Constructing Textures in Quark-Lepton Complementarity

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Outline

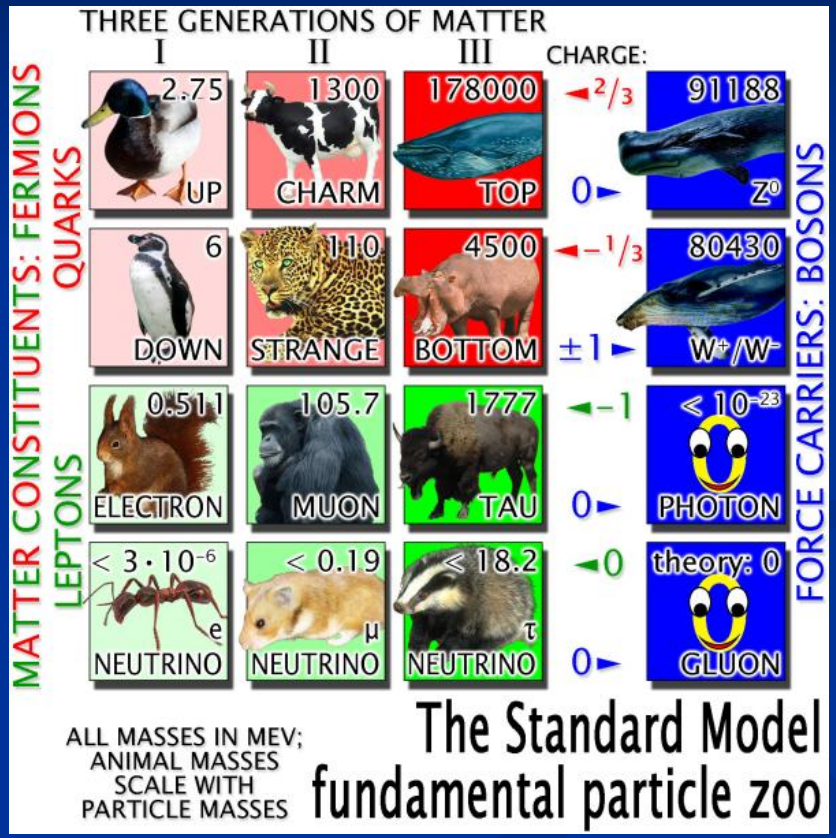
- The Standard Model and beyond
- Extended Quark-Lepton Complementarity
- Results
- Extension to seesaw mechanism
- Summary

Standard Model

SM contains 3 families of

- Up-type quarks
- Down-type quarks
- Neutrinos
- Charged leptons

→ Order principle



- Large mass hierarchy among generations observed.

Observations – Quark Sector

($\varepsilon \sim 0.2$)

- *Quarks:*

$$m_u : m_c : m_t = \varepsilon^6 : \varepsilon^3 : 1$$

$$m_d : m_s : m_b = \varepsilon^4 : \varepsilon^2 : 1$$

$$V_{CKM} \sim \begin{pmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix}$$

$\theta_C^2 = m_d/m_s$ (Gatto et al.,1968)

with $m_b : m_t = \varepsilon^2 : 1$,

$$\theta_{12} \sim \varepsilon,\ \theta_{13} \sim \varepsilon^3,\ \theta_{23} \sim \varepsilon^2,\ \delta \approx 63^\circ$$

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Observations – Lepton Sector

(ε~0.2)

- *Charged leptons:*

  $m_e:m_\mu:m_\tau=\varepsilon^4:\varepsilon^2:1$

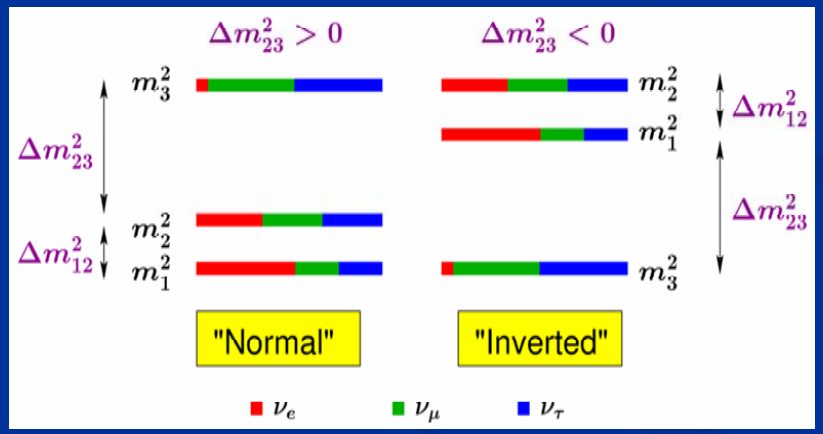
  $m_\tau:m_b=1:1$

- *Neutrinos:*

  $m_1:m_2:m_3=\varepsilon^2:\varepsilon:1$ (NH)

  $m_1:m_2:m_3=1:1:\varepsilon$ (IH)

  $m_1:m_2:m_3=1:1:1$ (DG)



- *PMNS Matrix:*

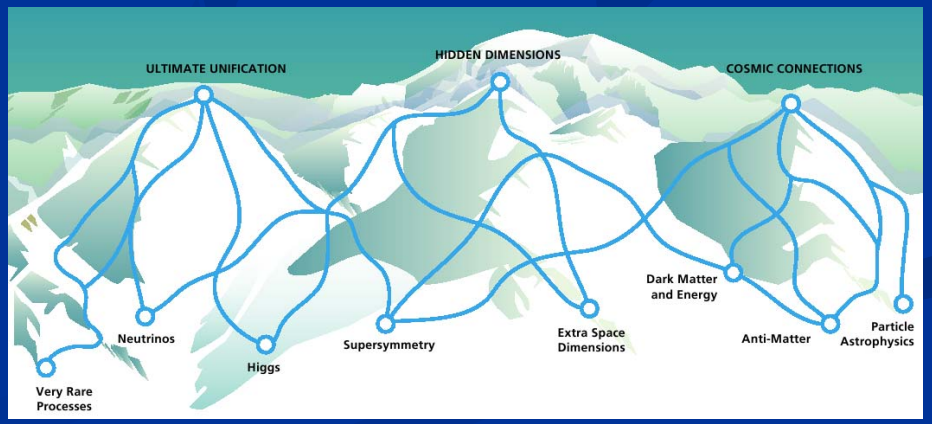
  $U_{PMNS}=U_l^+U_\nu$, $\theta_{23}\approx\pi/4$, $\theta_{12}\approx\pi/4-\varepsilon$, $\theta_{13}\leq\varepsilon$

