Automated On-Shell Methods for One-Loop Amplitudes.

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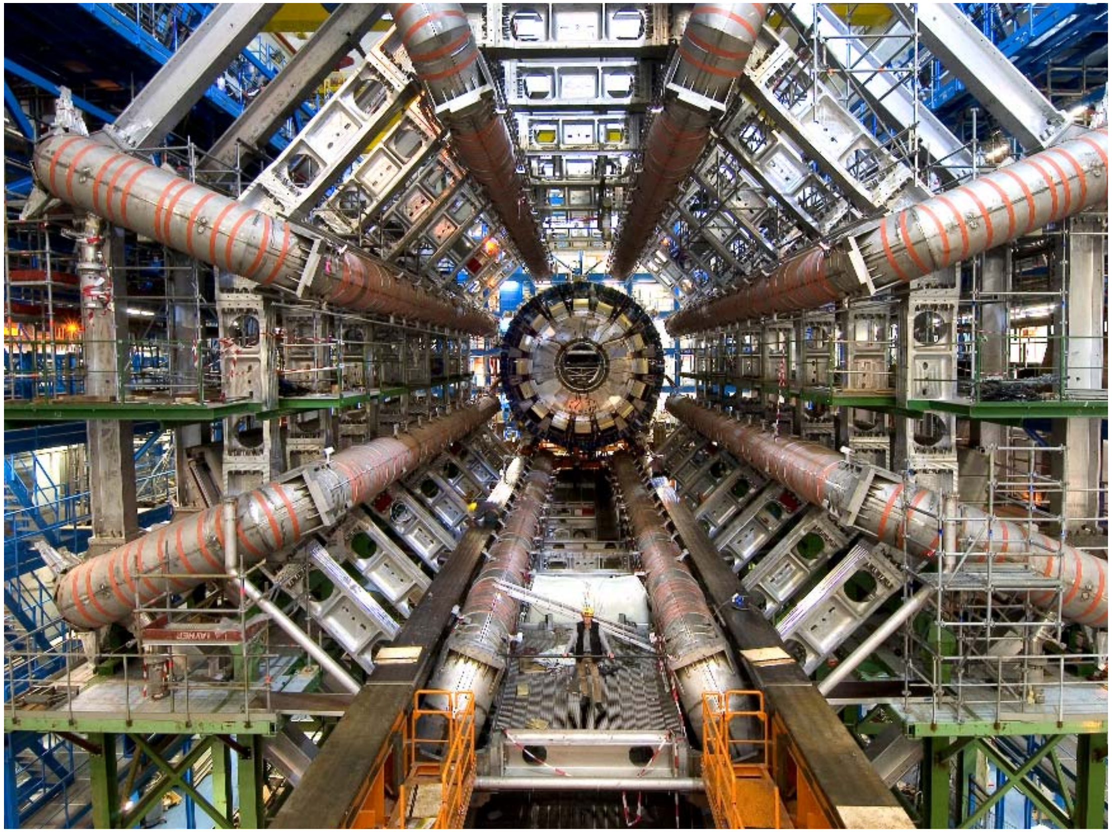
Santa Fee 08

based on work in collaboration with:

Carola Berger (SLAC), Zvi Bern (UCLA), Lance Dixon (SLAC), Fernando Febres Cordero (UCLA), Darren Forde (SLAC), David Kosower (Saclay), Daniel Maitre (SLAC).

LHC opens in: 29d15h0m0s



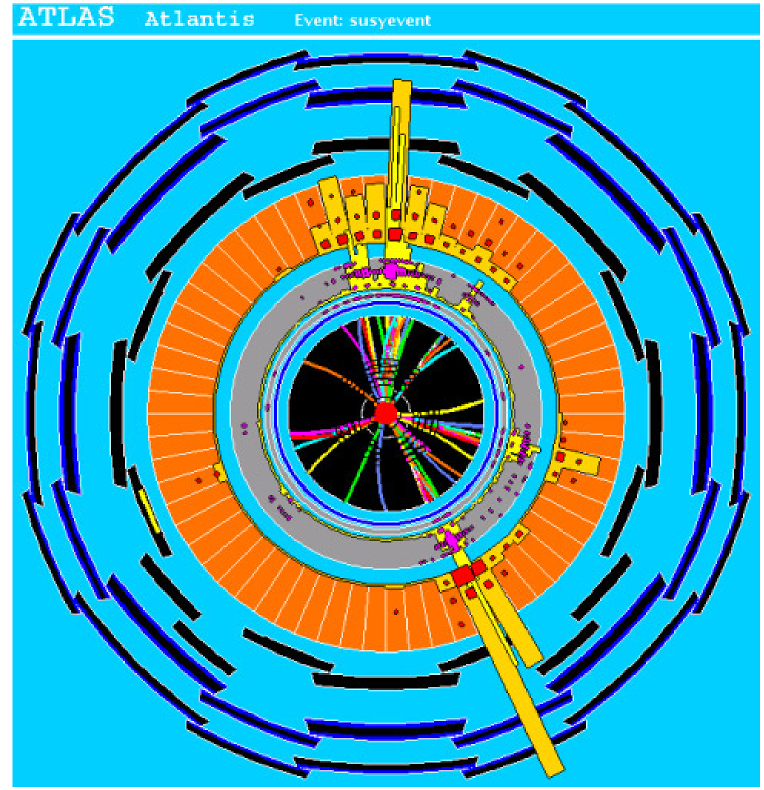
(http://www.lhcountdown.com/)

