Power of Kinematic Constraints

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Outline

What is kinematics constraints

Applications at the LHC

Conclusion and Outlook

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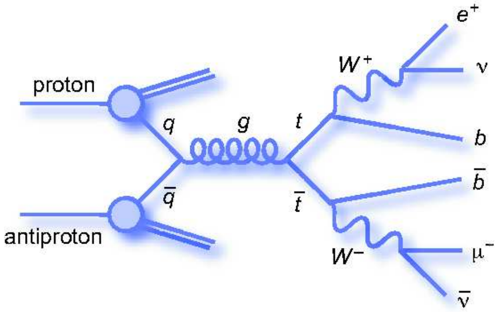
Applications at the LHC

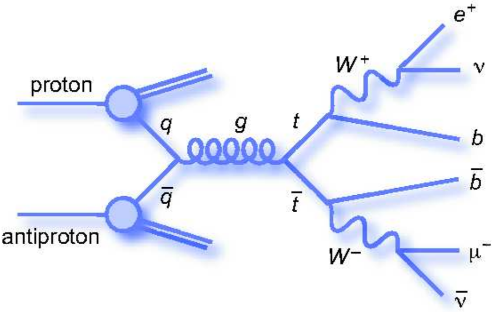
Conclusion and Outlook

An example, $t\bar{t}$ production.



Assuming $W^+, W^-, t, \bar{t}$ on-shell, no other invisible particles:

An example, $t\bar{t}$ production.



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$$p_\nu^2 = p_{\bar{\nu}}^2 = 0,$$

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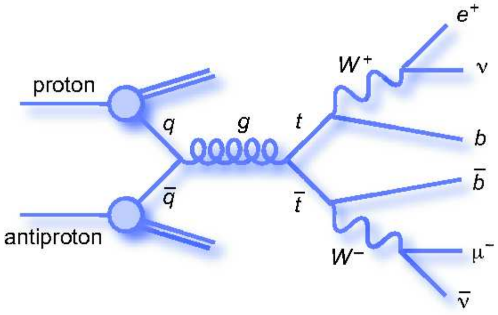

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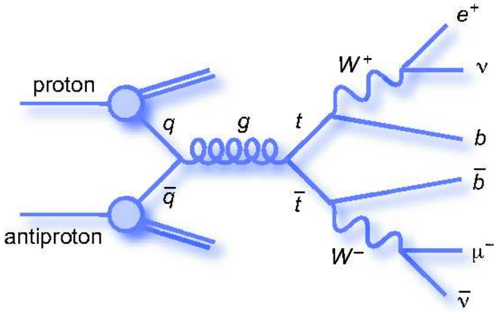

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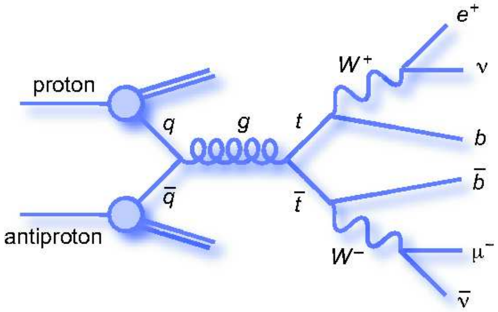

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(p_{l+} + p_\nu + p_b)^2 = (p_{l-} + p_{\bar{\nu}} + p_{\bar{b}})^2 = m_t^2, \\
p_\nu^x + p_{\bar{\nu}}^x = p_{miss}^x, p_\nu^y + p_{\bar{\nu}}^y = p_{miss}^y.
\end{gathered}
$$

8 unknowns: p_ν and $p_{\bar{\nu}}$. 8 equations. $\rightarrow$ We can solve the system.

What's the use?