- Unknown: θ_{13}, absolute mass, ordering and phases

Beyond the Standard Model

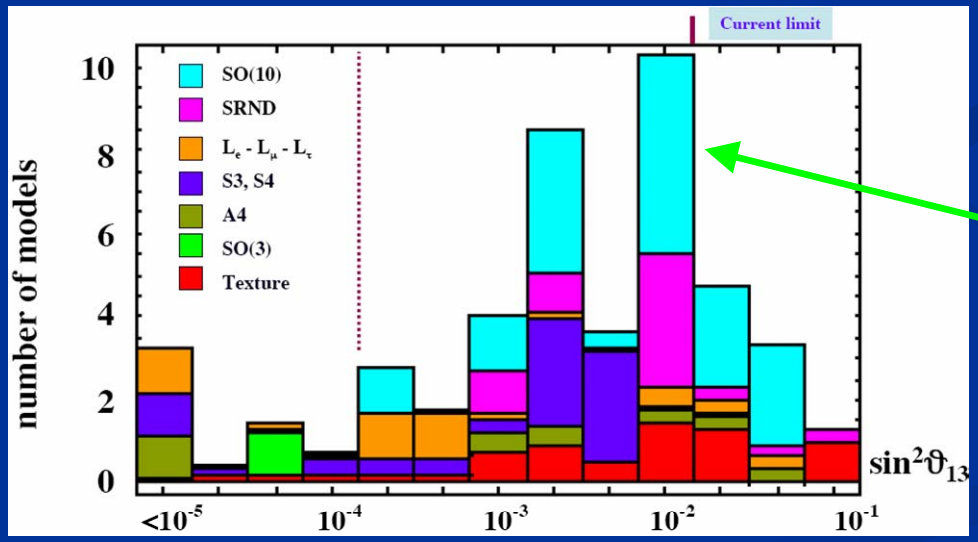
- SM augmented by massive neutrinos contain a lot of free parameters.
- Why is the quark and neutrino sector so radical different?
- Why are the neutrino masses so small?
- …

Quest to change this:



Usual Methods

- Symmetries (GUTs, flavor,…), anarchy arguments etc. used to predict PMNS mixings
- Example: literature research on θ_{13}



Generic or biased peak?

(Albright,Chen,2006)

New Systematic Approach

- Model independent bottom-up approach
- Systematic reconstruction of Yukawa couplings
- Extremely simple and efficient
- Extensive extraction of realizations and textures

Unbiased exploration of parameter space

Extended QLC

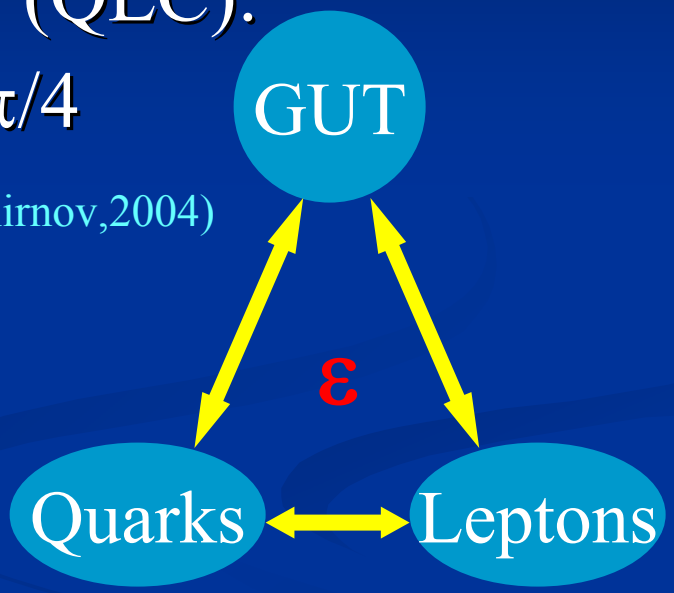
($\varepsilon \sim \theta_C \approx 0.2$)

- Quark-Lepton Complementarity (QLC):

$$\theta_{12}+\theta_C \approx \pi/4, \qquad \theta_{23}+\theta_{cb} \approx \pi/4$$

(Petcov,Smirnov,1993; Smirnov,2004; Raidal,2004; Minakata,Smirnov,2004)

E.g. from $U_{PMNS} \sim V_{CKM}{}^{+} U_{bimax}$
(U_{bimax}: $\theta_{12}=\theta_{23}=\pi/4$)



- *ε parameterizes everything?!*

→ Extended QLC:

- All mass ratios are powers of ε
- All mixing angles from $\{\pi/4, \varepsilon, \varepsilon^2, .., 0\}$

Example - $U_{CKM}^{+}\, U_{bimax}$

$$M_l = U_l\, M_l^{diag}\, U_l'^{+} \rightarrow \mathbb{1}_{3x3}$$

$$M_\nu^{Maj} = U_\nu\, M_\nu^{diag}\, U_\nu^{T}$$

$$M_\nu^{Dirac} = U_\nu\, M_\nu^{diag}\, U_\nu'^{+} \rightarrow \mathbb{1}_{3x3}$$

$$U_{PMNS} = U_l^{+} U_\nu$$

- *Mixing angles:*

$(\theta^l_{12},\theta^l_{13},\theta^l_{23})=(\varepsilon,0,\varepsilon^2)$, $(\theta^\nu_{12},\theta^\nu_{13},\theta^\nu_{23})=(\pi/4,\varepsilon,\pi/4)$

