Breaking scale invariance from a singular inflaton potential

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Inflation: the good / the bad (the not-good)

- I
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- C it y
- C vo u

- *Th* del?
- Be
- No
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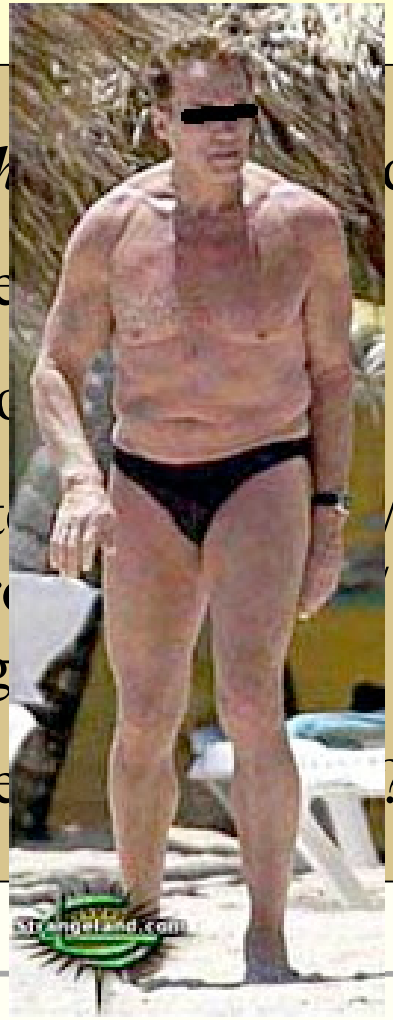
- Re ?



- Standard slow-roll approximation

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} = O(\xi)$$

$$\delta_1 \equiv \frac{\ddot{\phi}}{H\dot{\phi}} = O(\xi)$$

Approximately constant

$$\Rightarrow \quad P(k) = \left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{\dot{\phi}}\right)^2 \left[1 + O(\xi)\right]$$

$$n(k) - 1 = \frac{d\ln P}{d\ln k} = O(\xi)$$

$$\frac{dn}{d\ln k} = O(\xi^2)$$

$\Omega_{tot} = 1.02 \pm 0.02$

$w < -0.78$ (95% CL)

$\Omega_\Lambda = 0.73 \pm 0.04$

$\Omega_b h^2 = 0.0224 \pm 0.0009$

$\Omega_b = 0.044 \pm 0.004$

$n_b = (2.5 \pm 0.1) \times 10^{-7} \text{cm}^{-3}$

$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$

$\Omega_m = 0.27 \pm 0.04$

$\Omega_\nu h^2 < 0.0076$ (95% CL)

$\eta = (6.1^{+0.3}_{-0.2}) \times 10^{-10}$

$\Omega_b \Omega_m^{-1} = 0.17 \pm 0.01$

$\sigma_8 = 0.84 \pm 0.04$

$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$

$z_{dec} = 1089 \pm 1$

$\Delta z_{dec} = 195 \pm 2$

$h = 0.71^{+0.04}_{-0.03}$

$r_s = 147 \pm 2$ Mpc

$d_A = 14.0^{+0.2}$ Gpc

$$n(k) - 1 = \frac{d \ln P}{d \ln k} = O(\xi) \qquad \frac{dn}{d \ln k} = O(\xi^2)$$

$r(k_0 = 0.002 \text{ Mpc}^{-1}) < 0.71$ (95% CL)

$A(k_0 = 0.05 \text{ Mpc}^{-1}) = 0.8$

$n_\gamma = 410.4 \pm 0.9 \text{ cm}^{-3}$

$n_s = 0.99 \pm 0.04$ (WMAP only

$n_s(k_0 = 0.05 \text{ Mpc}^{-1}) = 0.9$

$t_{dec} = 379^{+3}_{-2}$ kyr

$^{194}_{210}$

.04

.031$^{+0.016}_{-0.018}$

Approximately c
onstant
slow-roll parame
ters

Dodelson and Stewart (2002)

- General slow-roll approximation

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} = O(\xi) \qquad \delta_1 \equiv \frac{\ddot{\phi}}{H\dot{\phi}} = O(\xi)$$

~~Approximately constant~~

$$O(|n(k)-1|) \sim O\left(\left|\frac{dn}{d\ln k}\right|\right)$$

Choe, ***JG*** and Stewart (2004)

- Primordial power spectrum $P(k)$

$$V(\phi) = V_0\left(1 - \frac{1}{2}\mu^2\phi^2 + \mathrm{L}\right)$$

$$f = \frac{3\pi}{H^2}\left(\sqrt{1+\frac{4}{3}\mu^2} - 1\right)x^{-\frac{3}{2}\left(\sqrt{1+\frac{4}{3}\mu^2}-1\right)}$$

$$f = \frac{2\pi}{k}(-k\eta)\frac{a\dot{\phi}}{H}$$

The story is not finished yet…

We would like to consider the cases wh
ere we cannot apply the usually taken sl
ow-roll conditions

Singularity

ϕ or ϕ is changing drastically

Sharp fea
tures

Starobinsky (1992) / ***JG*** (2005)