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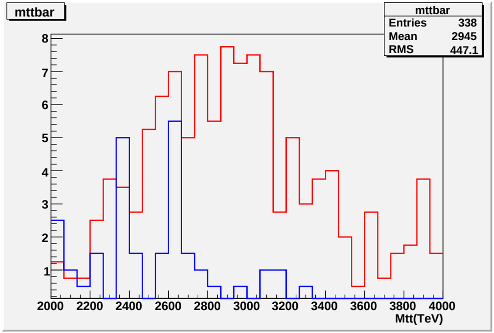
$t\bar{t}$ invariant mass

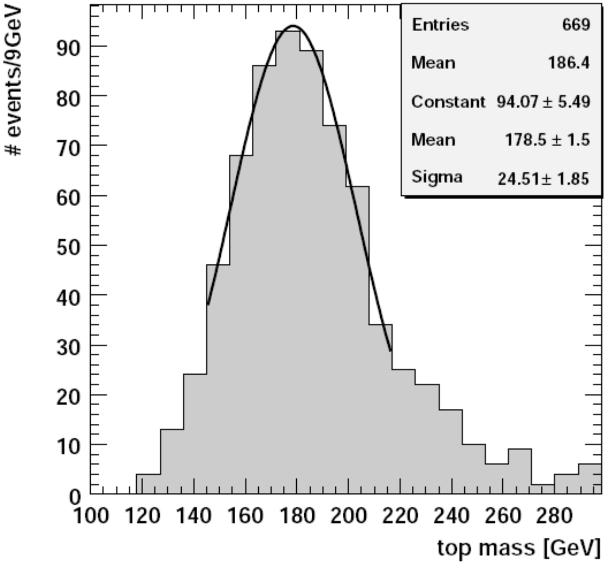
$t\bar{t}$ resonance. (*with Y. Bai, in progress*)



3TeV resonance with 300GeV width, 100 fb^{-1}, after detector simulation, dilepton channel. Red: signal. Blue: SM $t\bar{t}$. Other background not included yet.

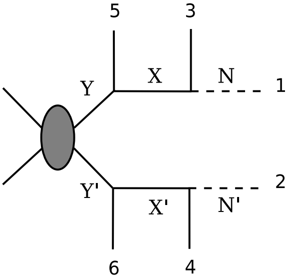
Measure top mass

- Vary top mass and try to solve the system.
- Look at number of solvable events (CMS note 2006/077):



Generalization

- Same topology, different processes.
- Assume $m_Y = m_{Y'}, m_X = m_{X'}, m_N = m_{N'}$.
- Example:$\tilde{\chi}_2^0 \to \tilde{l}l \to \tilde{\chi}_1^0 ll$.
- All masses unknown.

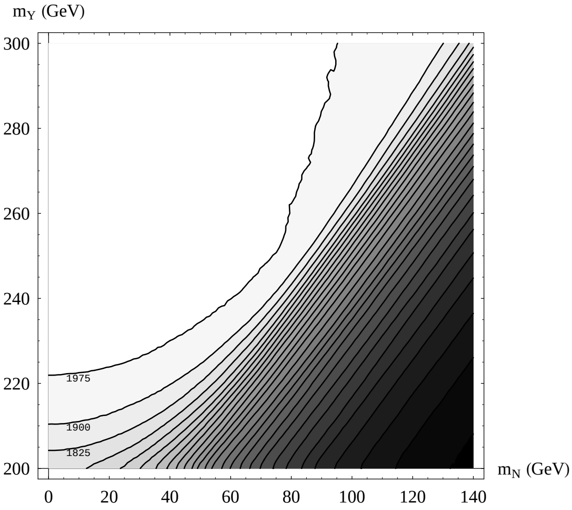


Can we determine the masses?

With *Cheng, Gunion, Manradella, McElrath*, arXiv:0707:0030.

Number of solvable events distribution

Solvable means having physical (real) solutions. 2000 events, with detector simulation. Input masses: 246.6, 182.4, 85.3.



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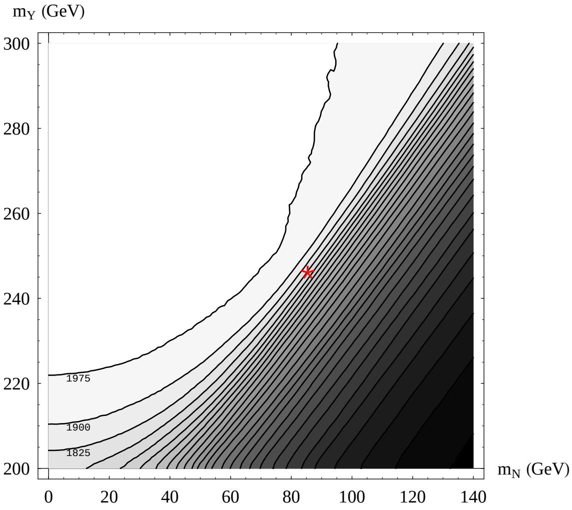
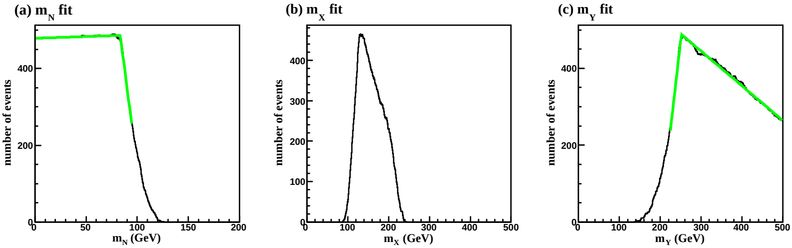