- *PMNS mixing angles:*

$U_l^{+} U_\nu \longrightarrow (\theta_{12},\theta_{13},\theta_{23})=(36.5^o, 3.6^o, 43.8^o)$

Example - $U_{CKM}^{+}\, U_{bimax}$

$$M_l = U_l\, M_l^{diag}$$
$$M_\nu^{Maj} = U_\nu\, M_\nu^{diag}\, U_\nu^T$$
$$M_\nu^{Dirac} = U_\nu\, M_\nu^{diag}$$

$$\begin{cases} M_l^{diag} = m_\tau\, \mathrm{diag}(\varepsilon^4,\varepsilon^2,1) \\ M_\nu = m_3\, \mathrm{diag}(\varepsilon^2,\varepsilon,1) \\ (\theta^l_{12},\theta^l_{13},\theta^l_{23}) = (\varepsilon,0,\varepsilon^2) \\ (\theta^\nu_{12},\theta^\nu_{13},\theta^\nu_{23}) = (\pi/4,\varepsilon,\pi/4) \end{cases}$$

$$M_\ell = m_\tau \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon^2 & \epsilon^2 \\ 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon^2 & \epsilon^2 \\ 0 & 0 & 1 \end{pmatrix},$$

$$M_\nu^{\mathrm{Maj}} = m_3 \begin{pmatrix} \frac{\epsilon}{2} + \frac{3\epsilon^2}{2} & \frac{3\epsilon}{2\sqrt{2}} - \frac{\epsilon^2}{\sqrt{2}} & \frac{\epsilon}{2\sqrt{2}} \\ \frac{3\epsilon}{2\sqrt{2}} - \frac{\epsilon^2}{\sqrt{2}} & \frac{1}{2} + \frac{\epsilon}{4} - \frac{3\epsilon^2}{4} & \frac{1}{2} - \frac{\epsilon}{4} - \frac{3\epsilon^2}{4} \\ \frac{\epsilon}{2\sqrt{2}} & \frac{1}{2} - \frac{\epsilon}{4} - \frac{3\epsilon^2}{4} & \frac{1}{2} + \frac{\epsilon}{4} + \frac{\epsilon^2}{4} \end{pmatrix} \to \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix},$$

$$M_\nu^{\mathrm{Dirac}} = m_3 \begin{pmatrix} -\frac{\epsilon^2}{\sqrt{2}} & -\frac{\epsilon}{\sqrt{2}} & \epsilon \\ \frac{\epsilon^2}{2} & -\frac{\epsilon}{2} + \frac{\epsilon^2}{2} & \frac{1}{\sqrt{2}} - \frac{\epsilon^2}{2\sqrt{2}} \\ -\frac{\epsilon^2}{2} & \frac{\epsilon}{2} + \frac{\epsilon^2}{2} & \frac{1}{\sqrt{2}} - \frac{\epsilon^2}{2\sqrt{2}} \end{pmatrix} \to \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix},$$

(FP,Seidl,Winter,2006)

Exended QLC Results

- 262.144 possibilities
- 2.468 in agreement with experimental data
- In 10 year limit: 20 textures

#		Normal Hierarchy		Inverted Hierarchy		Degenerate		
	M_ℓ	M_ν^{Maj}	M_ν^{Dirac}	M_ν^{Maj}	M_ν^{Dirac}	M_ν^{Dirac}	$(s_{12}^\ell, s_{13}^\ell, s_{23}^\ell)$ $(s_{12}^\nu, s_{13}^\nu, s_{23}^\nu)$ $(\delta^\ell, \delta^\nu, \widehat{\varphi}_1, \widehat{\varphi}_2)$	$(\theta_{12}, \theta_{13}, \theta_{23})$
17	$\begin{pmatrix} 0 & \epsilon^2 & \epsilon \\ 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon^2 & 1 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ 1 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon^2 & \epsilon^2 & 1 \\ 0 & \epsilon & 0 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & 0 \\ 1 & \epsilon & \epsilon \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & 1 \\ \epsilon & 1 & 0 \\ 1 & \epsilon & 1 \end{pmatrix}$	$(\frac{1}{\sqrt{2}}, \epsilon, \frac{1}{\sqrt{2}})$ $(\epsilon, \frac{1}{\sqrt{2}}, 0)$ $(\pi, \xi, \pi, \xi + \pi)$	$(35.2^\circ, 3.8^\circ, 50.8^\circ)$
18	$\begin{pmatrix} 0 & \epsilon^2 & \epsilon \\ 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon^2 & 1 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ 1 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon^2 & \epsilon^2 & 1 \\ 0 & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon^2 & 1 \\ \epsilon^2 & 1 & \epsilon^2 \\ 1 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & 0 \\ 1 & \epsilon & \epsilon \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & 1 \\ \epsilon & 1 & \epsilon^2 \\ 1 & \epsilon & 1 \end{pmatrix}$	$(\frac{1}{\sqrt{2}}, \epsilon, \frac{1}{\sqrt{2}})$ $(\epsilon, \frac{1}{\sqrt{2}}, \epsilon^2)$ $(\pi, \pi, \pi, 0)$	$(33.6^\circ, 3.1^\circ, 52.2^\circ)$

(FP,Seidl,Winter,2006)

Exended QLC Results

- New textures:

