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Finite Temperature Simulations using Real Time Dynamics?

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* Presenting

Motivation

Why study real time quantum field dynamics non-perturbatively?

- Quark-gluon plasma in heavy-ion collisions
- Early universe
- Finite chemical potential in QCD

How?

- Heisenberg / Schrödinger eqs. - impossible
- Monte Carlo - complex "probabilities"
- Revert to approximations
  - Fully classical - Non-perturbative, Rayleigh-Jeans type divergence
  - Homogeneous Hartree - No thermalization
  - Inhomogeneous Hartree - This talk

φ^4-model as play ground

- Heisenberg operator equations

$$\dot{\hat{\varphi}} = \hat{\pi}, \quad \dot{\hat{\pi}} = (\Delta - \mu^2)\hat{\varphi} - \lambda\hat{\varphi}^3$$

- Discretize φ field on $(1+1$ dim) lattice
- Focus on Green functions, assume Gaussian density matrix. Hartree approx. gives e.o.m. for 1 and 2-point functions
  - 1-point function: mean field $\langle\hat{\varphi}_x\rangle = \varphi_x$,
  - 2-point function can be expanded in mode functions f_x^α

$$\langle\hat{\varphi}_x\hat{\varphi}_y\rangle_{\text{conn}} = \sum_\alpha [(1+n_\alpha^0) f_x^\alpha f_y^{\alpha *} + n_\alpha^0 f_x^{\alpha *} f_y^\alpha]$$

  - Gaussian: higher-point functions follow
  - $n_\alpha^0 = 0$: modes start in vacuum state

Hartree equations of motion

- Equations for $\varphi = \langle\hat{\varphi}\rangle$ and the f^α's

$$\ddot{\varphi}_x = \Delta\varphi_x - [\mu^2 + \lambda(\varphi_x^2 + 3\sum_\alpha (2n_\alpha^0 + 1) f_x^\alpha f_x^{\alpha *})]\varphi_x$$

$$\ddot{f}_x^\alpha = \Delta f_x^\alpha - [\mu^2 + \lambda(3\varphi_x^2 + 3\sum_\alpha (2n_\alpha^0 + 1) f_x^\alpha f_x^{\alpha *})] f_x^\alpha$$

  - Requires only mass renormalization
  - The mean field φ is *inhomogeneous*
  - There are $O(N^2)$ d.o.f.
  - Classically, no f's, only $O(N)$ d.o.f.

- Hamiltonian dynamics: too much like classical? UV-catastrophe?

- Gaussian: too semi-classical? no tunneling

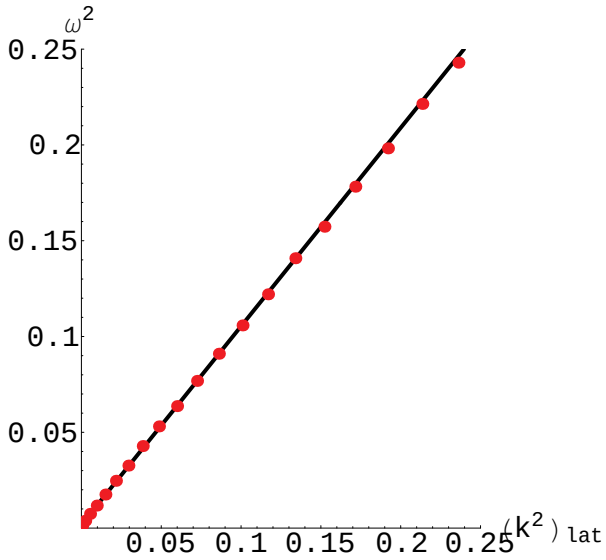
- How accurate is Hartree dynamics?
  - Homogeneous φ: no thermalization
  - Inhomogeneous φ: "Scattering on mean field φ". Does it thermalize?
- Use free field formula to compute mode energy ω_k and particle density n_k from two-point functions