New ground:

- Proton-proton collisions @ **14 TeV** center-of-mass energy,7 times the previous one (Tevatron)
- Luminosity (collision rate) **10-100** times greater

Opportunity to understand physics at the next smallest distance scales.
Maximize discovery potential!

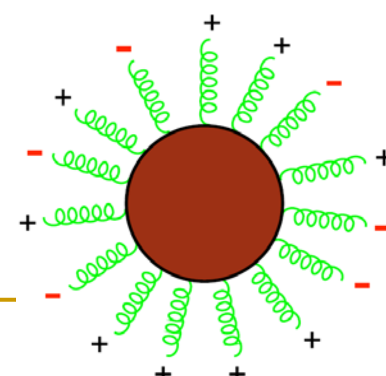
Challenge is complexity.



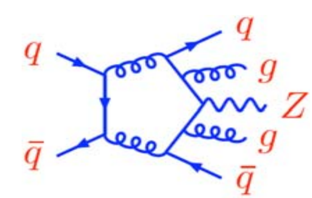
- Electron-positron colliders – clean events.
- Proton colliders – large backgrounds.
- Higher energy, new open thresholds, QCD at work, jets…

Focus here in high mulitiplicity processes: e.g.: susy:

- Signal: jets + missing energy
- SM background: jets + Z (Z→ν)



e.g.:



NLO needed.

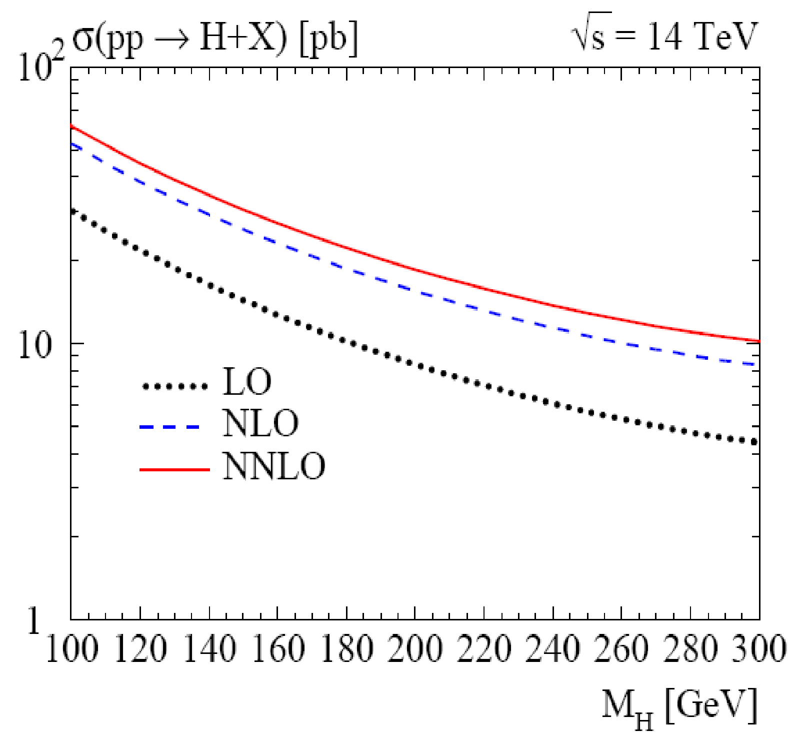
- Tree-level (LO) predictions qualitative, due to poor convergence of expansion in strong coupling,

$$\alpha_s(2\mu)^n \sim \alpha_s^n(\mu)(1 - n\,\alpha_s(\mu))\,, \quad \alpha_s(100GeV) \sim 0.12\,.$$

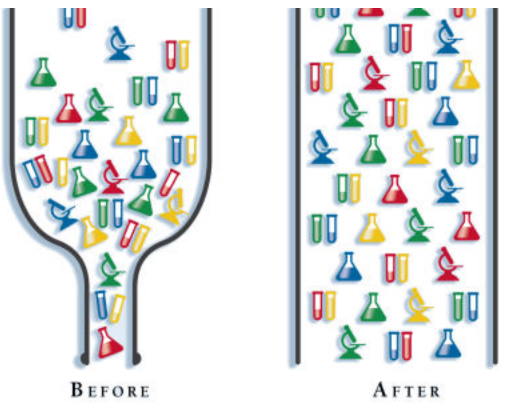
implies scale uncertainty:
  - •1 jet about 12%
  - •2 jets about 24%
  - •3 jets about 36%

- large NLO corrections can be 30% - 80% of LO.