Example: “Diamond” textures

“Diamond”: $\theta_{13}=\pi/4$

#	M_ℓ	Normal Hierarchy $M_\nu^{\rm Maj}$	Normal Hierarchy $M_\nu^{\rm Dirac}$	Inverted Hierarchy $M_\nu^{\rm Maj}$	Inverted Hierarchy $M_\nu^{\rm Dirac}$	Degenerate $M_\nu^{\rm Dirac}$	$(s_{12}^\ell, s_{13}^\ell, s_{23}^\ell)$ $(s_{12}^\nu, s_{13}^\nu, s_{23}^\nu)$ $(\delta^\ell, \delta^\nu, \widehat{\varphi}_1, \widehat{\varphi}_2)$	$(\theta_{12}, \theta_{13}, \theta_{23})$
17	$\begin{pmatrix} 0 & \epsilon^2 & \epsilon \\ 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon^2 & 1 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ 1 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon^2 & \epsilon^2 & 1 \\ 0 & \epsilon & 0 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & 0 \\ 1 & \epsilon & \epsilon \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & 1 \\ \epsilon & 1 & 0 \\ 1 & \epsilon & 1 \end{pmatrix}$	$(\frac{1}{\sqrt{2}}, \epsilon, \frac{1}{\sqrt{2}})$ $(\epsilon, \frac{1}{\sqrt{2}}, 0)$ $(\pi, \xi, \pi, \xi+\pi)$	$(35.2^\circ, 3.8^\circ, 50.8^\circ)$
18	$\begin{pmatrix} 0 & \epsilon^2 & \epsilon \\ 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon^2 & 1 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ 1 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon^2 & \epsilon^2 & 1 \\ 0 & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon^2 & 1 \\ \epsilon^2 & 1 & \epsilon^2 \\ 1 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & 0 \\ 1 & \epsilon & \epsilon \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & 1 \\ \epsilon & 1 & \epsilon^2 \\ 1 & \epsilon & 1 \end{pmatrix}$	$(\frac{1}{\sqrt{2}}, \epsilon, \frac{1}{\sqrt{2}})$ $(\epsilon, \frac{1}{\sqrt{2}}, \epsilon^2)$ $(\pi, \pi, \pi, 0)$	$(33.6^\circ, 3.1^\circ, 52.2^\circ)$

(FP,Seidl,Winter,2006)

Results

- Diamond textures reveal also non-trivial sum rules.
- New sum rules:

$$\theta_{12} + \frac{3}{5+2\sqrt{2}}\,\epsilon = \arctan(2-\sqrt{2}).$$

$$\theta_{13} = \arcsin\left(\frac{1}{4}(2-\sqrt{2})\right) - \frac{1}{\sqrt{5+2\sqrt{2}}}\,\epsilon\ ,$$

$$\theta_{23} = \arctan\left(1+\frac{1}{\sqrt{2}}\right) + \frac{1}{17}(2-11\sqrt{2})\,\epsilon\ .$$

(FP,Seidl,Winter,2006)

QLC:

$\theta_{12}+\varepsilon=\pi/4$

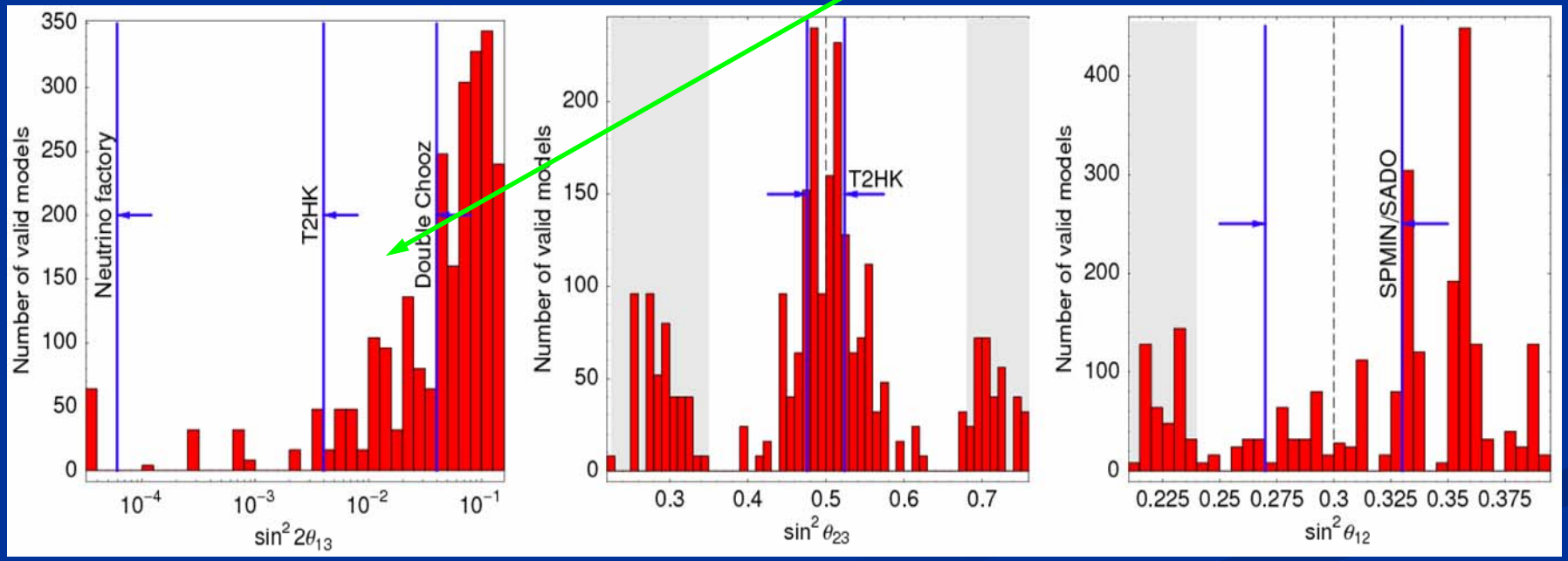
$\theta_{13}=\mathcal{O}(\varepsilon)$

$\theta_{23}=\pi/4+\mathcal{O}(\varepsilon^2)$

Results

- Generic distributions:

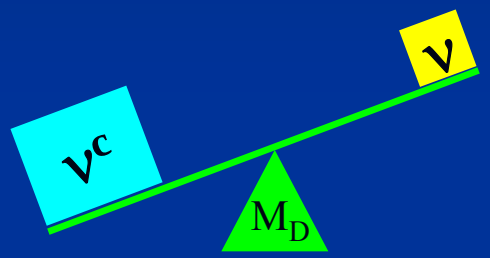
No peak at $\sin^2 2\theta_{13} \sim 0.04$



(FP,Seidl,Winter,2006)

Extension to Seesaw Mechanism

- Seesaw (Type I): $M_D = U_D\, M_D^{diag}\, U_{D'}^{+}$ $\quad M_R = U_R\, M_R^{diag}\, U_R^{+}$



$$\mathcal{L}_{mass} \sim \left(\nu,\ \nu^c\right) \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \end{pmatrix}$$

$$M_l = U_l\, M_l^{diag}\, U_{l'}^{+} \qquad U_{l'}^{+} \rightarrow \mathbb{1}_{3x3}$$

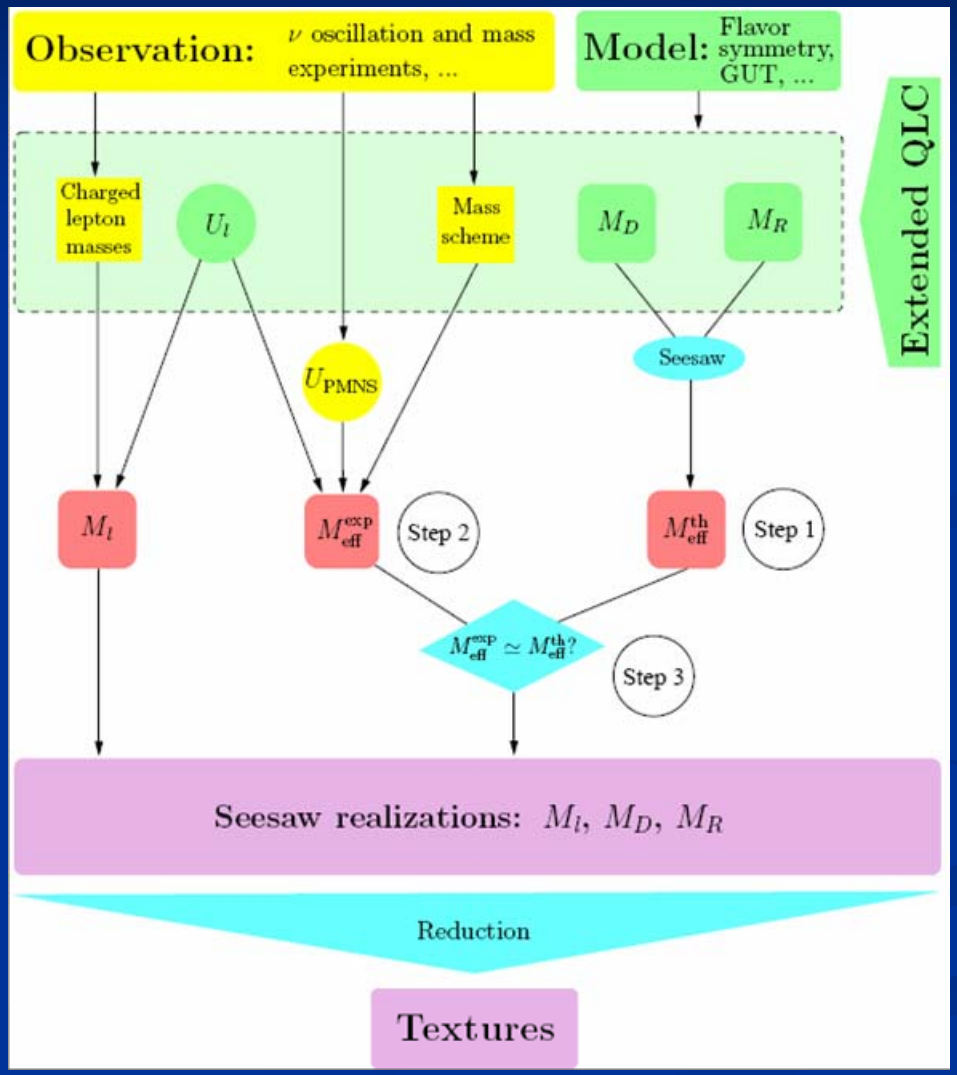
$$\rightarrow \quad M_{eff}^{Maj} = -M_D\, M_R^{-1}\, M_D^T$$

$$U_{PMNS} = U_l^{+} U_\nu$$

ok → Viable realization

not ok → Drop it

Extension to Seesaw Mechanism



(FP,Seidl,Winter,2007)