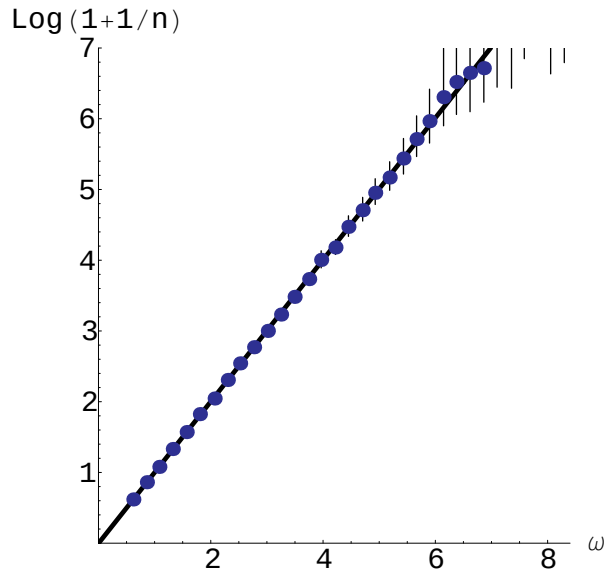
$$\sum_x \langle \widehat{\varphi}_{x+z} \widehat{\varphi}_x \rangle_{\text{conn}} = \sum_k \frac{n_k + \frac{1}{2}}{\omega_k} e^{ikz}$$

$$\sum_x \langle \widehat{\pi}_{x+z} \widehat{\pi}_x \rangle_{\text{conn}} = \sum_k (n_k + \tfrac{1}{2}) \omega_k \, e^{ikz}$$

- Test for Bose-Einstein distribution $n_k = 1/(e^{\omega_k/T} - 1)$. Classically, $n_k = T/\omega_k$
- Checked using Monte Carlo (imaginary time)

Monte Carlo simulation





- Measure $\langle \widehat{\varphi}_x \widehat{\varphi}_y \rangle_{\text{conn}}$
- Assuming BE distribution, compute dispersion relation $\omega(k^2_{lat})$
- Same parameters as in Hartree, $T/m = 1$, $Lm = 25.6$, $1/am = 10$ ($N_t = 20$)
- Free field form holds very well

- Facilitate Hartree: $\widehat{\rho} = \sum_i p_i \widehat{\rho}_i^{gauss}$
- Average over initial conditions and/or time

$$\langle \widehat{\varphi}_x \widehat{\varphi}_y \rangle_{\text{conn}} =$$

$$\sum_i p_i [\langle \widehat{\varphi}_x \widehat{\varphi}_y \rangle_{\text{conn}}^{(i)} + \varphi_x^{(i)} \varphi_y^{(i)}] - (\text{disconn.})$$

- Gaussian initial states (modes in vacuum)
  - Far from equilibrium: few plane waves, random phase α

$$n_\alpha^0 = 0, \ \varphi_x = v, \ \pi_x = \sum_j^{k_{max}} A_j \cos(k_j x + \alpha_j),$$

  - Closer to equilibrium: mean field from BE distribution

$$P(\varphi_k, \pi_k) \propto \exp[-(e^{\omega_k/T_0} - 1)(\pi_k^2 + \omega_k^2 \varphi_k^2)/2\omega_k]$$

Thermalization?