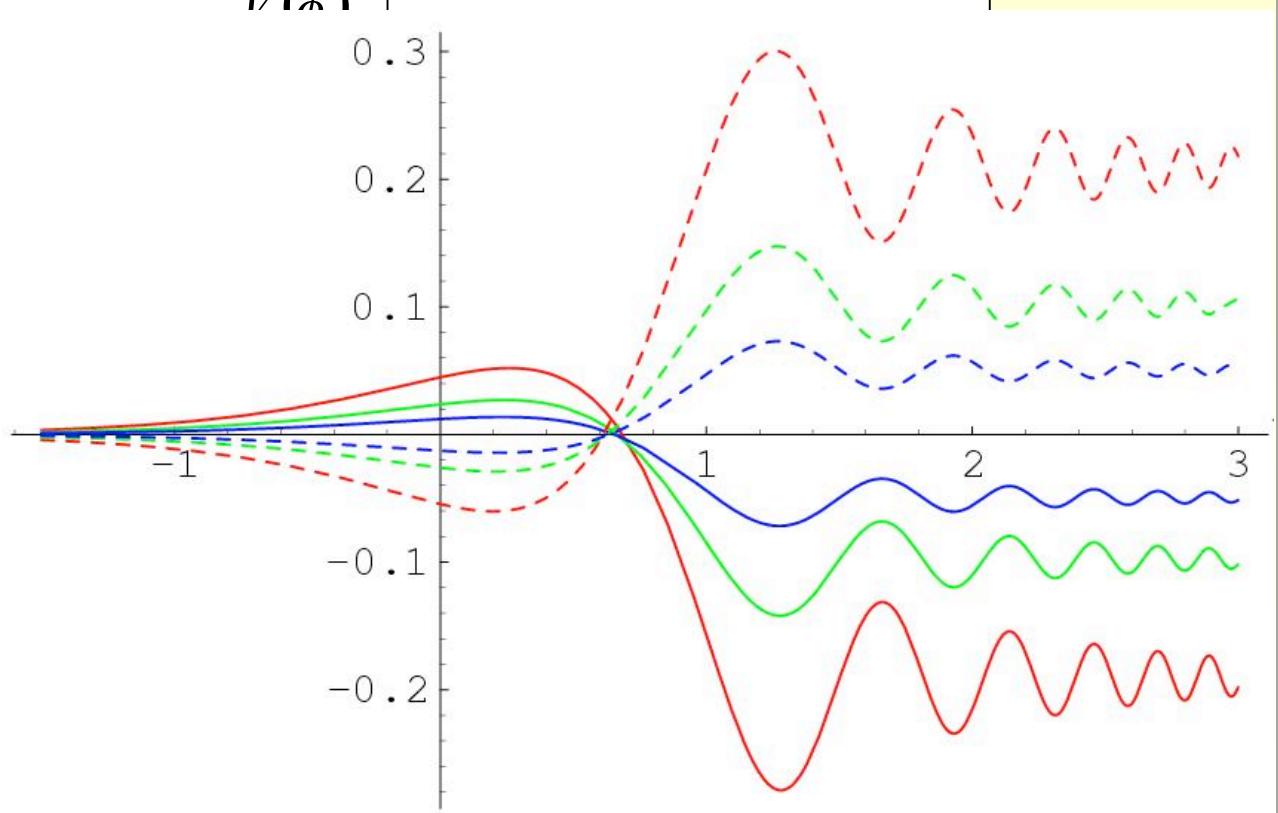
$$V(\phi) = V_0 \left\{ 1 - \left[A + \theta(\phi - \phi_0) \Delta A \right] (\phi - \phi_0) \right\}$$

$$\ln P = \ln$$

$$+$$

$$W($$

$$X(x) = -$$



JG (2005)

$$V(\phi) = V_0\left[1 - A(\phi - \phi_s) - a\theta(\phi - \phi_s)\right]$$

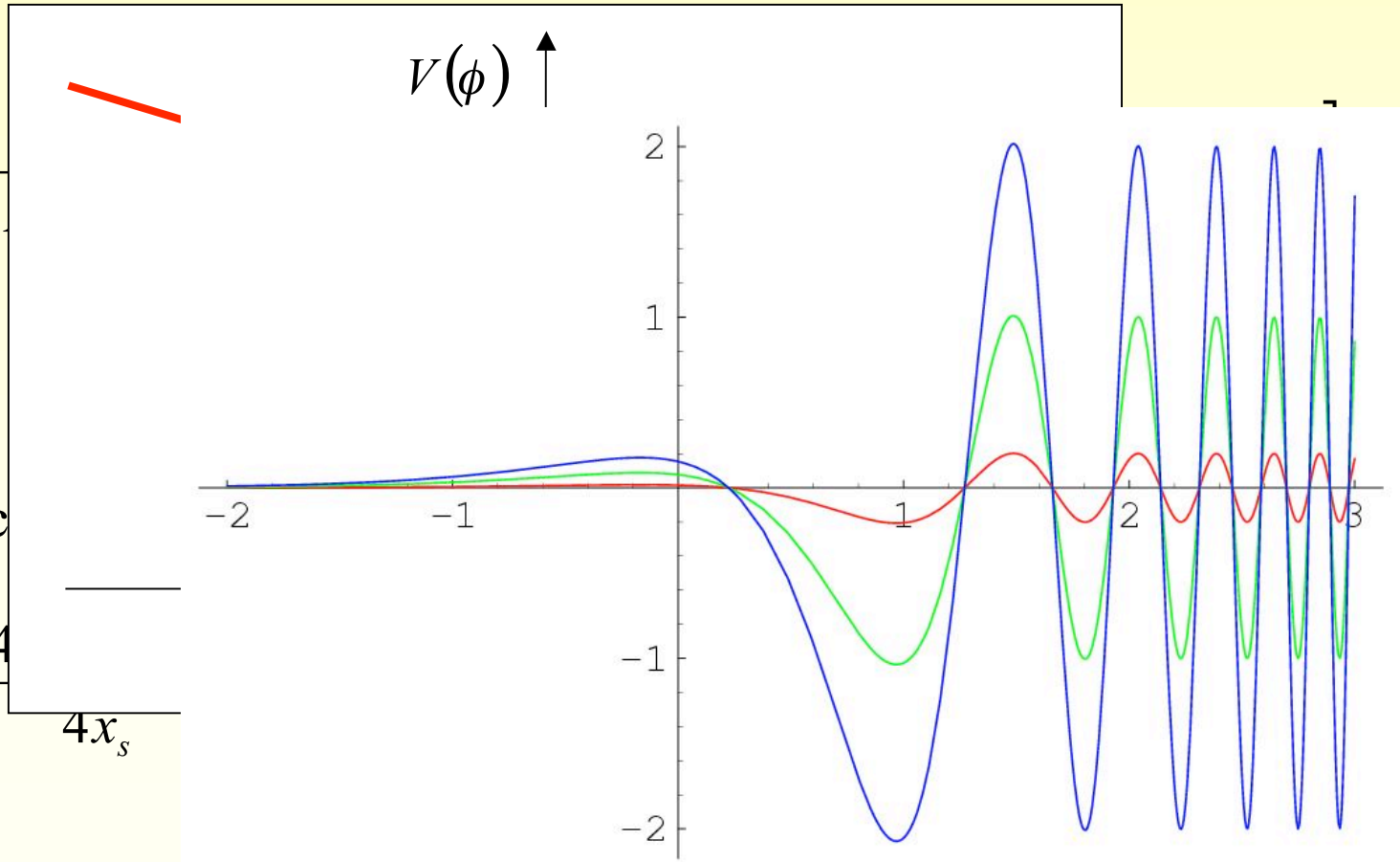
$$\ln P = \ln\left(\frac{V_0}{12\pi^2}\right.$$