Seesaw Results

- Over 20 trillion possibilities (CP conserving)
- 1.981 textures found

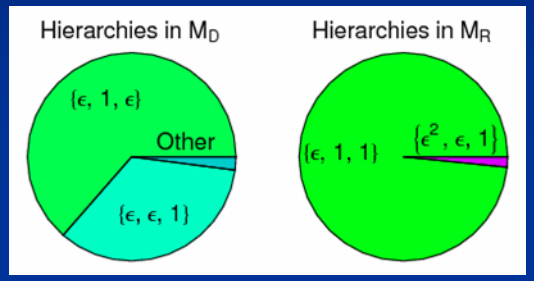
#	M_ℓ	M_D	M_R	$M_D^{\rm diag}/m_D$ $M_R^{\rm diag}/M_{B-L}$	$(\theta_{12}^\ell,\theta_{13}^\ell,\theta_{23}^\ell)$ $(\theta_{12}^D,\theta_{13}^D,\theta_{23}^D)$ $(\theta_{12}^{D'},\theta_{13}^{D'},\theta_{23}^{D'})$ $(\theta_{12}^R,\theta_{13}^R,\theta_{23}^R)$
3	$\begin{pmatrix} 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon & \epsilon^2 & \epsilon \\ \epsilon^2 & 1 & \epsilon \\ \epsilon & \epsilon & \epsilon \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & 1 \\ \epsilon & 1 & \epsilon \\ 1 & \epsilon & 1 \end{pmatrix}$	$(\epsilon,1,\epsilon)$ $(\epsilon,1,1)$	$(\frac{\pi}{4},\frac{\pi}{4},\frac{\pi}{4})$ $(0,\frac{\pi}{4},\epsilon)$ $(0,0,\epsilon)$ $(\epsilon,\frac{\pi}{4},0)$
12	$\begin{pmatrix} 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon & 0 & \xi \\ 0 & \epsilon & \epsilon \\ \xi & \epsilon & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon & \xi & \xi \\ \xi & 1 & 0 \\ \xi & 0 & 1 \end{pmatrix}$	$(\epsilon,\epsilon,1)$ $(\epsilon,1,1)$	$(\frac{\pi}{4},\frac{\pi}{4},\frac{\pi}{4})$ (ξ,ξ,ϵ) (ξ,ξ,ϵ) (ξ,ξ,ϵ)
33	$\begin{pmatrix} 0 & 0 & \xi \\ 0 & \epsilon^2 & \xi \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon & \xi & \xi \\ \xi & 1 & 0 \\ \xi & 0 & 1 \end{pmatrix}$	$(\epsilon,1,\epsilon)$ $(\epsilon,1,1)$	(ξ,ξ,ξ) $(\xi,\frac{\pi}{4},\frac{\pi}{4})$ $(0,\epsilon,\frac{\pi}{4})$ (ξ,ξ,ξ)

#	$(\delta^l,\alpha_1^l,\alpha_2^l)$	$(\delta^D,\varphi_1^D,\varphi_2^D,\varphi_3^D)$	$(\delta^{D'},\alpha_1^{D'},\alpha_2^{D'})$	$(\delta^R,\varphi_1^R,\varphi_2^R,\varphi_3^R)$	$(\theta_{12},\theta_{13},\theta_{23})$	χ^2	Cases
3	$(\pi,0,0)$	$(0,0,\pi,0)$	$(0,0,0)$	$(\pi,0,\pi,0)$	$(33.5°,0.2°,51.3°)$	4.9	26
12	$(0,\pi,\pi)$	$(0,0,0,0)$	$(0,\pi,0)$	$(\pi,0,0,\pi)$	$(33.4°,0.0°,51.3°)$	4.81	475
33	(π,π,π)	$(\pi,0,0,0)$	$(0,0,0)$	$(0,0,\pi,0)$	$(33.6°,0.1°,51.5°)$	5.31	83

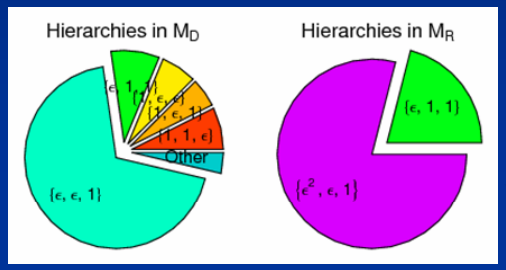
(FP,Seidl,Winter,2007)

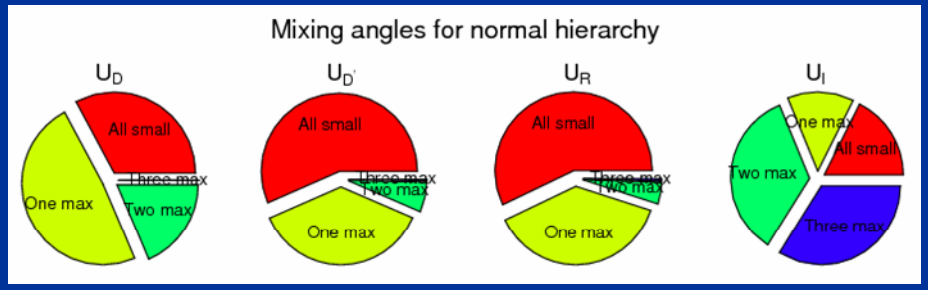
Seesaw Results

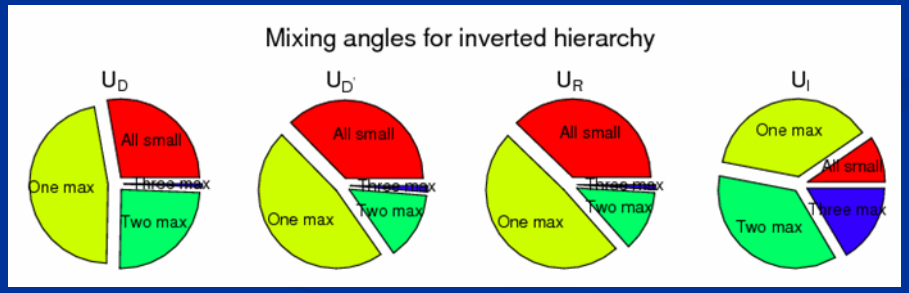
Normal hierarchy:

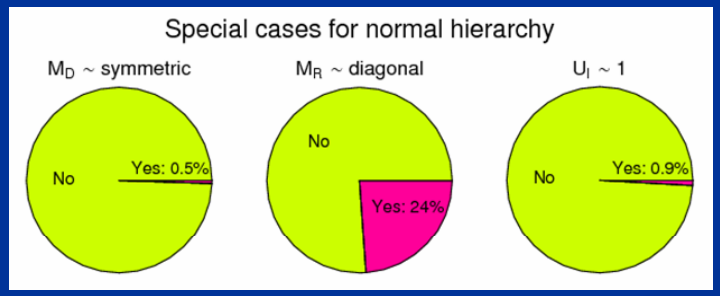


Inverted hierarchy:









(FP,Seidl,Winter,2007)

