1 Introduction to Supersymmetry

1.1 The Unreasonable Effectiveness of the Standard Model

loop corrections make the Higgs mass diverge quadratically

$$\mathcal{L}_{\text{Yukawa}} = -\frac{\lambda_t}{\sqrt{2}} H^0 \overline{t_L} t_R + h.c. \tag{1.1}$$

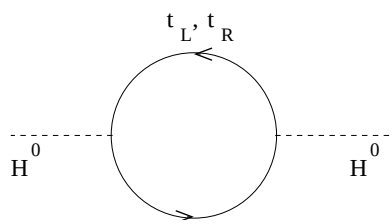

Figure 1:

$$= \frac{2iN_c|\lambda_t|^2}{16\pi^2}\left[\Lambda^2 - 3m_t^2 \ln\left(\frac{\Lambda^2 + m_t^2}{m_t^2}\right) + \ldots\right]. \tag{1.2}$$

invent new particles ϕ_L and ϕ_R with the following interactions:

$$\mathcal{L}_{\text{scalar}} = -\frac{\lambda}{2}(H^0)^2(|\phi_L|^2 + |\phi_R|^2) - \mu H^0(|\phi_L|^2 + |\phi_R|^2). \tag{1.3}$$

$$= \frac{-i\lambda N}{16\pi^2}\left[2\Lambda^2 - m_L^2 \ln\left(\frac{\Lambda^2 + m_L^2}{m_L^2}\right) - m_R^2 \ln\left(\frac{\Lambda^2 + m_R^2}{m_R^2}\right) + \ldots\right]. \tag{1.4}$$

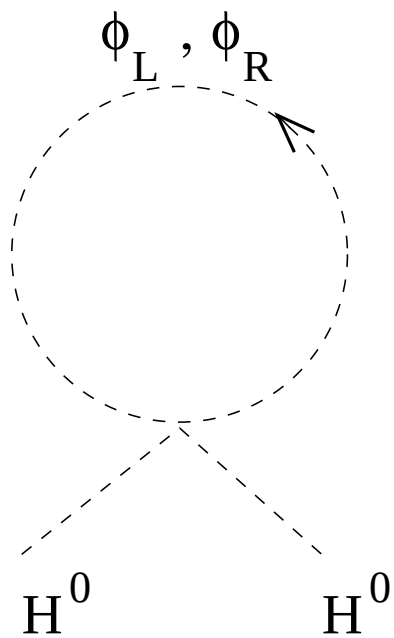

Figure 2:

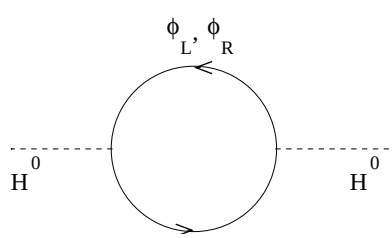

Figure 3:

$$= \frac{i\mu^2 N}{16\pi^2}\left[\ln\left(\frac{\Lambda^2+m_L^2}{m_L^2}\right)+\ln\left(\frac{\Lambda^2+m_R^2}{m_R^2}\right)+\ldots\right]. \tag{1.5}$$

if $N = N_c$ and $\lambda = |\lambda_t|^2$ the quadratic divergence is cancelled, if we also have $m_t = m_L = m_R$ and $\mu^2 = 2\lambda m_t^2$ the logarithmic divergence cancels. SUSY will garauntee these conditions.

1.2 SUSY Algebra

The generators of SUSY are complex, anti-commuting spinors Q and $Q^\dagger$:

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu, \tag{1.6}$$
$$\{Q_\alpha, Q_\beta\} = \{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0 \tag{1.7}$$
$$[P_\mu, Q_\alpha] = [P_\mu, Q^\dagger_{\dot\alpha}] = 0 \tag{1.8}$$
$$[Q_\alpha, R] = Q_\alpha \tag{1.9}$$
$$[Q^\dagger_{\dot\alpha}, R] = -Q^\dagger_{\dot\alpha} \tag{1.10}$$
$$\tag{1.11}$$

where P_μ is the momentum operator and R is a $U(1)$ charge.

$$\sigma^\mu_{\alpha\dot\alpha} = (1, \sigma^i) \tag{1.12}$$

$$\overline{\sigma}^{\mu\alpha\dot\alpha} = (1, -\sigma^i) \tag{1.13}$$

note:

$$H = P^0 = \frac{1}{4}(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2) \tag{1.14}$$

for the subspace of states with the same p_μ:

$$\begin{aligned}
4\sum_i \langle i|(-1)^F P^0|i\rangle &= \sum_i \langle i|(-1)^F Q Q^\dagger|i\rangle + \sum_i \langle i|(-1)^F Q^\dagger Q|i\rangle \\
&= \sum_i \langle i|(-1)^F Q Q^\dagger|i\rangle + \sum_i \sum_j \langle i|(-1)^F Q^\dagger|j\rangle\langle j|Q|i\rangle \\
&= \sum_i \langle i|(-1)^F Q Q^\dagger|i\rangle + \sum_j \langle j|Q(-1)^F Q^\dagger|j\rangle \\
&= \sum_i \langle i|(-1)^F Q Q^\dagger|i\rangle - \sum_j \langle j|(-1)^F Q Q^\dagger|j\rangle \\
&= 0.
\end{aligned} \tag{1.15}$$

therefore: $n_B = n_F$ for each supermultiplet with $p^0 \neq 0$
vacuum properties: $H \geq 0$,
$|0\rangle$ is supersymmetric $\Rightarrow Q_\alpha|0\rangle = 0 \Rightarrow \langle 0|H|0\rangle = 0$;
$|0\rangle$ is non-supersymmetric $\Rightarrow Q_\alpha|0\rangle \neq 0 \Rightarrow \langle 0|H|0\rangle \neq 0$

