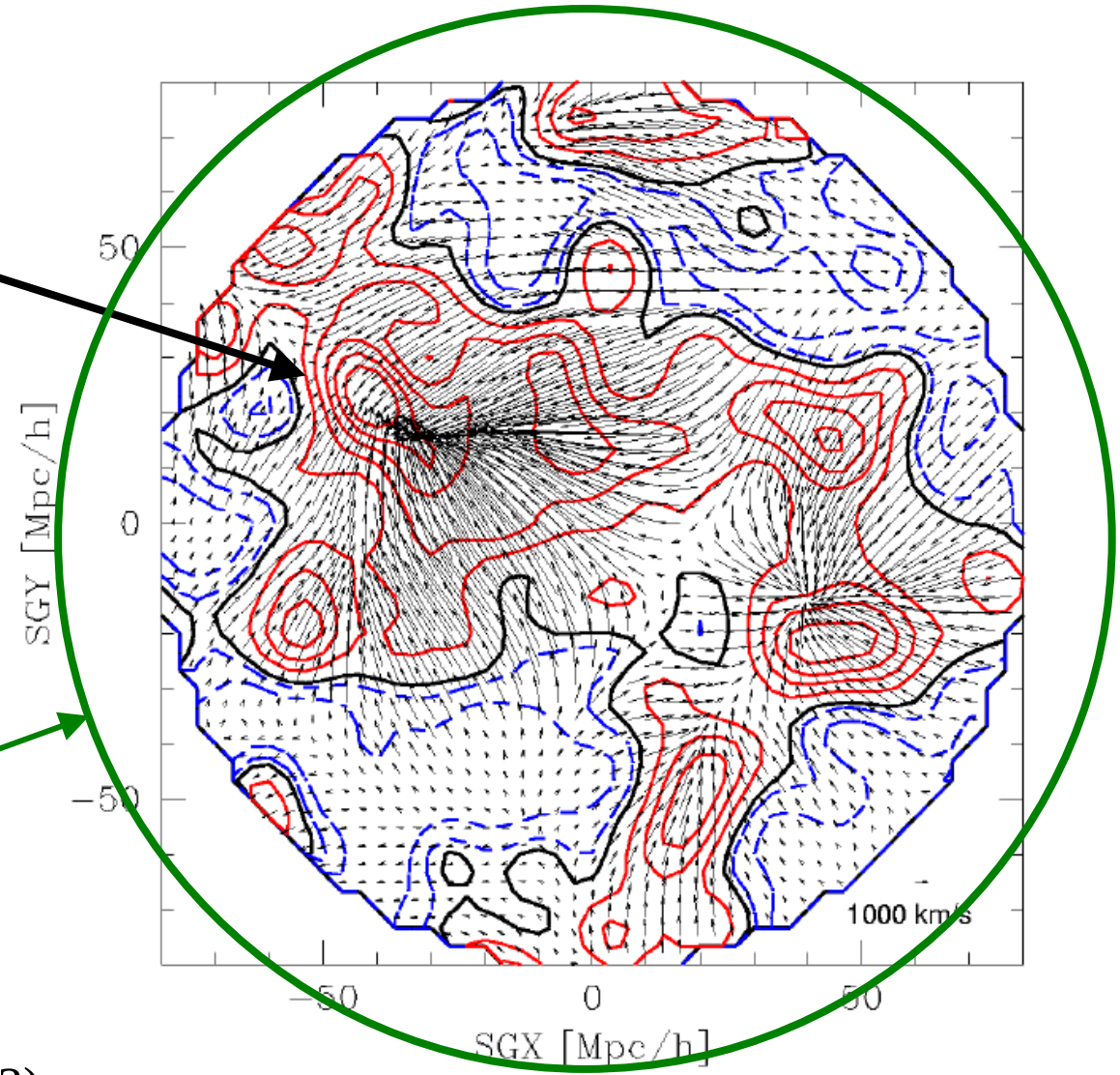


17,000□°

The future: 6DF

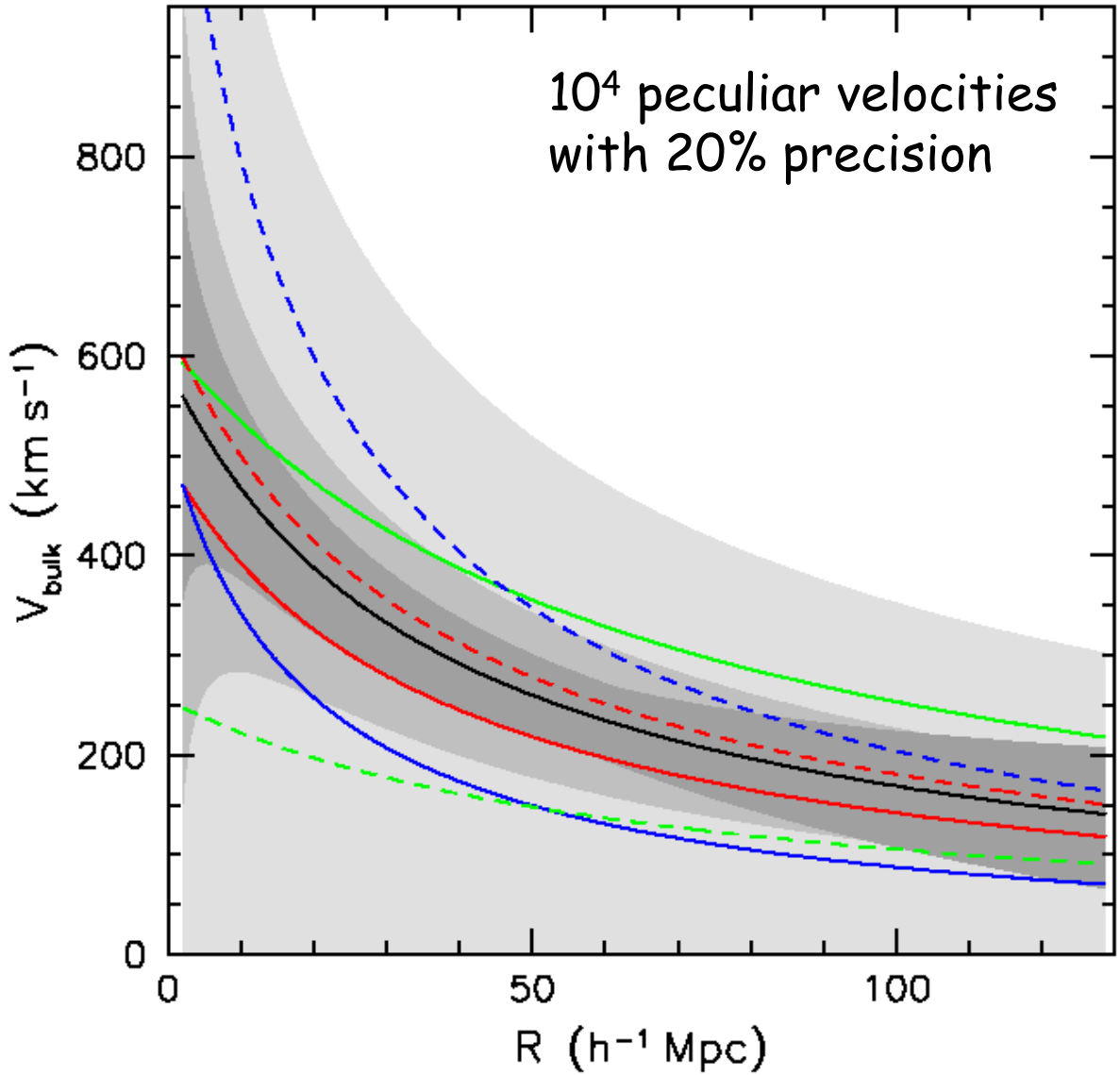
PSCz predicted velocity field (15,000 galaxies)

6dFGS observed velocity field (15,000 galaxies)



From Colless (2003)

Peculiar velocities & cosmology



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Conclusions

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Conclusions

- Peculiar velocities useful cosmological probes, but **big** problems to be overcome from distance indicator noise
- Major issues of sparseness, noise and bias still affect their calibration and use – **handle with care!**
- Can tackle via *robust* methods: important cross-check
- Field has been quiet since late 1990s, but 6dF + SDSS may trigger a revival – **methodology well developed**
- Useful adjunct to CMBR + z-surveys

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