$$+ B^2\left[1 + \right.$$

$$+ c$$

$$+ -$$

$$4x_s$$



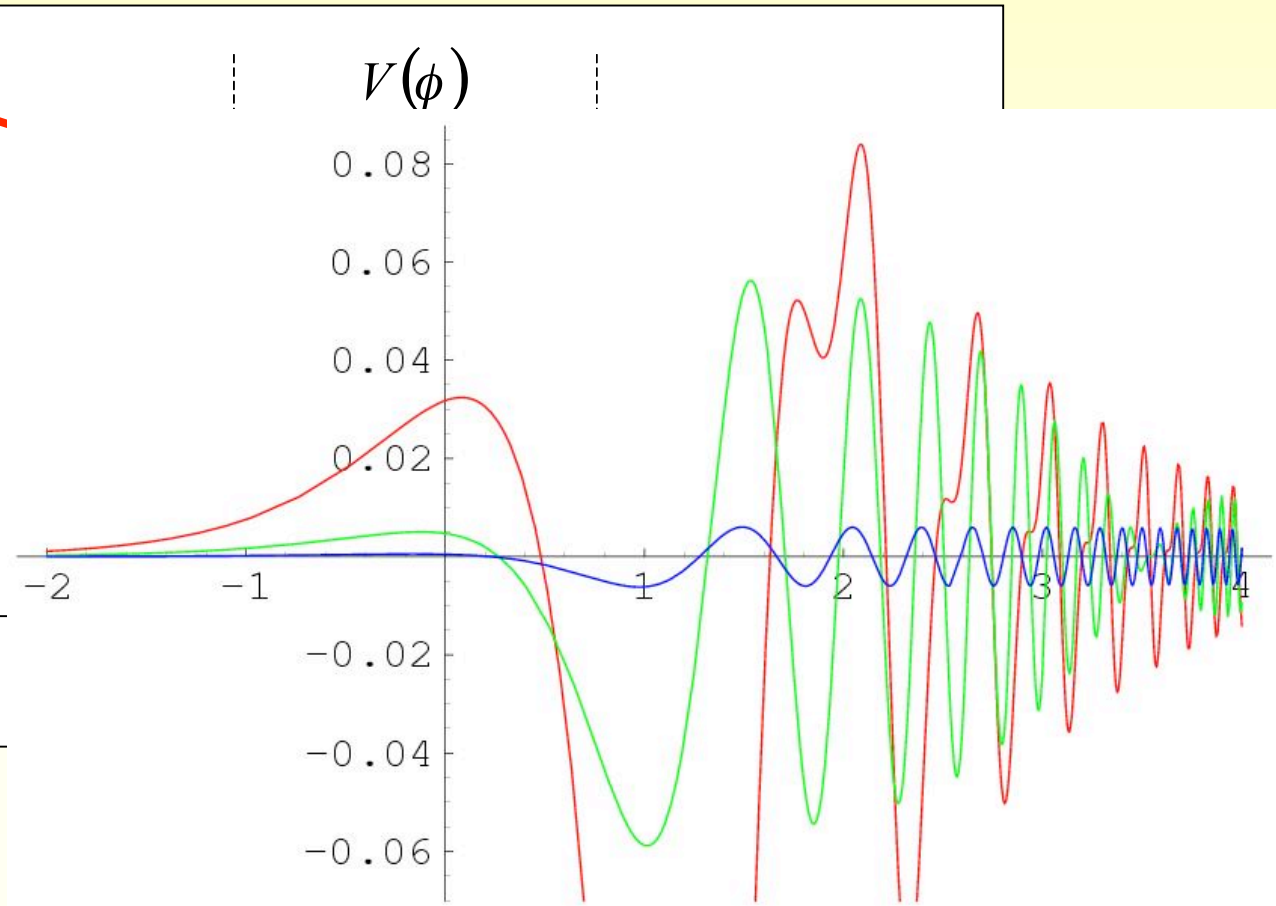
JG (2005)

$$V(\phi) = V_0 \left\{ 1 - A(\phi - \phi_1) - \Delta A \left[\theta(\phi - \phi_1)(\phi - \phi_1) - \theta(\phi - \phi_2)(\phi - \phi_2) \right] \right\}$$

$$\ln P = \ln\left(\frac{V_0}{12\pi^2} \right.$$

$$- \frac{6\cos(2}{x_1^2}$$

$$+ O\left[\left(\frac{\ddot{A}}{A} \right. \right.$$



JG (2005)

$$V(\phi) = V_0\left[1 - \frac{1}{2}\mu^2\phi^2 - a\theta(\phi - \phi_s)\right]$$