Harlander, Kilgore (2002), Anastasiou, Melnikov (2002)



Bottleneck:



- LO information long available.
- subtraction method (*Catani, Seymour*) available. (*Gleisberg, Krauss;Seymour, Tevlin*)
- Slowdown: IR finite parts of one-loop virtual corrections.

No complete NLO QCD computation with at least 4 final state objects available!

(Electroweak a bit better: e+e- 4 fermions by *Denner, Dittmaier*)

"... extremely time and theorist consuming... " (*quote from Les Houches Summary Report 2007*)

process wanted at NLO ($V \in \{Z, W, \gamma\}$)	background to
1. $pp \to VV +$ jet	$t\bar{t}H$, new physics
2. $pp \to H + 2$ jets	H production by vector boson fusion (VBF)
3. $pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$
4. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
5. $pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to VV + 2$ jets	VBF $\to H \to VV$
7. $pp \to V + 3$ jets	new physics
8. $pp \to VVV$	SUSY trilepton

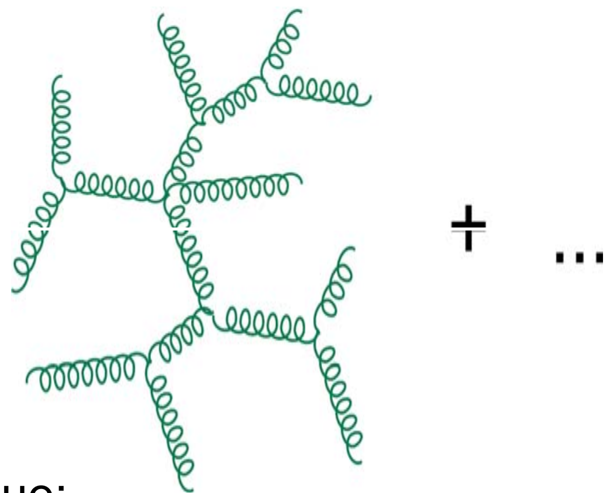
Dittmaier, Kallweit, Uwer; Campbell, Ellis, Zanderighi

Campbell, Ellis, Zanderighi; Ciccolini, Denner, Dittmaier

Lazopoulos, Melnikov, Petriello; Hankele, Zeppenfeld

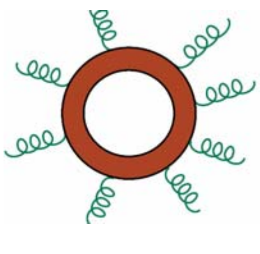
A more efficient way to compute?

Feynman rules tell us how to do computations…?!



Problems:

•Large expressions due to redundant information: cancellations between gauge non-invariant parts.



pure glue:

	3 jets	4 jets	5 jets	6 jets
# digrams	1000	10.000	150.000	3.000.000

• Numerical reduction of tensor integrals unstable.

Recent progress in on-shell methods.

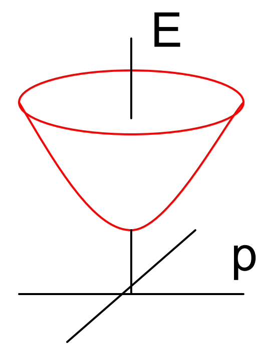
Many contributed to recent (string inspired) progress:

Anastasiou, Badger, Berger, Bern, Britto, Bjerrum-Bohr, Brandhuber, Cachazo, Del Duca, Dixon, Dunbar, Feng, Forde, Febres Cordero, Giele, Glover, Kosower, Kunszt, Mastrolia, McNamara, Melnikov, Ossola, Papadopoulos, Perkins, Pittau, Risager, Spence, Travaglini, Witten, ...

Tools now available. Will focus here on a particular approach. Keep others in mind.