Two stages in time development

1. Creation and scattering of particles, Equilibration towards BE distribution

2. Gradual emergence of equipartition, starting with low ω particles

- Rates depend on interaction strength

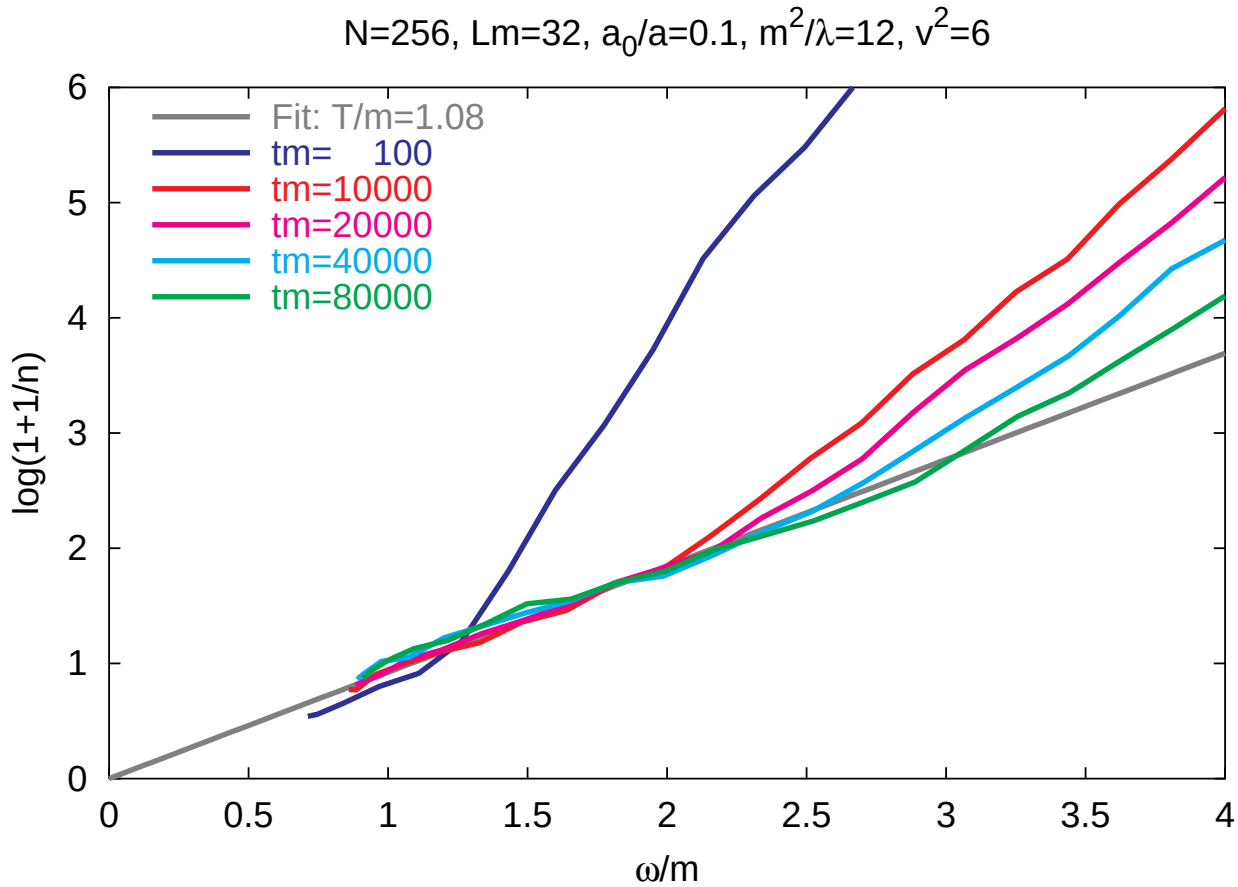
Starting far out-of-equilibrium

- Thermalization extends too slowly towards large ω particles.

Starting with BE distribution for mean field

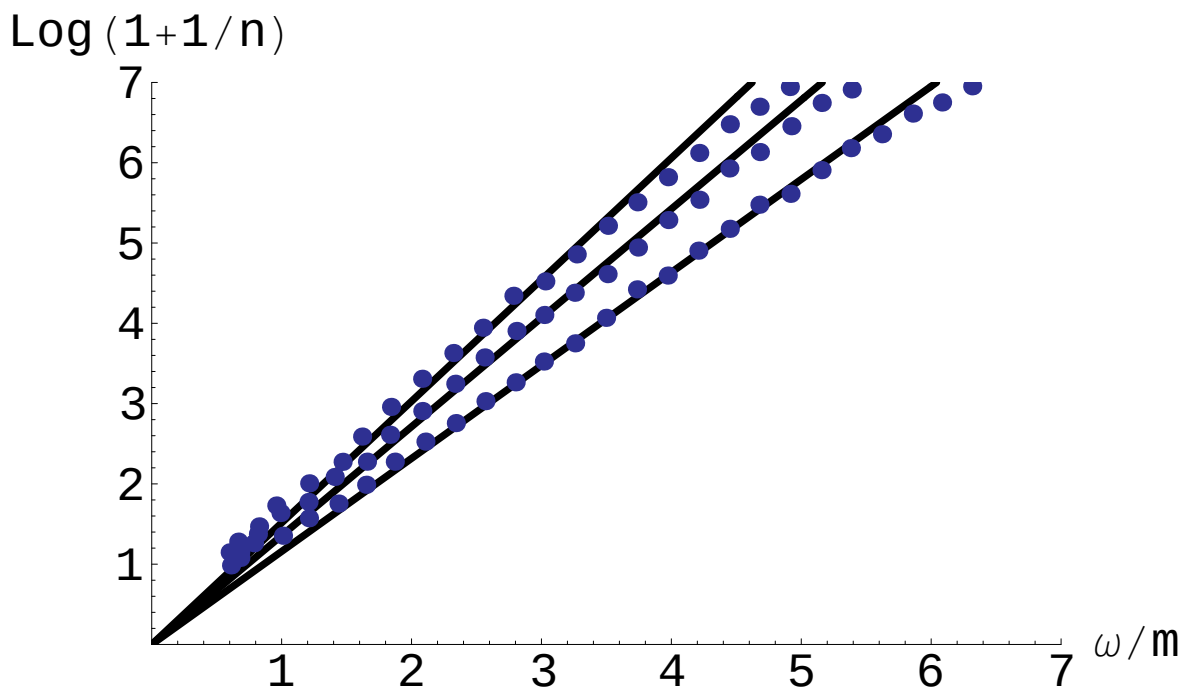
- BE-equilibrium sets in very fast
- But equipartition is inevitable