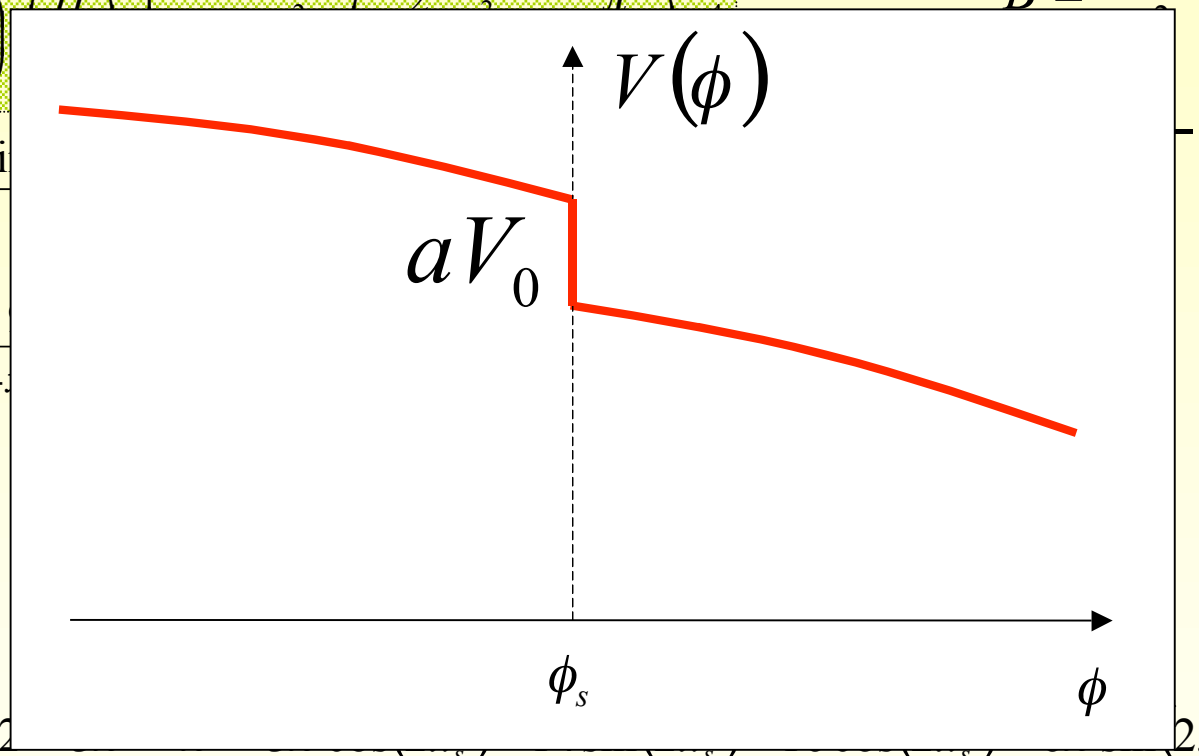
$$\ln P = \ln\left[\left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{\ldots}\right)^2\right] \ldots$$

$$B = \frac{aV_0}{\ldots}$$

$$\ldots - \ln 2 - \gamma - \ln x_*$$

$$+ B\left[-\frac{3\,\mathrm{si}\ldots}{\ldots}\right.$$

$$+ B^2\left[1 + \frac{\ldots}{4\ldots}\right.$$



$$+ \mu^2 B\left[-\frac{2}{3} - \frac{\ldots}{x_s^3} - \frac{\ldots}{x_s} + \frac{\ldots}{x_s^3} - \frac{\ldots}{x_s^3} + \frac{\ldots}{x_s^2} + \frac{\ldots(2x_s)}{x_s^2}\right.$$

$$\left. - \frac{5\pi\cos(2x_s)}{x_s} + \frac{20\sin(2x_s)}{3x_s} - \frac{2}{3}\cos(2x_s) - 2\pi\sin(2x_s) + \frac{2Si(2x_s)}{x_s}\left(1 + \frac{3}{x_s^2}\right)\right]$$

JG (2005)