Summary

- Generalization of QLC to Extended QLC:
  $\varepsilon \sim \theta_C$ parameterizes everything (masses, mixings)
- Efficient way to reconstruct Yukawa matrices
- Systematic generation of possibilities:
  Complete extraction of textures (*e.g.* diamond texture)
  Reveal general features (*e.g.* new sum rule)

References

- FP, G. Seidl and W. Winter,
  "Systematic Parameter Space Search of Extended Quark-Lepton-Complementarity",
  hep-ph/0612169 (submitted to Nucl.Phys.B)
- http://theorie.physik.uni-wuerzburg.de/%7Ewinter/Resources/Textures/index.html
- FP, G. Seidl and W. Winter,
  "Constructing Seesaw Textures in Quark-Lepton Complementarity",
  0707.xxxx (hep-ph)

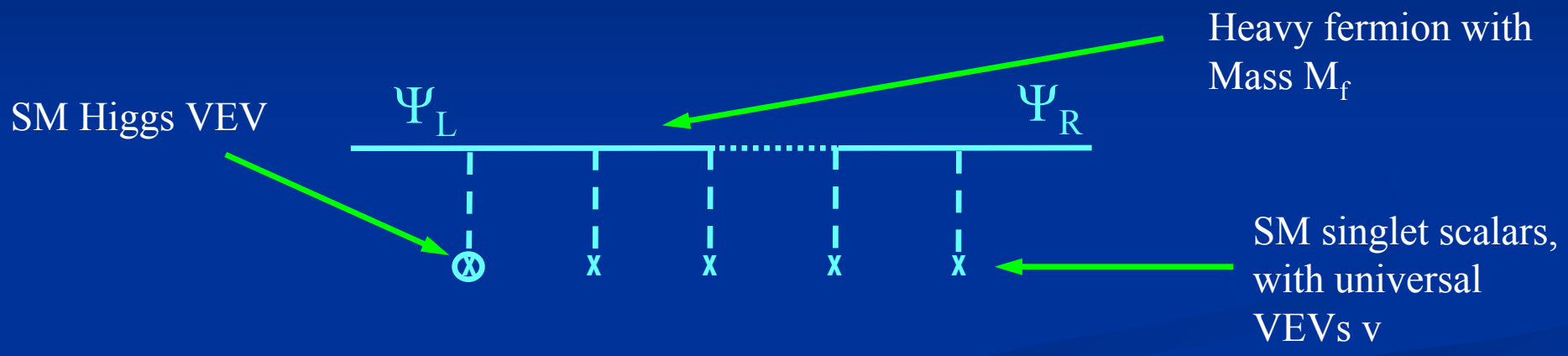
References

- Particle zoo:

  http://e4.physik.uni-dortmund.de/bin/view/ATLAS/SmBilder
- Slopes to unification:

  high-energy physics in the 21st century, *"The Science Ahead, the Way to Discovery."*

Froggatt-Nielson Mechanism



- v breaks flavor symmetry successively
- After integrating out heavy fermions:

$$\mathcal{L}^m_{eff} = \langle H \rangle \varepsilon^n \overline{\Psi}_L \Psi_R, \qquad \varepsilon = v/M_f$$

Example – Z_n Flavor Symmetries

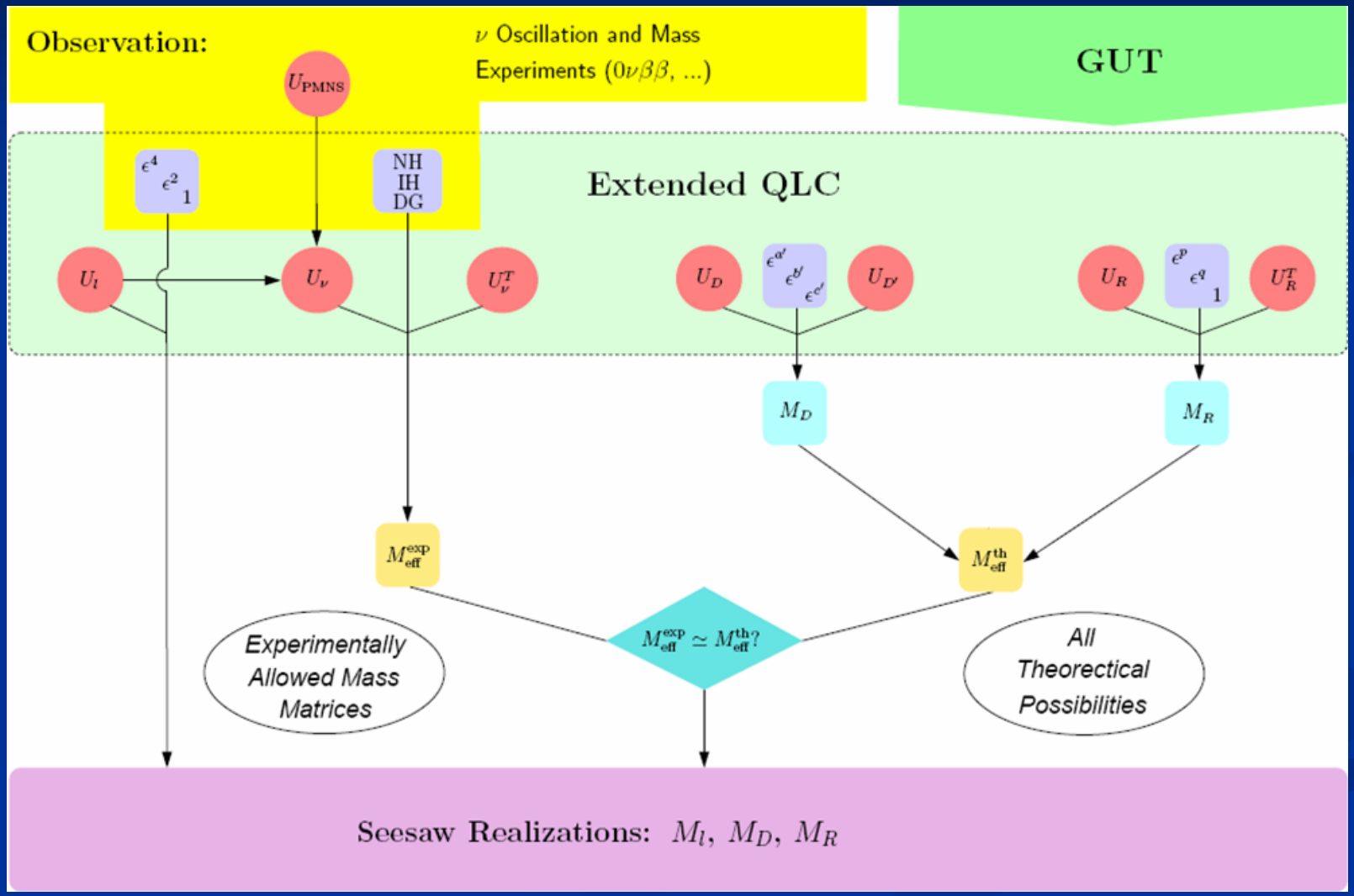
- $G_F = Z_4^{(1)} \times Z_4^{(2)} \times \ldots \times Z_4^{(7)}$
- Flavon charges: $f_i \sim 1$, $f_i' \sim 2$ under $Z_4^{(i)}$
- $\langle f_i \rangle \approx \langle f_i' \rangle \approx v = \varepsilon M_f$