Towards equilibrium



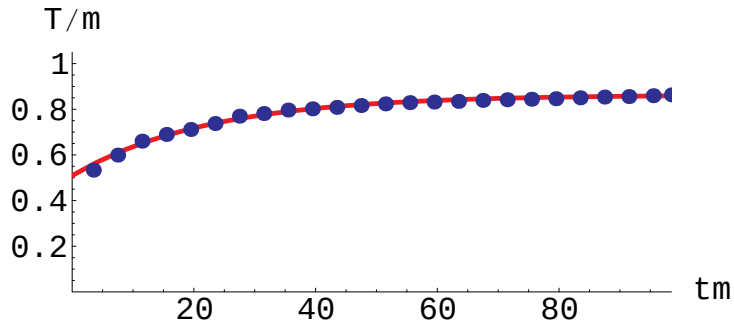
- Far out-of-equilibrium initial fields
- Particle distribution plotted as $\mathrm{Log}\,(1+1/n)$
- Weak coupling $\lambda/m^2 = 1/12$, $T/m = 1.1$
- Particles with $\omega_k/m \lesssim 2$ equilibrate quickly, further progress slow

Fast thermalization

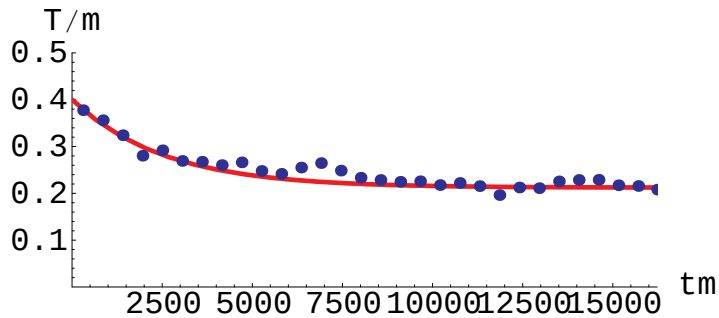


- Early times $tm = 10, 30$ and 100.
- Bose-Einstein distribution for all ω's
- The straight lines are fits with temperature $T/m = 0.66, 0.78$ and 0.86 (top to bottom)
- Stronger coupling $\lambda/m^2 = 1/4$, $Lm = 26$