One dimensional fits

- Number of solvable events, fixing two masses.
- Take the “turning” point as the estimation of the mass.



Recursive fits

Starting from some random masses satisfying $m_N < m_X < m_Y$, apply one-dimensional recursive fits.
Update the masses after each fit.

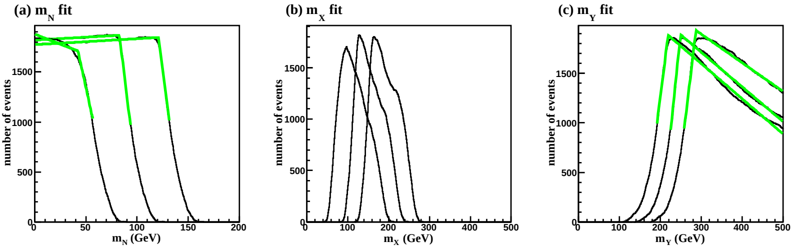
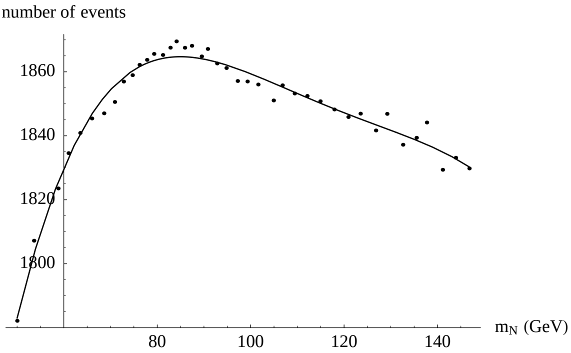


Figure: After cuts $|\eta|_\mu < 2.5, p_{T\mu} > 10$ GeV, $\not{p}_T > 50$ GeV

.

The masses go up, but the fits in general do not converge. However, the number of events at the "turning" points are maximized around the correct mass.

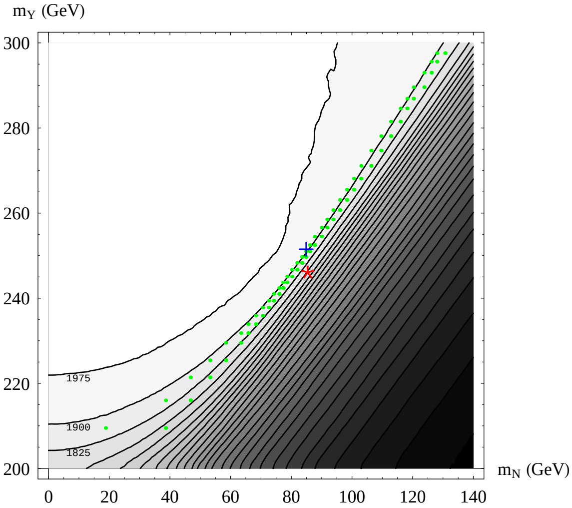
Number of events at the turning points



- Record the number of events at the turning points after each fit of m_N. Fit the plot to a polynomial and take the peak position as the estimation for m_N.
- Do a few one-dimensional fits for m_X and m_Y with fixed m_N until they are stablized.

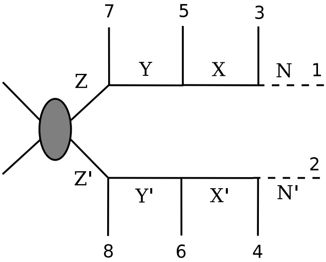
Number of solvable events distribution

The “+” is the fitted mass.