Physical properties useful to compress:

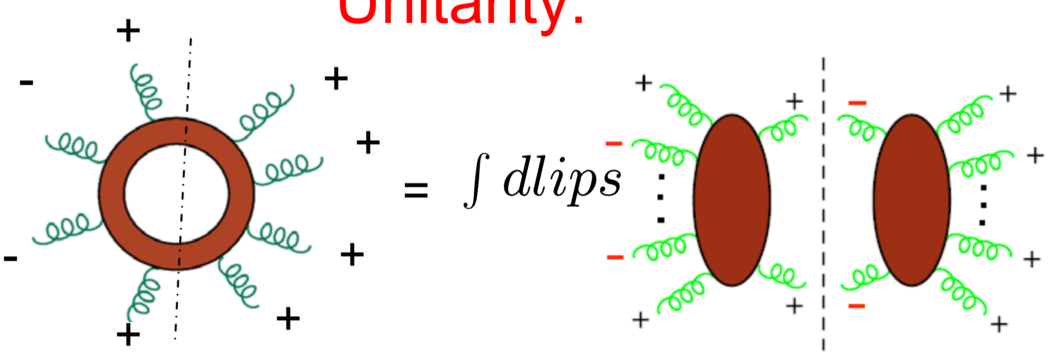
- Think off-shell work on-shell!



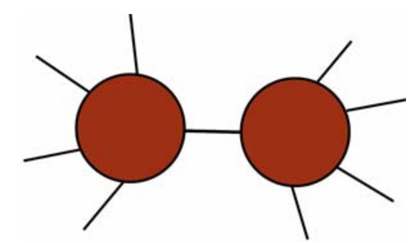
- Exploit field theory properties:

Unitarity:

Disc



Factorization:



- Use right variables.

Good variables for trees.

helicity basis:

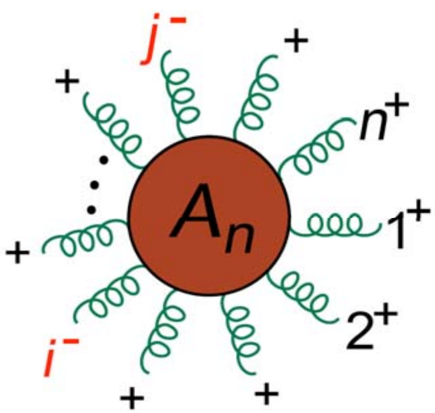
Exploit analytic properties:

- The right variables: spinor-helicity gives simplicity for trees.

spinor variables:

$$
\begin{aligned}
SO(4,\mathcal{C}) &\sim SL(2,C)\times SL(2,C)\,,\\
k^2=0: &\quad k^\mu=\sigma^\mu_{\alpha\dot\alpha}\lambda^\alpha\tilde\lambda^{\dot\alpha}\,,\\
\langle 12\rangle &\equiv \varepsilon_{\alpha\beta}\lambda_1^\alpha\lambda_2^\beta\,.
\end{aligned}
$$

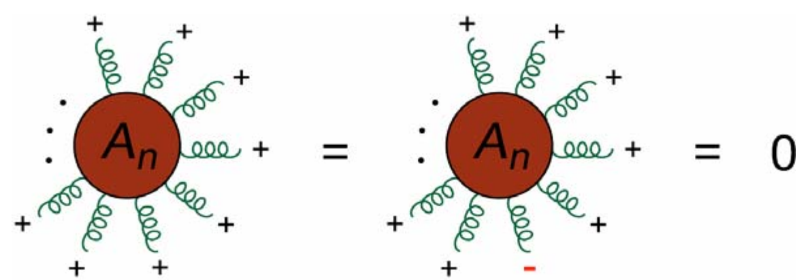
simple trees:



$$= \frac{\langle i\, j\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\cdots\langle n\,1\rangle}$$

Parke-Taylor formula (1986)

vanishing trees:



The one loop challenges: R, b, c, d

Bern, Dixon, Dunbar, Kosower, hep-ph/9403226

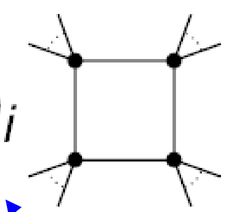
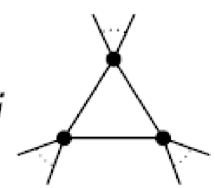
All external momenta are in *D=4*, loop momenta in *D=4-2ε* (dimensional regularization), one can write:

Rational part

Cut part

Known basis of scalar integrals:

$$A = R + C$$

$C = \sum_i b_i$  $+ \sum_i c_i$  $+ \sum_i d_i$ 

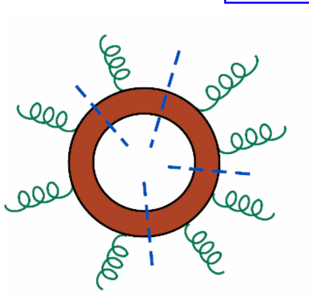
Process dependent D=4 rational integral coefficients

Cut parts from generalized unitarity.