Time scales



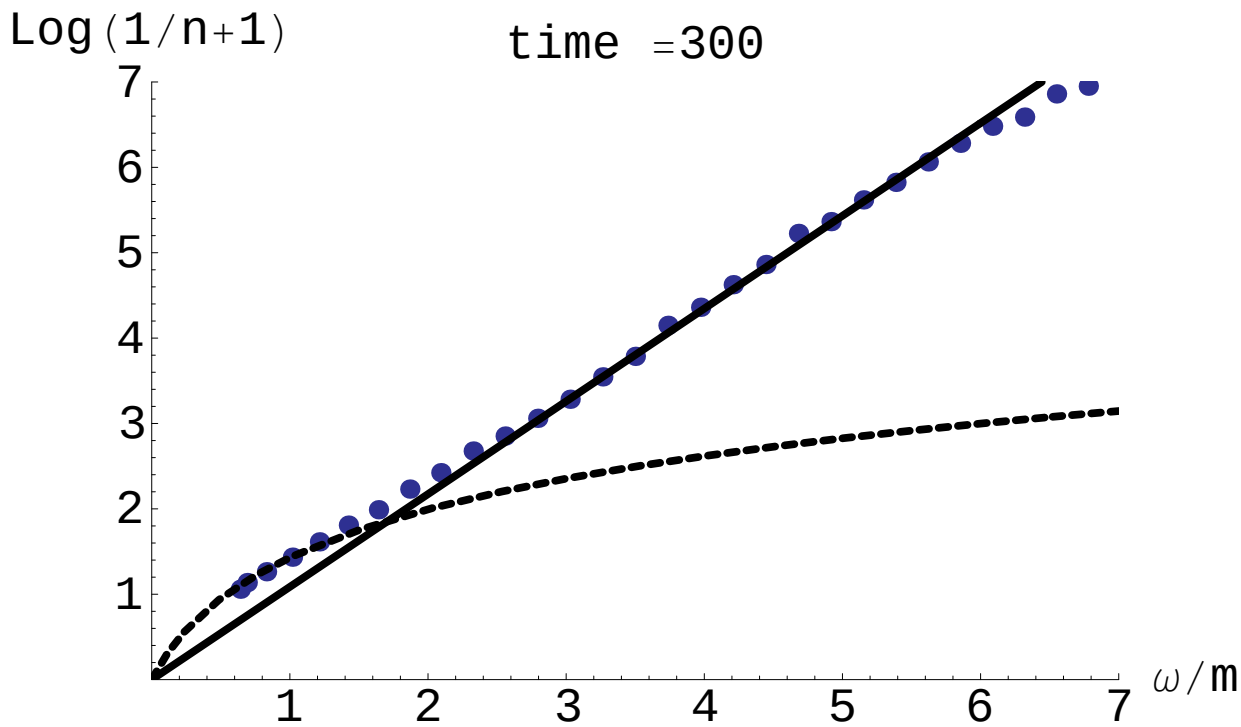
1 Warming up (Bose-Einstein)



2 Cooling down (Classical)

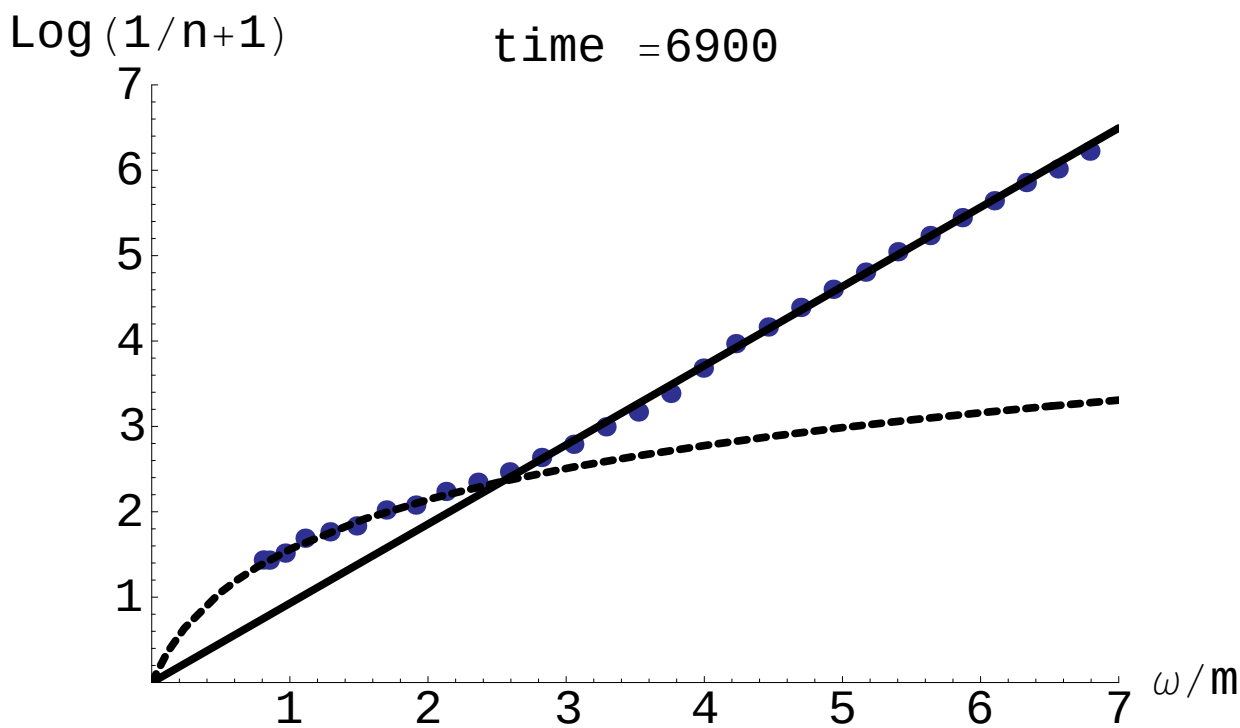
- Time dependence fitted to exponential form $T(t) = T_\infty \pm A \exp(-t/\tau)$
- Warming-up time is short, $\tau/m \approx 20$
- Cooling-down time is long, $\tau/m \approx 2500$