More complicated topology with *Cheng, Engelhardt, Gunion, McElath*. arXiv: 0802.4290

Example: $\tilde{q} \to q\tilde{\chi}_2^0 \to q\tilde{l}l \to \tilde{q}\chi_1^0 ll$
Assume $m_Z = m_{Z'}, m_Y = m_{Y'}, m_X = m_{X'}, m_N = m_{N'}$
Same unknowns p_1, p_2, more equations.



Count the constraints

One event, 6 constraints:

$$
\begin{aligned}
(M_N^2 &=) & p_1^2 &= p_2^2, \\
(M_X^2 &=) & (p_1+p_3)^2 &= (p_2+p_4)^2, \\
(M_Y^2 &=) & (p_1+p_3+p_5)^2 &= (p_2+p_4+p_6)^2, \\
(M_Z^2 &=) & (p_1+p_3+p_5+p_7)^2 &= (p_2+p_4+p_6+p_8)^2.
\end{aligned}
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$$p_1^x + p_2^x = p_{miss}^x, \quad p_1^y + p_2^y = p_{miss}^y.$$

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$$p_1^x + p_2^x = p_{miss}^x, \quad p_1^y + p_2^y = p_{miss}^y.$$

But 8 unknowns: p_1, p_2. Number of equations less than number of unkowns.

A pair of events.

Add one event,

$$
\begin{aligned}
q_1^2 &= q_2^2 \\
(q_1 + q_3)^2 &= (q_2 + q_4)^2 \\
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8 more unknowns q_1, q_2, but 10 more equations. 16 unknowns vs 16 equations, we can solve the system and obtain discrete solutions.

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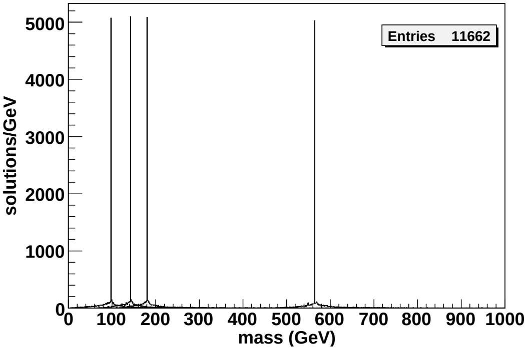
The system can be reduced to 13 linear equations+ 3 quadratic equations, so generally we have 8 complex solutions. Only keep real and positive ones for masses.

An ideal example

$\tilde{q}\tilde{q} \rightarrow q\tilde{\chi}_2^0 q\tilde{\chi}_2^0 \rightarrow q\tilde{l}lq\tilde{l}l \rightarrow q\tilde{\chi}_1^0 llq\tilde{\chi}_1^0 ll$

SPS1a, masses: (97.4, 142.5, 180.3, 564.8) GeV



2 solutions per pair on average.

Realistic case

- Wrong combinations. One event, 8 combinations for $2\mu 2e$ channel, 16 for 4μ or $4e$ channel. A pair of events, 64, 128 or 256 combinations.

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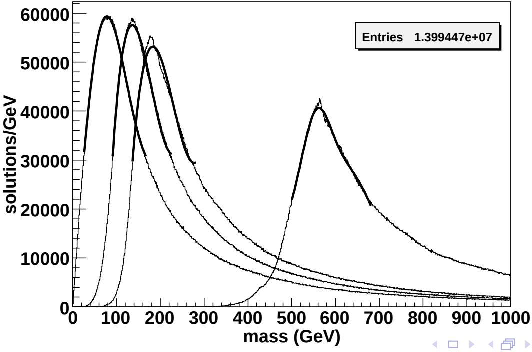
- Initial/final state radiation.
- Extra jets. $\tilde{g} \to q\tilde{q}_L$.
- Experimental resolutions. Simulated with ATLFAST.
- Background events.

Realistic solution distributions

Cuts:

1. 4 isolated leptons with $p_T > 10\ \mathrm{GeV}$, $|\eta| < 2.5$, consistent flavors and charges.
2. No b-jet, $\geq$ 2 jets with $p_T > 100\ \ \mathrm{GeV}$, $|\eta| < 2.5$. Take 2 highest-p_T jets as partiles 7 and 8.
3. $p_{Tmiss} > 50\ \mathrm{GeV}$.