Bern, Dixon, Kosower, hep-ph/9708239;
Britton, Cachazo, Feng, hep-th/0412308

Compare discontinuities:

Lhs:



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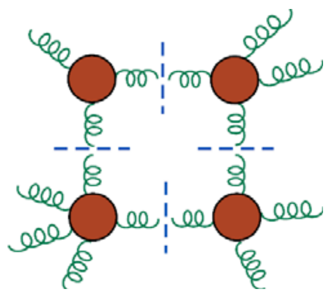

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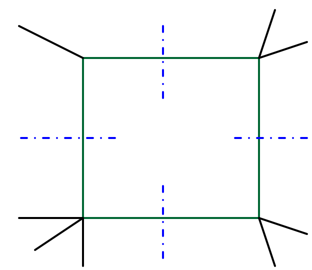
Rhs:

d_i 

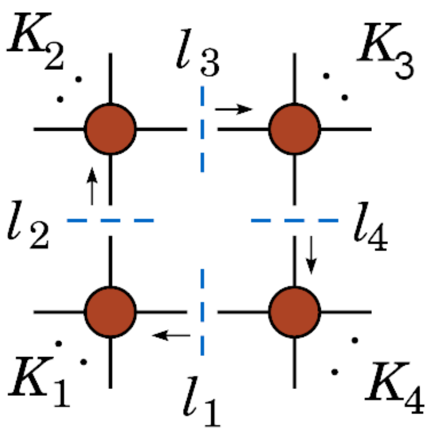
$$\int d^4l\,\delta(l^2)\delta((l-K_1)^2)\delta((l-K_2)^2)\delta((l-K_3)^2)\,A^{\text{1-loop}}(l)$$
$$=\sum_{\pm} A_1^{\text{tree}}(l_\pm)A_2^{\text{tree}}(l_\pm)A_3^{\text{tree}}(l_\pm)A_4^{\text{tree}}(l_\pm)=d_i\,.$$

D=4: 4 constraints=4 components of l $\rightarrow$ frozen integral (2 solutions)

Box-example:

Berger, Bern, Dixon, Fabres, Forde,H.I., Kosower, Maitre 0803.4180 [hep-ph]; Risager 0804.3310[hep-th]

Simplified analytic solutions for improved numerical stability.



$$(l_1^{(\pm)})^\mu = \frac{\langle 1^\mp | \not{K}_2 \not{K}_3 \not{K}_4 \gamma^\mu | 1^\pm \rangle}{2 \langle 1^\mp | \not{K}_2 \not{K}_4 | 1^\pm \rangle}, \qquad (l_2^{(\pm)})^\mu = -\frac{\langle 1^\mp | \gamma^\mu \not{K}_2 \not{K}_3 \not{K}_4 | 1^\pm \rangle}{2 \langle 1^\mp | \not{K}_2 \not{K}_4 | 1^\pm \rangle},$$

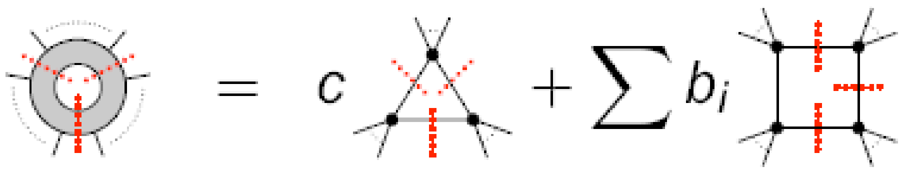
$$(l_3^{(\pm)})^\mu = \frac{\langle 1^\mp | \not{K}_2 \gamma^\mu \not{K}_3 \not{K}_4 | 1^\pm \rangle}{2 \langle 1^\mp | \not{K}_2 \not{K}_4 | 1^\pm \rangle}, \qquad (l_4^{(\pm)})^\mu = -\frac{\langle 1^\mp | \not{K}_2 \not{K}_3 \gamma^\mu \not{K}_4 | 1^\pm \rangle}{2 \langle 1^\mp | \not{K}_2 \not{K}_4 | 1^\pm \rangle}.$$

Non-physical (=spurious) singularities from parametrization.
Have to cancel eventually: task of the rational term c,b & R.