From BE to equipartition



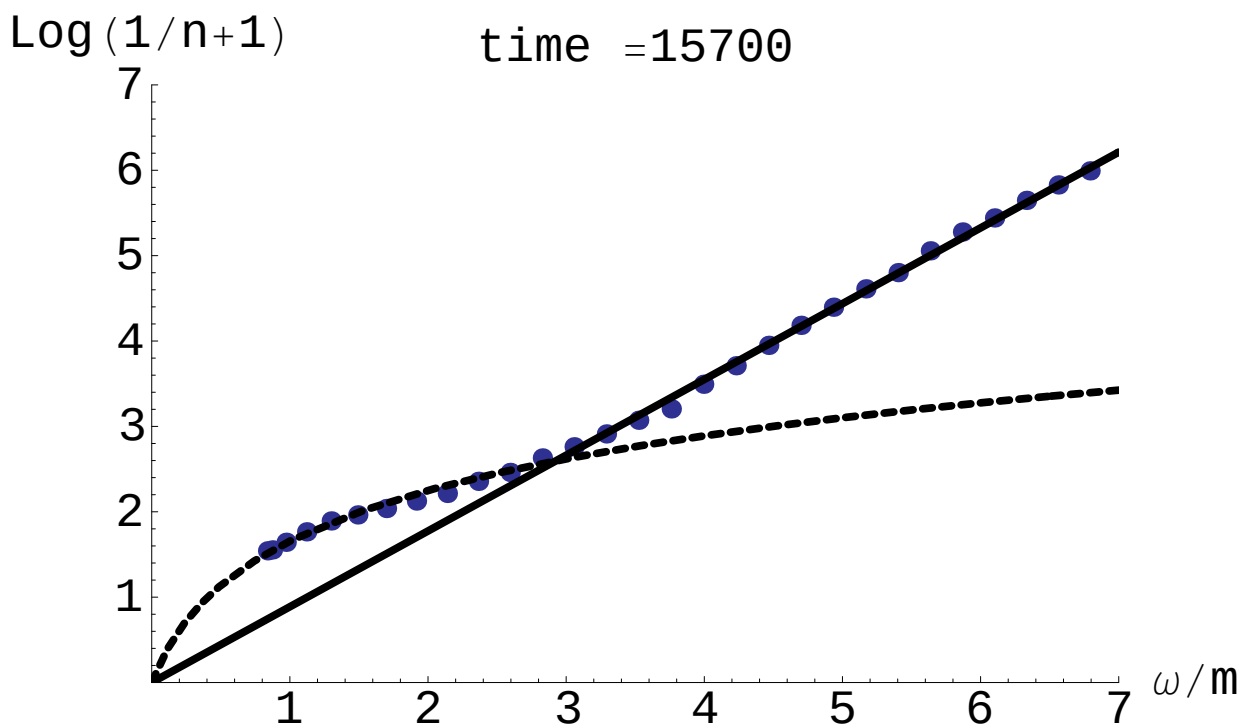
- Particle distribution $\mathrm{Log}\,(1 + 1/n)$ for a later time $tm = 300$
- The straight line is a Bose-Einstein distribution with temperature $T/m = 0.92$
- The curved line fits $n = T_{\text{class}}/\omega$ with a "classical" temperature $T_{\text{class}}/m = 0.31$ for the particles with $\omega \lesssim 1.6$

