Renormalization of the S Parameter in Holographic Theories of EWSB

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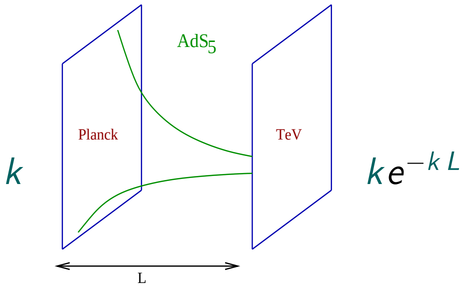
Santa Fe Summer Workshop, July 7 2008

Outline

Solution to the Hierarchy in AdS_5

- One compact extra dimension. Non-trivial metric induces small energy scale from Planck scale.



- For $kR \simeq (11 - 12) \Rightarrow$

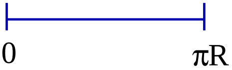
$$\boxed{\Lambda_{\mathrm{TeV}} \sim M_{\mathrm{Planck}}\, e^{-kL}} \qquad \text{with } k \text{ the curvature}$$

AdS$_5$

- RS in AdS$_5$

$$ds^2 = e^{-2\kappa|y|}\,\eta^{\mu\nu}dx_\mu dx_\nu - dy^2$$

- Compactified on S_1/Z_2 with $L = \pi R$



and $k \lesssim M_P$, AdS_5 curvature.

- Or, in conformal coordinates

$$ds^2 = \frac{1}{(k\,z)^2}\,\left(\eta^{\mu\nu}dx_\mu dx_\nu - dz^2\right)$$

AdS_5

Original RS: only gravity propagates in the bulk.
But

- Higher-dimensional operators are only suppressed by the TeV scale.
  Flavor-violation, GUT-induced proton decay, etc.
- But to solve the Hierarchy Problem we only need the Higgs to be localized near the TeV brane.

Bulk RS Models

If we allow gauge fields and fermions in the 5D bulk

What is the bulk gauge theory ?

- Cannot be $SU(2)_L \times U(1)_Y$. Breaks isospin at tree-level.
  $\Rightarrow$ Bulk gauge theory must include isospin symmetry.
  Minimal choice:
  $$SU(2)_L \times SU(2)_R \times U(1)_X$$
  Corresponds to having a global $SU(2)_C$ in the dual 4D theory.

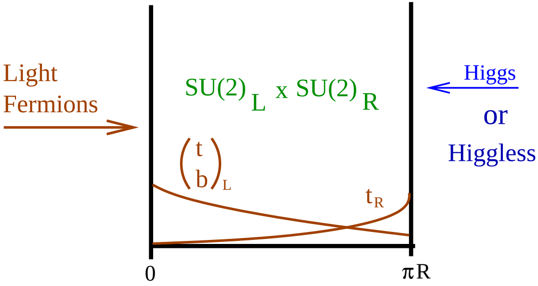
Bulk RS Models

How are the zero-mode fermions localized ?

- To avoid effects of higher-dimensional operators, flavor violation, etc.
  $\Rightarrow$ Light fermions localized near the Plank brane
- Get fermion masses without fine-tuning: $O(1)$ bulk mass parameters
  $\Rightarrow$ Heavier fermions $(t_R, (t_L, b_L))$ localized near TeV brane

Bulk RS Models

The Minimal Bulk RS Model:



- Solution to the Hierarchy Problem
- Fermion localization ⇒ fermion mass hierarchy

Electroweak Precision Constraints

The T parameter:

- As long as bulk gauge symmetry is isospin preserving ($SU(2)_L \times SU(2)_R$, $SO(5)$, ...)
  $\Rightarrow$ No large T parameter contributions
- There are still some loop-induced contributions to T

Electroweak Precision Constraints

But there is a tree-level S parameter contribution

$$S_{\mathrm{tree}} \simeq 2\pi v^2 z_1^2 \simeq 12\pi \frac{v^2}{M_{\mathrm{KK}}^2}$$

E.g. for $M_{\mathrm{KK}} = 2.5$ TeV, $S_{\mathrm{tree}} \simeq 0.3$

- S_{tree} can be made smaller by de-localizing light fermions from Plank brane
- But we loose theory of flavor

S to One Loop

Why compute S to one loop in these theories ?

- S_{tree} is large, important constraint on RS bulk models. Can there be cancellations ?
- Counter-term $\Rightarrow$ there could be infinities $\Rightarrow$ need to renormalize S

To compute S at one loop, write the low-energy effective theory by integrating out the bulk

The Holographic Description

The bulk gauge theory is $SU(2)_L \times SU(2)_R \times U(1)_X$

$$\mathcal{S} = \int d^4x\, dz \sqrt{g} \left\{ -\frac{1}{2}\mathrm{Tr}\left[L_{MN}L^{MN}\right] - \frac{1}{2}\mathrm{Tr}\left[R_{MN}R^{MN}\right] - \frac{1}{4}X_{MN}X^{MN} \right\}$$