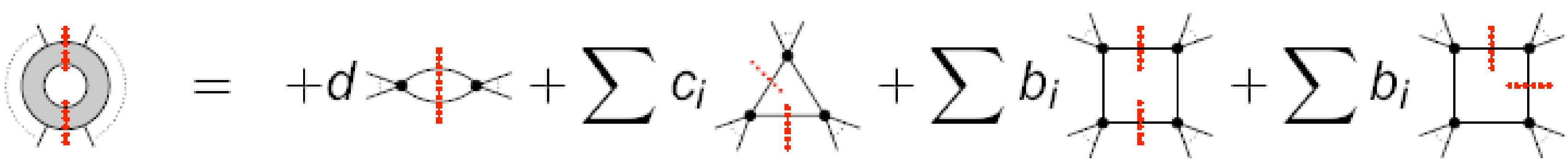
Bubble & triangle coefficients.

Britto et al. (2005,2006); Mastrolia (2006); Ossola, Papadopoulos, Pittau (2007); Ellis, Giele, Kunszt, 0708.2398[ph]; Forde (2007);..

Triangle coefficients from triple cuts. Subtract boxes.



Bubbles from double cuts. Subtract boxes & triangles.



Minor Complications:

- Continuous solutions to cut-constraints.
- Work under the integral. Ossola, Papadopoulos, Pittau: classified terms that integrate to zero and work at the integrand level.

Rational parts.

By definitions no logs and thus no cuts in D=4. Can get them by D=4-2ε unitarity.

Bern, Morgan (1996); Bern, LD, Kosower (1996);
Brandhuber, McNamara, Spence, Travaglini hep-th/0506068;
Anastasiou et al., hep-th/0609191, hep-th/0612277;
Britto, Feng, hep-ph/0612089, 0711.4284 [ph];
Giele, Kunszt, Melnikov, 0801.2237 [ph];
Britto, Feng, Mastrolia, 0803.1989 [ph];
Britto, Feng, Yang, 0803.3147 [ph];
Ossola, Papadopolous, Pittau, 0802.1876 [ph];
Mastrolia, Ossola, Papadopolous, Pittau, 0803.3964 [ph];
Giele, Kunszt, Melnikov (2008);
Giele, Zanderighi, 0805.2152 [ph];
Ellis, Giele, Kunszt, Melnikov, 0806.3467 [ph]

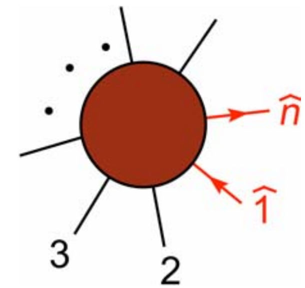
HERE: we discuss on-shell recursion.

Explore factorization in complex momentum space. Trees first.

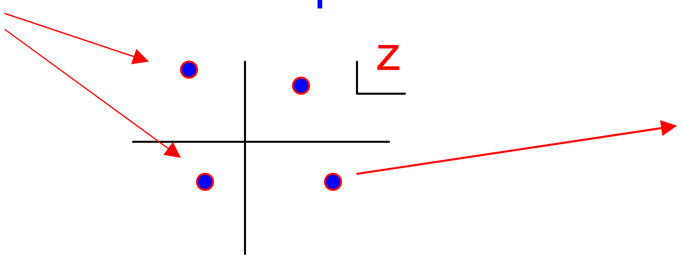
Britto, Cachazo, Feng, Witten, hep-th/0501052

1. Add extra momentum at leg 1 and extract it at leg n.

$$k_1(z) + k_n(z) = k_1 + k_n\,,\qquad k_1(z)^2 = 0\,,\quad k_n(z)^2 = 0\,. \longrightarrow A_n(0) \to A_n(z)$$



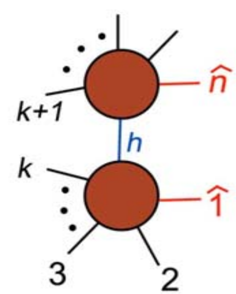
2. Poles in z correspond to factorization limits of trees.



$$\frac{i}{(k_1(z) + k_2 + \ldots k_m)^2} \sim \frac{1}{z - z_k} \to \infty$$

3. Residues on poles are given by product of on-shell tree amplitudes.

$$res_{z_k}\, A_n(z) \; =$$



BCF on-shell recursions.