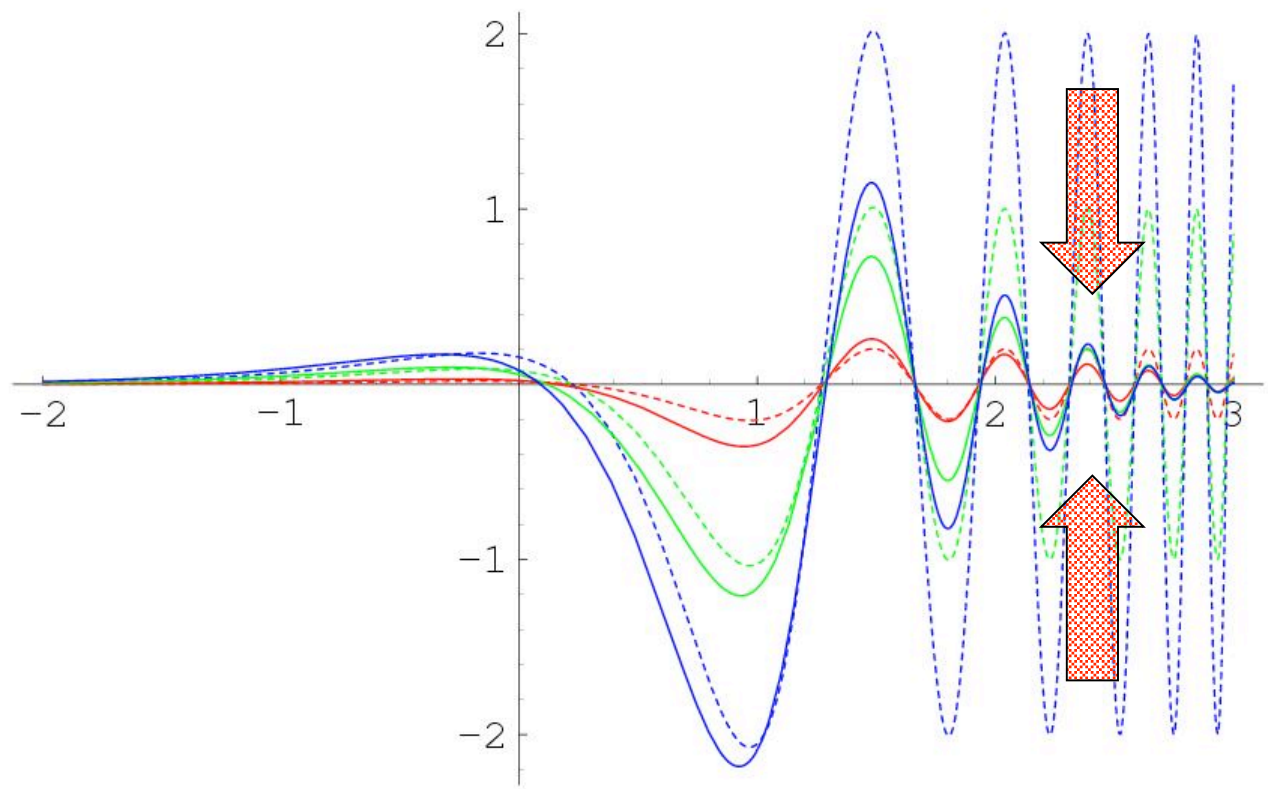
$$V(\phi) = V_0\left[1 - A\phi - a\tan^{-1}\left(\frac{\phi - \phi_s}{b}\right)\right]$$

$$\frac{V''}{V} = \frac{2aA}{b^3}$$

$$\ln P = \ln\left(\right.$$

$$= \ln\left(\right.$$

$$+ O\left[\left(\frac{}{b^3}\right)\right]$$



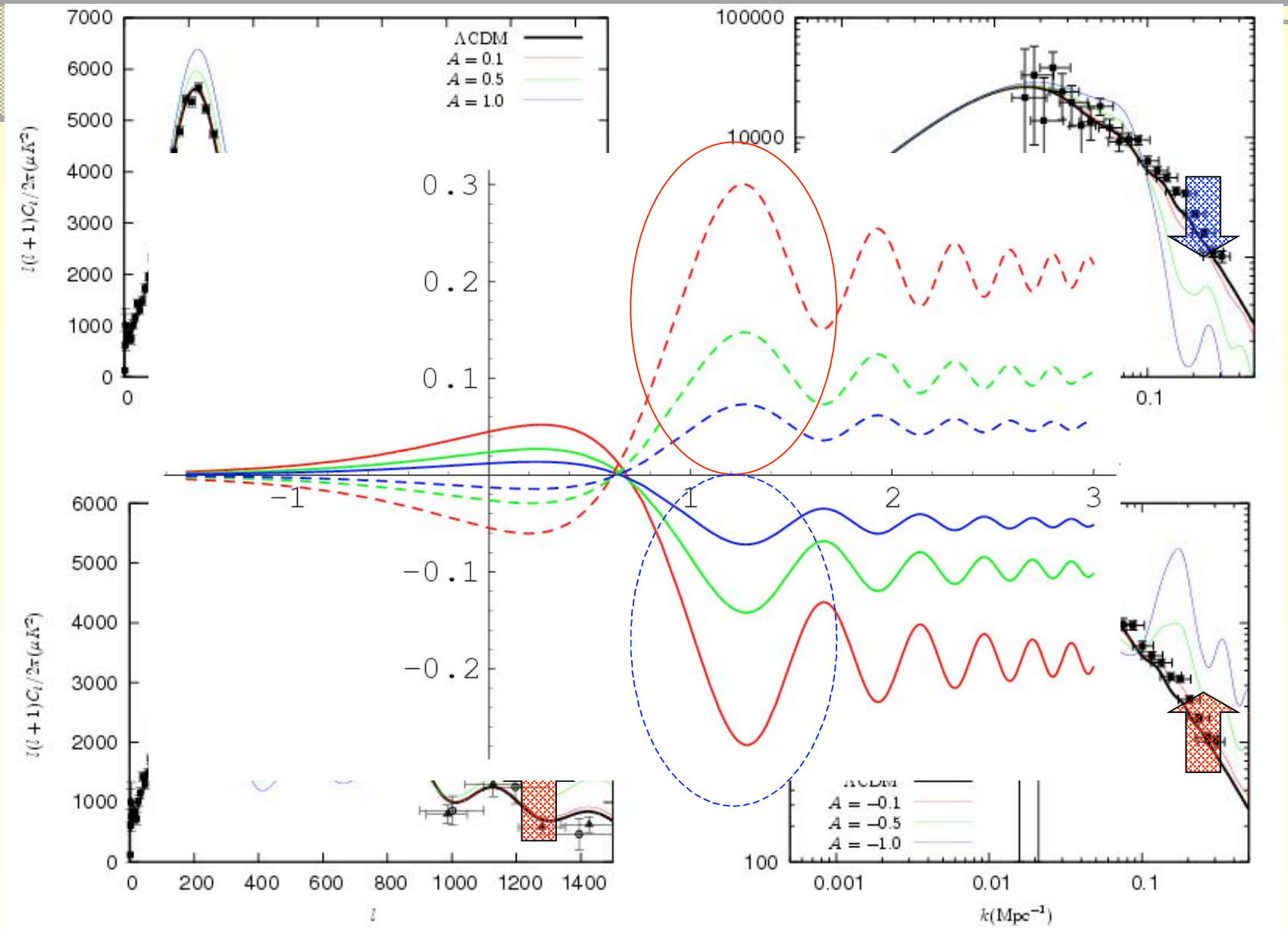
How do these spectra look like i
n the present?