1.3 SUSY Representations

Massive particles are labeled by mass, total spin, and one component of the spin: $|m, s, s_3\rangle$

rest frame $p_\mu = (m, \vec{0})$

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = 2m\delta_{\alpha\dot\alpha} \tag{1.16}$$

SUSY algebra reduces to Clifford algebra

"Clifford vacuum" state:

$$|\Omega_s\rangle = Q_1 Q_2 |m, s, s_3\rangle \tag{1.17}$$

$$Q_1|\Omega_s\rangle = Q_2|\Omega_s\rangle = 0 \tag{1.18}$$

massive supermultiplet:

$$\begin{array}{c} |\Omega_s\rangle \\ Q_1^\dagger|\Omega_s\rangle, Q_2^\dagger|\Omega_s\rangle \\ Q_1^\dagger Q_2^\dagger|\Omega_s\rangle \end{array} \tag{1.19}$$

massive chiral multiplet:

$$\begin{array}{rc} \text{state} & s_3 \\ |\Omega_0\rangle & 0 \\ Q_1^\dagger|\Omega_0\rangle, Q_2^\dagger|\Omega_0\rangle & \pm\frac{1}{2} \\ Q_1^\dagger Q_2^\dagger|\Omega_0\rangle & 0 \end{array} \tag{1.20}$$

massive vector multiplet:

$$\begin{array}{rc} \text{state} & s_3 \\ |\Omega_{\frac{1}{2}}\rangle & \pm\frac{1}{2} \\ Q_1^\dagger|\Omega_{\frac{1}{2}}0\rangle, Q_2^\dagger|\Omega_{\frac{1}{2}}\rangle & 0, 1, 0, -1 \\ Q_1^\dagger Q_2^\dagger|\Omega_{\frac{1}{2}}\rangle & \pm\frac{1}{2} \end{array} \tag{1.21}$$

Massless particles are labeled by energy and helicity: $|E, \lambda\rangle$

choose the frame where $p_\mu = (E, 0, 0, E)$

$$\{Q_1, Q_1^\dagger\} = 4E, \tag{1.22}$$

$$\{Q_2, Q_2^\dagger\} = 0. \tag{1.23}$$

$$|\Omega_\lambda\rangle = Q_1|E, \lambda\rangle, \tag{1.24}$$

$$Q_1|\Omega_\lambda\rangle = 0 \tag{1.25}$$

$$\langle\Omega_\lambda|Q_2Q_2^\dagger|\Omega_\lambda\rangle + \langle\Omega_\lambda|Q_2^\dagger Q_2|\Omega_\lambda\rangle = 0 \tag{1.26}$$

$$\Rightarrow \langle\Omega_\lambda|Q_2Q_2^\dagger|\Omega_\lambda\rangle = 0 \tag{1.27}$$

so $Q_2^\dagger$ produces states of zero norm.

massless supermultiplet:

$$\begin{array}{rc} \text{state} & \text{helicity} \\ |\Omega_\lambda\rangle & \lambda \\ Q_1^\dagger|\Omega_\lambda\rangle & \lambda+\frac{1}{2} \end{array} \tag{1.28}$$

CPT requires the presence of states with helicity $-\lambda$ and $-\lambda-\frac{1}{2}$ as well:

$$\begin{array}{rc} \text{state} & \text{helicity} \\ |\Omega_{-\lambda-\frac{1}{2}}\rangle & -\lambda-\frac{1}{2} \\ Q_1^\dagger|\Omega_{-\lambda-\frac{1}{2}}\rangle & -\lambda \end{array} \tag{1.29}$$

massless chiral multiplet:

$$\begin{array}{cc} \text{state} & \text{helicity} \\ |\Omega_0\rangle & 0 \\ Q_1^\dagger|\Omega_0\rangle & \frac{1}{2} \end{array} \tag{1.30}$$

+ CPT conjugate:

$$\begin{array}{cc} \text{state} & \text{helicity} \\ |\Omega_{\frac{-1}{2}}\rangle & -\frac{1}{2} \\ Q_1^\dagger|\Omega_{\frac{-1}{2}}\rangle & 0 \end{array} \tag{1.31}$$

massless vector multiplet:

$$\begin{array}{cc} \text{state} & \text{helicity} \\ |\Omega_{\frac{1}{2}}\rangle & \frac{1}{2} \\ Q_1^\dagger|\Omega_{\frac{1}{2}}\rangle & 1 \end{array} \tag{1.32}$$

+ CPT conjugate:

$$\begin{array}{cc} \text{state} & \text{helicity} \\ |\Omega_{-1}\rangle & -1 \\ Q_1^\dagger|\Omega_{-1}\rangle & -\frac{1}{2} \end{array} \tag{1.33}$$

Superpartner names:

fermion	$\leftrightarrow$	sfermion
quark	$\leftrightarrow$	squark
gauge boson	$\leftrightarrow$	gaugino
gluon	$\leftrightarrow$	gluino

References

[1] S. Martin, "A Supersymmetry Primer," hep-ph/9709356.

[2] S. Coleman and J. Mandula, *Phys. Rev.* **159**, 1251 (1967).

[3] R. Haag, J. Lopuszanski, and M. Sohnius, *Nucl. Phys.* **B88**, 257 (1975).

[4] J. Lykken, "Introduction to Supersymmetry," hep-th/9612114.