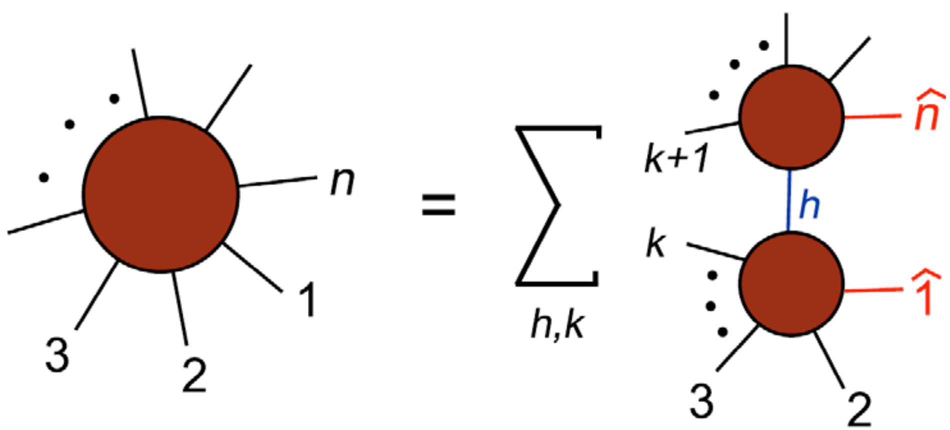
Britto, Cachazo, Feng, hep-th/0412308

- Cauchy theorem allows to assemble residues into trees.

$$A_n(\infty) = 0$$

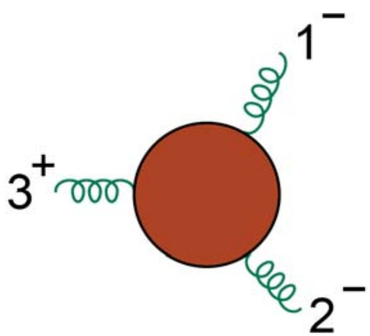
$$A(0) = \oint_{z\to 0} \frac{dz}{z} A_n(z) = -\oint_{z\to\infty} \frac{dz}{z} A_n(z) \sim \sum_k res_{z_k} A_n(z)\,.$$

- Full amplitude given in terms of on-shell factor amplitudes:



All gravity & gluon amplitudes built from 3-point vertex!

Gluons: Gravitons:



$$= \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}. \qquad \left(\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\right)^2.$$

Off-shell vertices:

$$\mathcal{R}_{\mu\nu}$$

Recursions for one-loop amplitudes

Bern, LD, Kosower, hep-th/0501240, hep-ph/0505055, hep-ph/0507005;
Bern, Bjerrum-Bohr, Dunbar, H.I, hep-ph/0507019
Berger, et al., hep-ph/0604195, hep-ph/0607014, 0803.4180 [hep-ph],

Applicable provided:

- Rational parts of loop amplitudes:
  - Coefficients of logs.
  - Rational parts of loop amplitudes.

INTERESTING THEORETICAL QUESTIONS:

- Need to know the poles beforehand.
  - Physical poles.
  - Spurious poles.
  - Factorization in complex momenta.

Consider physical subparts of amplitudes.

Properties of unreal poles.

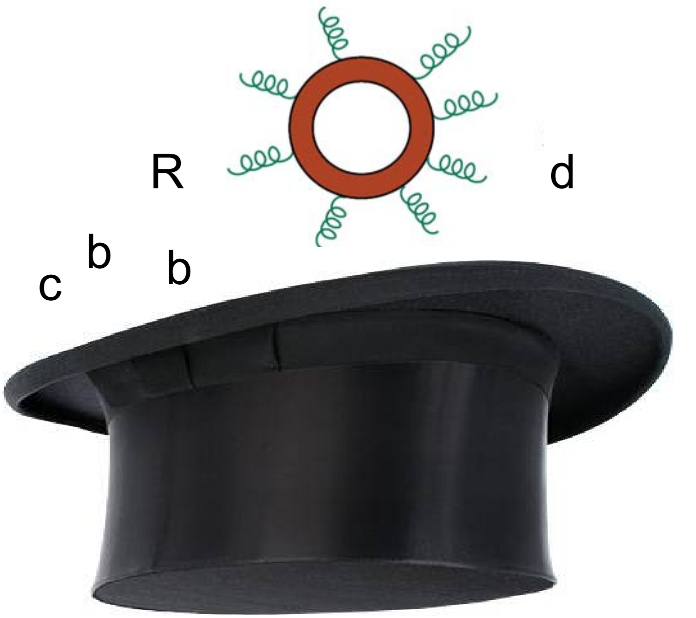
- Need to know residues beforehand:
  - physical poles from factorization theorems.
  - spurious poles from consistency: must cancel in full amplitude.

Can we invert factorization?

Automation in: BLACKHAT

C.F. Berger, Z. Bern, L.J. Dixon, F. Febres Cordero, D. Forde, HI, D.A. Kosower, D. Maitre; T.Gleisberg

- C++ implementation.
- Framework for automated NLO computations.



Numerical stability

Test for loss of precision:

- Compare non logarithmic singularities to tree amplitude.
- Cancelling of spurious poles in singularity.

$$A_n^{1-\text{loop}}|_{1/\epsilon, non-log} = \frac{1}{\epsilon}\sum_i b_i = -\left[\frac{1}{\epsilon}\left(\frac{11}{3} - \frac{2}{3}\frac{n_f}{N_c}\right)\right] A_n^{\text{tree}}\,.$$

Rescue strategy: re-compute at higher precision:16 to 32 (to 64) digits.