About 1000 events ($\sim$ 700 signals) after cuts for 300 fb^{-1}.



Fit the masses

Fitting each curve using a sum of a Gaussian and a quadratic polynomial and take the peak positions as the estimated masses, we get $\{77.8, 135.6, 182.7, 562.0\}\ \mathrm{GeV}$.
Averaging over 10 different data sets:

$$m_N = 76.7 \pm 1.4\ \mathrm{GeV}, \quad m_X = 135.4 \pm 1.5\ \mathrm{GeV},$$
$$m_Y = 182.2 \pm 1.8\ \mathrm{GeV}, \quad m_Z = 564.4 \pm 2.5\ \mathrm{GeV}.$$

The statistical errors are very small, but the masses are biased.
Inputs: (97.4, 142.5, 180.3, 564.8) GeV

Some model-independent techniques
I. Cut off “bad” combinations

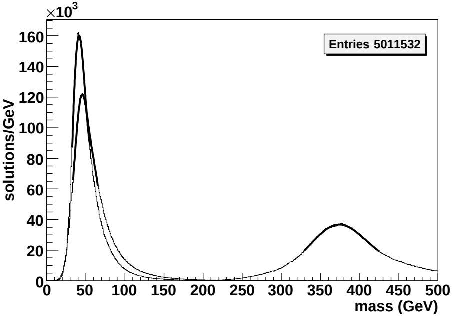
- For the ideal case, the correct combination of one event can always pair with any other event and yield solutions. So the number of events that pair with this combination is maximized as $N_{evt} - 1$.
- After smearing, this is no longer true, but the correct combinations still have statistically larger number of events to pair.
- We cut on this number so that we have about 4 combinations per event left (originally 11).

II. Number of solutions weighting.

A pair with many solutions enters with a large weight, although at most one of the solutions can be the true masses.
$\longrightarrow$ Treat each pair equally, weight the solutions by $1/n$, n=number of solutions for the pair.

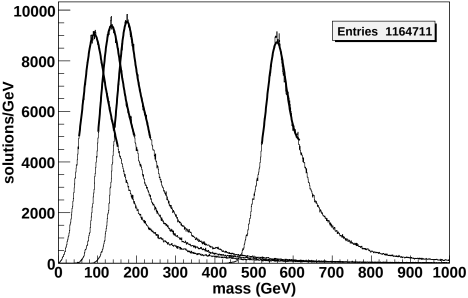
III. Cut on mass difference

Some solutions may have one or more, but not all four masses to be close to the true masses. Remove these solutions by a mass window cut.



Require all three mass differences to be within the mass window defined by $0.7\times$ peak height.

Mass peaks with smaller biases
SPS1a

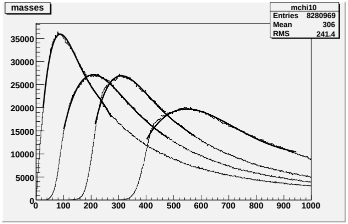


10 sets:

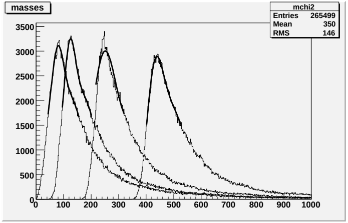
$$m_N = 94.1 \pm 2.8 \text{ GeV}, \quad m_X = 138.8 \pm 2.8 \text{ GeV},$$
$$m_Y = 179.0 \pm 3.0 \text{ GeV}, \quad m_Z = 561.5 \pm 4.1 \text{ GeV}.$$

Compare: $\{$ 97.4, 142.5, 180.3, 564.8 $\}$ GeV

Another mass point



Inputs $\{85.3, 128.4, 246.6, 431.1/438.6\}$ GeV.



10 sets:

$$m_N = 85 \pm 4 \text{ GeV}, m_X = 131 \pm 4 \text{ GeV}, m_Y = 251 \pm 4 \text{ GeV}, m_Z = 444 \pm 5 \text{ GeV}.$$

With this precision, we know more about the “model” and thus know how to Monte Carlo. The remaining biases can be removed by comparing with simulation.

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- Simple but Powerful.
- Pure kinematic, don't assume a model.
- A good start point for a more complicated method, ex, a full likelyhood fit with matrix element, PDF. . .
- Other topologies? Multiple channels?
- Spin determination?