The Higgs is IR-localized and a $(\mathbf{2},\mathbf{2})_0$

$$\mathcal{S}_H = \int d^4x\, dz\, \sqrt{g}\, \delta(z - z_1)\, \left\{ \mathrm{Tr}\left[|D_\mu H|^2\right] - V(H) \right\}$$

with

$$D_\mu = \partial_\mu + i g_{5L} L_\mu H - i g_{5R} H R_\mu$$

The Holographic Description

The Higgs VEV at the IR brane

$$\langle H \rangle = \frac{1}{2} \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}$$

breaks $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$

Go to the (V, A) basis:

$$\begin{aligned} V_M &= \frac{1}{\sqrt{g_{5L}^2 + g_{5R}^2}} (g_{5R} L_M + g_{5L} R_M)\,, \\ A_M &= \frac{1}{\sqrt{g_{5L}^2 + g_{5R}^2}} (g_{5L} L_M - g_{5R} R_M) \end{aligned}$$

The Holographic Description

Solve the bulk EOM, enforcing the IR boundary conditions

$$\partial_z V_\mu|_{z_1} = V_5|_{z_1} = \partial_z X_\mu|_{z_1} = X_5|_{z_1} = 0$$

$$\left(\frac{1}{kz}\partial_z + \frac{g_{5L}^2 + g_{5R}^2}{4} v^2\right) A_\mu|_{z_1} = A_5|_{z_1} = 0$$

The solutions can be written in terms of the UV fields $V_\mu^0(p) = V_\mu(p, z_0)$, $A_\mu^0(p) = A_\mu(p, z_0)$ and $X_\mu^0(p) = X_\mu(p, z_0)$, as

$$V_\mu(p, z) = V_\mu^0(p) f_V(p, z) \qquad A_\mu(p, z) = A_\mu^0(p) f_A(p, z)$$

$$X_\mu(p, z) = X_\mu^0(p) f_V(p, z)$$

The Holographic Description

- Solutions back into $\mathcal{S} \Rightarrow$ UV boundary theory from UV boundary terms
- UV dynamical fields are

$$L_\mu^{a(0)}(p) = W_\mu^a \ , \qquad B_\mu = \frac{g_{5X} R_\mu^3 + g_{5R} X_\mu}{\sqrt{g_{5R}^2 + g_{5X}^2}}$$

- Effective 4D theory with UV degrees of freedom

The Effective Low Energy Theory

In terms of UV boundary fields

$$\mathcal{L}_{\rm eff} = \frac{P_{\mu\nu}}{2}\left[W^a_\mu \Pi_L(p^2) W^a_\nu + 2W^3_\mu \Pi_{3Y}(p^2) B_\nu + B_\mu \Pi_B(p^2) B_\nu\right] + \dots$$

with

$$\Pi_L(p^2) = \frac{g_{5R}^2 \Pi_V + g_{5L}^2 \Pi_A}{g_{5L}^2 + g_{5R}^2}$$

$$\Pi_B(p^2) = \frac{(g_{5L}^2 g_{5X}^2 + g_{5L}^2 g_{5R}^2 + g_{5R}^4)\Pi_V + g_{5R}^2 g_{5X}^2 \Pi_A}{(g_{5L}^2 + g_{5R}^2)(g_{5R}^2 + g_{5X}^2)}$$

The Effective Low Energy Theory

And

$$\Pi_{3Y}(p^2) \quad = \quad \frac{g_{5L} g_{5R} g_{5X}}{(g_{5L}^2 + g_{5R}^2)\sqrt{g_{5R}^2 + g_{5X}^2}}(\Pi_V - \Pi_A)$$

with $\Pi_V(p^2)$ and $\Pi_A(p^2)$ determined by $f_V(p,z)$ and $f_A(p,z)$.

Bulk $\Rightarrow$ tree-level misalignment of UV gauge basis w.r.t. IR gauge basis:

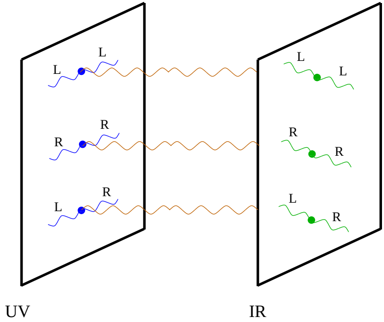
$$g\, g'\, S_{\rm tree} = -16\pi \Pi'_{3Y}(0)$$

The S Parameter – Tree Level

The tree-level contribution

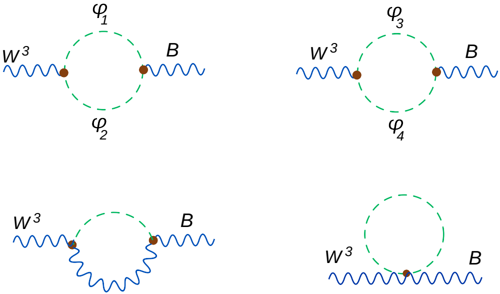
$$S_{\rm tree} = 4\pi v^2 z_1^2 \frac{32 + 3(g^2 + g'^2) v^2 z_1^2}{(8 + (g^2 + g'^2) v^2 z_1^2)^2} \simeq 2\pi v^2 z_1^2$$

for $v z_1 \ll 1$.