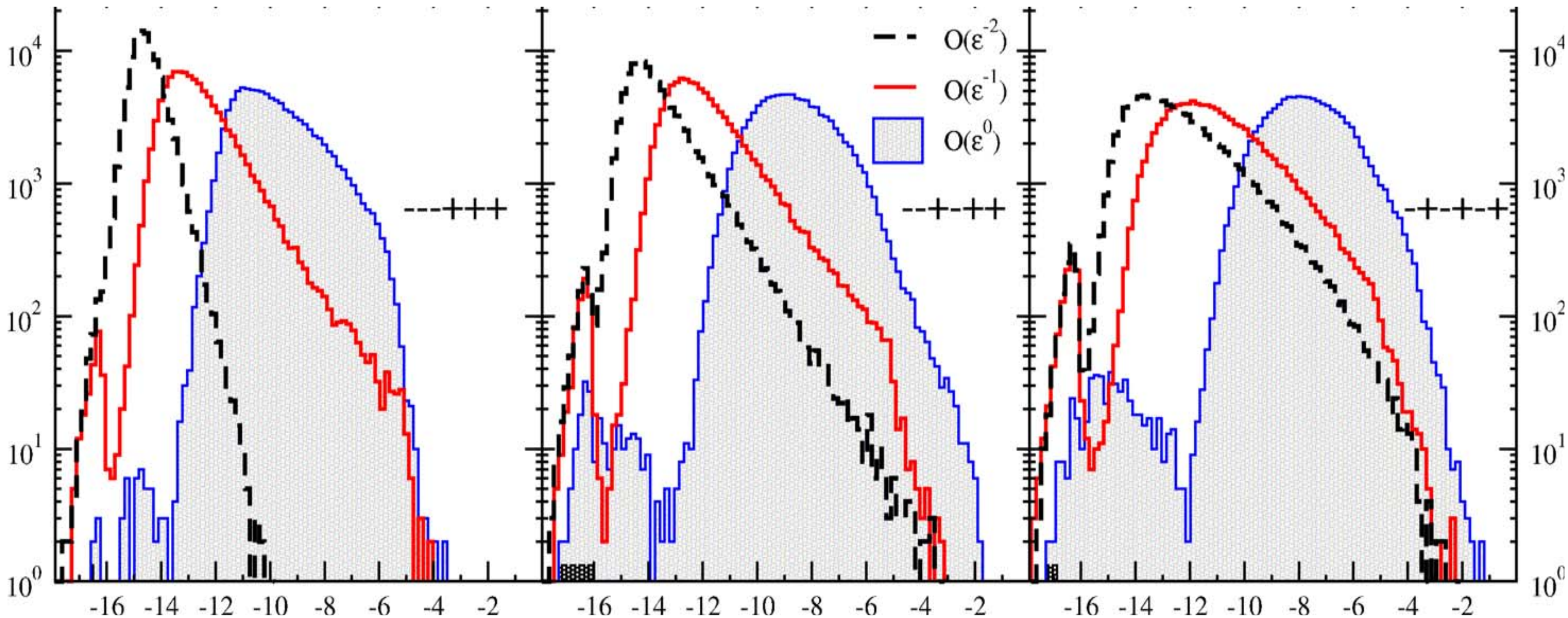
- local calls for re-computation.
- robust to the source, simple to implement.

We do much better than needed for phenomenological studies: demonstrates control over instabilities.

Precision study:

Berger, Bern, Dixon, Febres Cordero, Forde, HI, Kosower, Maître, 0803.4180[ph]

log(# of pts)



log(rel. error)

100 000 PS points, ET>0.01 s(1/2), pseudorapidity<3, separation cut >0.4

Efficiency:

Pure gluon scattering:

Scaling with number scattering gluons.

Scaling with complexity of helicity structure.

6 pt MHV	8 ms
7 pt MHV	14 ms
8 pt MHV	34 ms
(-+-+++)	24 ms
(-++-++)	76 ms
(---+++)	16 ms
(--+-++)	48 ms
(-+-+-+)	80 ms

Conclusion

- On-shell algorithms and an their automated implementation very satisfying.
- Numerical stability under control.
- Moderate scaling of computer time with complexity or number of partons.

Tasks:

- External fermions, massive quarks & vector bosons
- Towards cross sections: BlackHat + automation of Catani-Seymour (*Gleisberg, Krauss*)
- BlackHat publicly available in about one year