From BE to equipartition



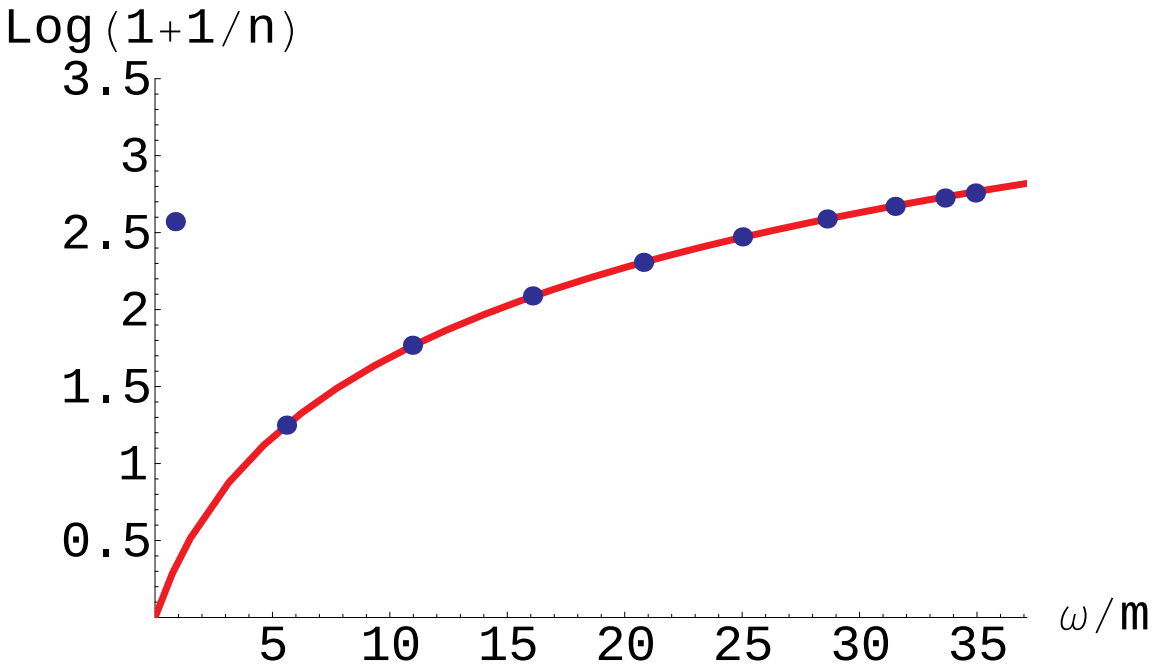
- Same as previous but at a much later time $tm = 6900$
- Bose-Einstein temperature $T/m = 1.08$
- "classical" temperature $T_{\text{class}}/m = 0.27$ for the particles with $\omega \lesssim 2.4$

From BE to equipartition



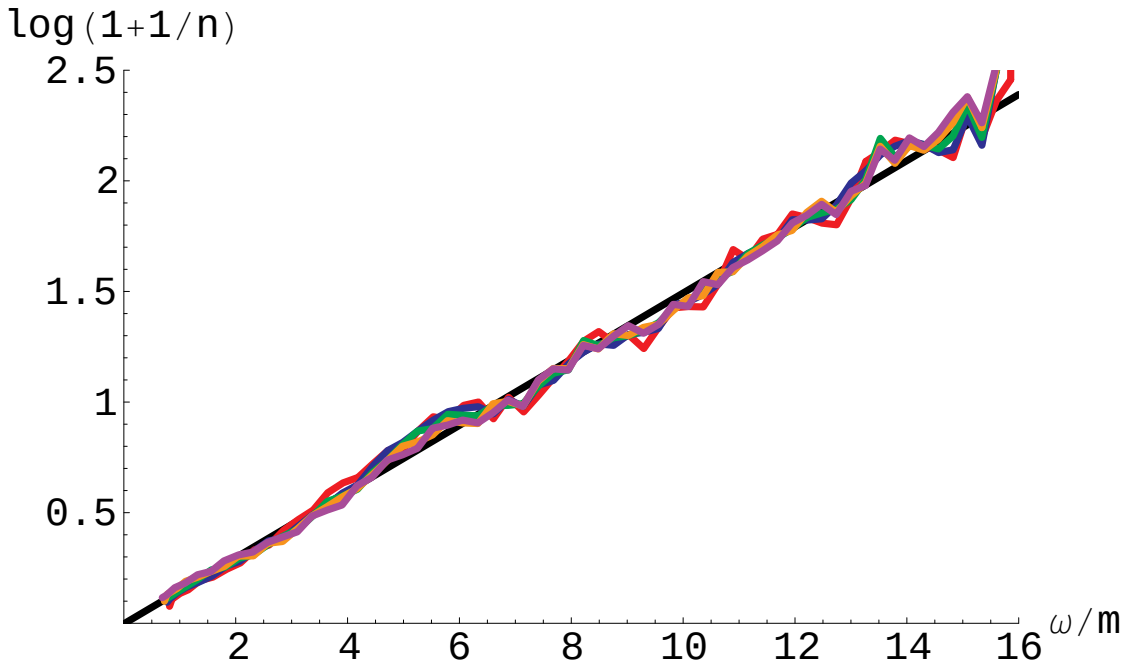
- Same as previous but at a much later time $tm = 15700$
- Bose-Einstein temperature $T/m = 1.13$
- "classical" temperature $T_{\text{class}}/m = 0.24$ for the particles with $\omega \lesssim 3$