The S Parameter – One Loop

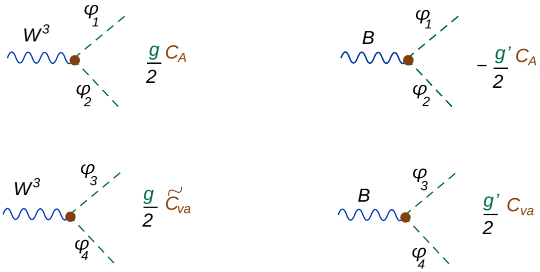
Want to compute Higgs-Gauge sector loop contributions to S



The S Parameter – One Loop

Insert solutions of EOM in

$$\mathcal{S}_H = \int d^4x\, dz\, \sqrt{g}\, \delta(z - z_1)\, \left\{ \mathrm{Tr}\left[|D_\mu H|^2 \right] + \ldots) \right\}$$



The S Parameter – One Loop

With

$$\begin{aligned} C_A(p) &= f_A(p,z)\Big|_{\mathrm{IR}} \\ \tilde{C}_{\mathrm{VA}}(p) &= \frac{2g_{5R}^2 f_V(p,z) + (g_{5L}^2 - g_{5R}^2) f_A(p,z)}{2(g_{5L}^2 + g_{5R}^2)}\Bigg|_{\mathrm{IR}} \\ C_{\mathrm{VA}}(p) &= \frac{2g_{5L}^2 f_V(p,z) + (g_{5R}^2 - g_{5L}^2) f_A(p,z)}{2(g_{5L}^2 + g_{5R}^2)}\Bigg|_{\mathrm{IR}} \end{aligned}$$

The S Parameter – One Loop

Sorting out the one-loop contributions

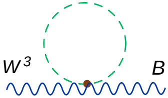
- S is divergent !
- But *some* contributions/divergences absorbed in renormalization of $S_{\rm tree}$

$$S_{\rm tree} \simeq 2\pi v^2 z_1^2$$

Which are the *genuine* contributions to S, and which renormalizations of v appearing in $S_{\rm tree}$?

The S Parameter – One Loop

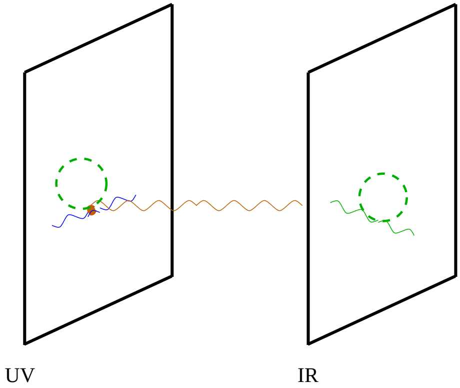
Example:



Contributions to S (e.g. p dependence) from

- External momentum dependence in the "4D" loop
- Momentum dependence in the couplings:
  Renormalization of v^2 in IR, picks up p dependence from bulk

The S Parameter – One Loop



$\Rightarrow$ This p-dependent $W^3 - B$ mixing is absorbed by v^2 in S_{tree}.

The S Parameter – One Loop

The rule is then

- Keep the contributions to S coming from loops with "4D" external momentum dependence
- Discard contributions where the external momentum dependence comes from the bulk. They correspond to renormalizations of v

The S Parameter – One Loop

Still, S is divergent.

$$
\begin{aligned}
S_{\rm loop} &= \frac{1}{12\pi}\left(N_\epsilon - 1\right)\left[\tilde{C}_{\rm VA}\, C_{\rm VA} - C_A^2\right] \\
&\quad + \frac{1}{12\pi}\left[C_A^2 \ln\left(\frac{m_h^2}{\mu^2}\right) - \tilde{C}_{\rm VA}\, C_{\rm VA} \ln\left(\frac{M_W^2}{\mu^2}\right)\right]
\end{aligned}
$$

with $N_\epsilon = 2/\epsilon + \ldots$

The S Parameter – Divergence

In the $v^2 z_1^2 \ll 1$ limit,

$$S_{\mathrm{loop}} \simeq \frac{(g_{5L}^2 + g_{5R}^2) k v^2 z_1^2}{48\pi} \ln\left(\frac{\Lambda^2}{\mu^2}\right) + \textit{finite terms}$$

Divergences in S

How to proceed ?

- Since $\Lambda \simeq O(1)$ TeV, "divergent" contribution is relatively small

$$S_{\text{loop}} \lesssim +0.1$$

or

- Renormalize S:
  - Introduce a counter-term to cancel $\ln \Lambda^2$
  - Renormalization condition for S fixes the finite terms

Renormalizing S

What should be the renormalization condition for S ?

- Counter-term cancels Log divergence only ?
- Finite terms from loops also cancel ? (Typical sizes $\simeq 0.1$)
- Can we choose any renormalization condition for S ?

Conclusions

- S is Log divergent in bulk RS models of EWSB
- One-loop divergences are likely present in any Holographic model of EWSB
- What is the "correct" renormalization condition for S?
- Fermions ?