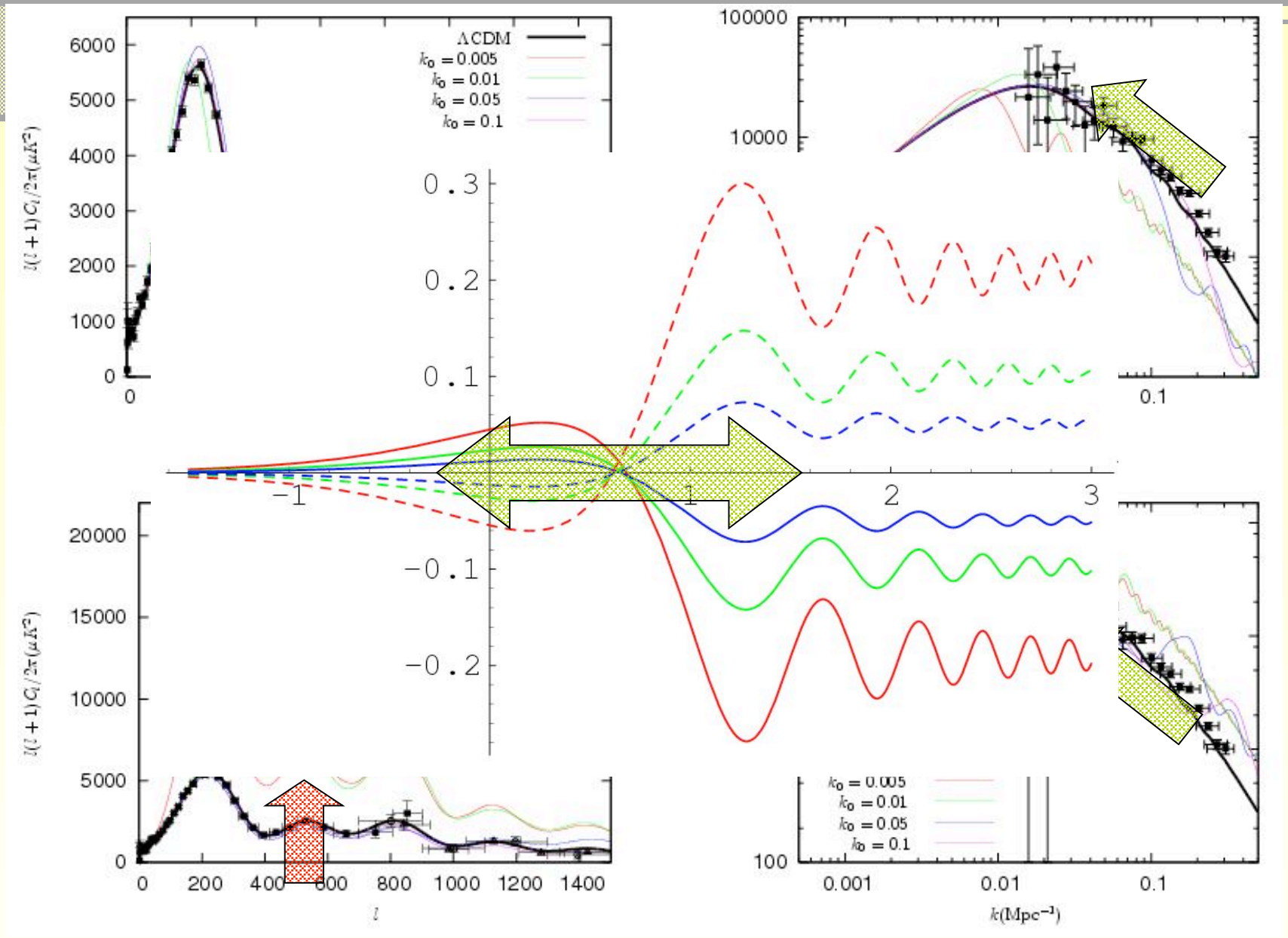
CMB anisotrop Matter inhomo
y tru
m

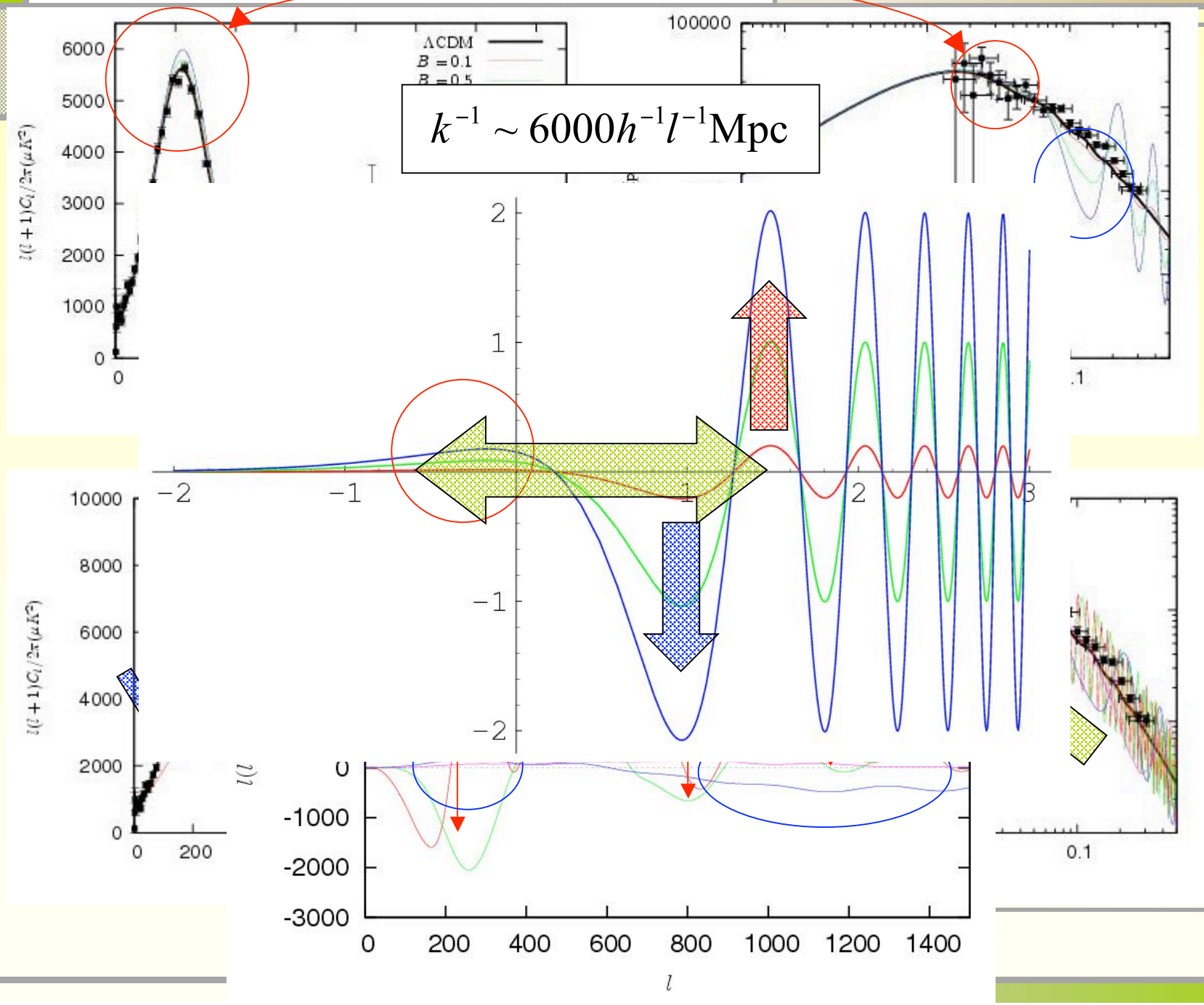
Currently favoured LCDM