Equipartition



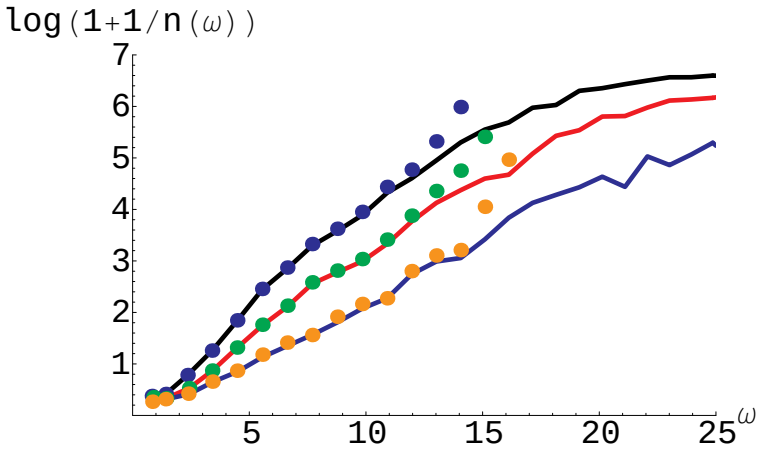
- Very large time $tm = 4\ 10^6$
- Small toy model $Lm = 1.1$, $(N = 20)$
- Dashed line $n = T/\omega$ fits data with $T/m = 2.2$
- Only zero mode and alternating mode deviate

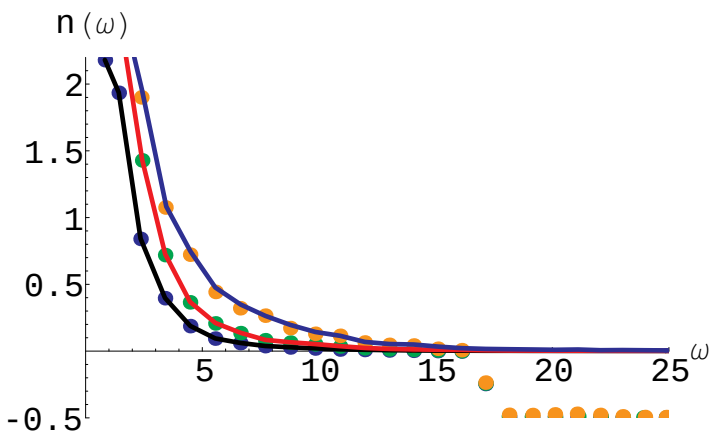
Finite temperature simulation



- Times in range $100 < tm < 4000$
- Weak coupling $\lambda/m^2 = 1/12$, $Lm = 23$
- Modes started in vacuum ($T = 0$), $\tau_W \approx 200$
- Particles are created fast, with BE distribution and constant temperature, $T/m \approx 6.5$

Reduced number of modes





- Full lines: simulation with all modes
- Dots: Only 32 of 128 modes with initial (plane wave) energies $\omega/m \lesssim 17$
- $\omega_{max}/T \approx 3$ can be much below cut-off

Conclusions

- Thermalization with inhomogeneous Hartree approximation
  - Slowly moving towards classical
- Long period of thermal equilibrium?
  - If not starting too far out of equilibrium
    If interactions not too strong
    If semi-classical good enough, (high T)
- Number of mode functions can be kept fixed for cut-off $\to \infty$
  - Number needed depends on T
  - Saves factor N in CPU time
- How about QCD at finite μ?