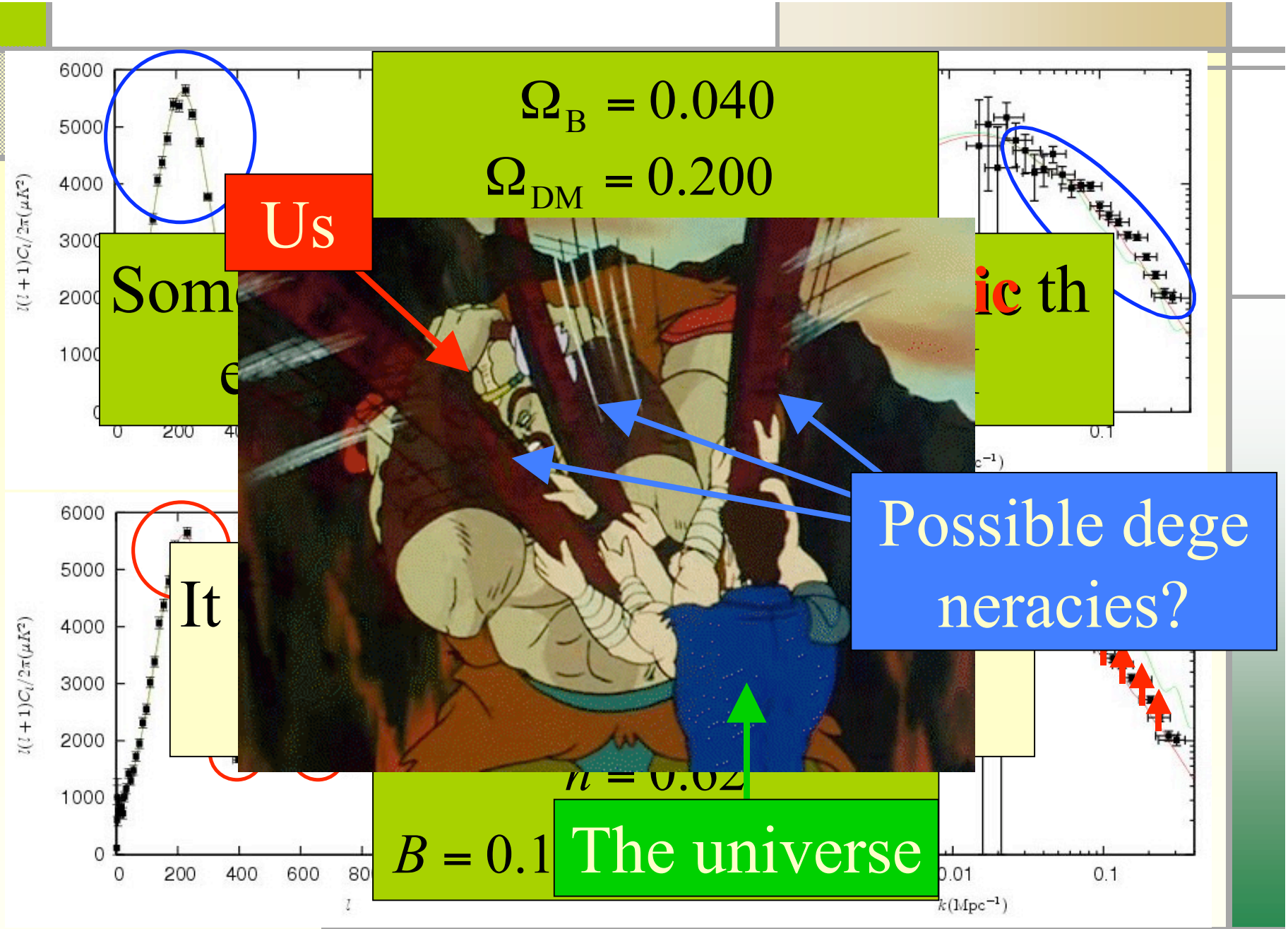
$$\Omega_B = 0.046$$
$$\Omega_{DM} = 0.224$$
$$\Omega_\Lambda = 0.730$$
$$h = 0.72$$

Scale invariant $P(k)$


ΛCDM
A = 0.1
A = 0.5
A = 1.0
A = −0.1
A = −0.5
A = −1.0
l(l + 1)C_l/2π(μK²)
l
k(Mpc⁻¹)


ΛCDM
$k_0 = 0.005$
$k_0 = 0.01$
$k_0 = 0.05$
$k_0 = 0.1$
$l(l+1)C_l/2\pi(\mu K^2)$
l
$k(\mathrm{Mpc}^{-1})$


$k^{-1} \sim 6000h^{-1}l^{-1}$Mpc
ΛCDM
B = 0.1
B = 0.5
$l(l+1)C_l/2\pi(\mu K^2)$
l



$\Omega_B = 0.040$
$\Omega_{DM} = 0.200$
Us
Som
ic th
It
Possible dege
neracies?
$B = 0.1$
The universe
$l(l+1)C_l/2\pi(\mu K^2)$
$k(\mathrm{Mpc}^{-1})$

Conclusions

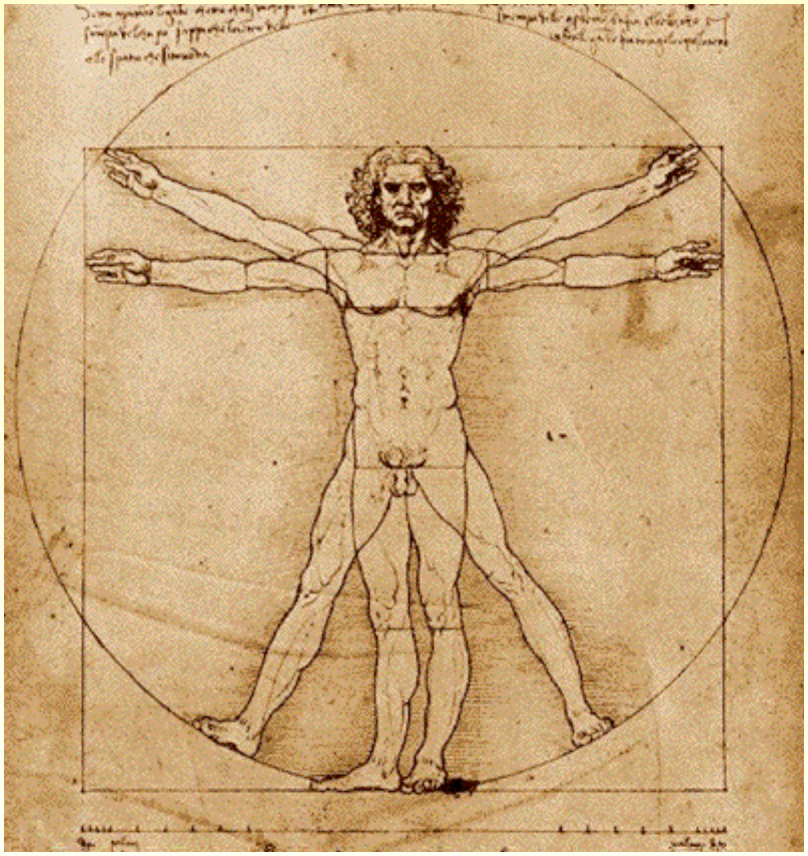
- Straightforward calculation for the power sp ectrum is available, motivated from the observed $|n-1|$ and $\left|\frac{dn}{d\ln k}\right|$ of comparable size.

- We can obtain an **explicit** and **analytic** expr ession for the curvature power spectrum.

- Generally, a feature in otherwise flat and slo wly varying inflaton potential induces a scal e dependent **oscillation** and possibly a **modu lation** in power across the feature.

- It seems the current ly observ m with featu red inflat add to est arameters.

What we should do is…



STRUGGLE