Field	Model 1	Model 2
ν_1^c	(0,0,0,1,0,1,1)	(2,0,0,2,0,0,1)
ν_2^c	(2,0,0,1,0,1,1)	(2,0,0,2,0,0,1)
ν_3^c	(0,0,0,1,0,1,1)	(0,2,0,0,2,1,0)
l_1	(0,2,0,0,1,0,1)	(2,0,2,2,2,1,0)
l_2	(0,0,0,0,1,1,0)	(2,2,0,2,2,1,0)
l_3	(0,0,2,1,0,0,1)	(0,2,2,2,2,0,1)
e_1^c	(2,2,2,1,1,1,1)	(0,0,0,0,0,1,1)
e_2^c	(2,2,2,0,0,1,0)	(2,2,2,0,0,3,3)
e_3^c	(0,2,2,0,0,0,0)	(2,2,2,2,2,3,3)

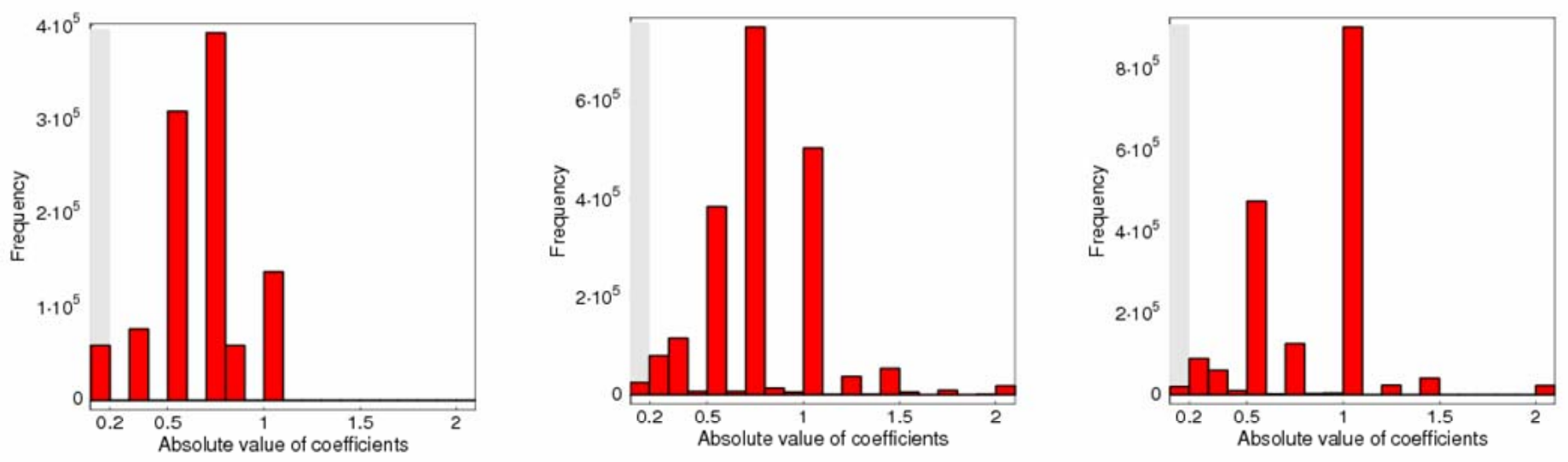
#	M_ℓ	M_D	M_R	$M_D^{\rm diag}/m_D$ $M_R^{\rm diag}/M_{B-L}$	$(\theta_{12}^\ell, \theta_{13}^\ell, \theta_{23}^\ell)$ $(\theta_{12}^D, \theta_{13}^D, \theta_{23}^D)$ $(\theta_{12}^{D'}, \theta_{13}^{D'}, \theta_{23}^{D'})$ $(\theta_{12}^R, \theta_{13}^R, \theta_{23}^R)$
17	$\begin{pmatrix} 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & \epsilon \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon & \epsilon^2 & \epsilon \\ 1 & \epsilon & 1 \\ 1 & \epsilon & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & 1 \\ \epsilon & 1 & \epsilon \\ 1 & \epsilon & 1 \end{pmatrix}$	$(\epsilon, \epsilon, 1)$ $(\epsilon, 1, 1)$	$(\frac{\pi}{4}, \frac{\pi}{4}, \epsilon)$ $(\epsilon, 0, \frac{\pi}{4})$ $(0, \frac{\pi}{4}, 0)$ $(\epsilon, \frac{\pi}{4}, 0)$
18	$\begin{pmatrix} 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon & \epsilon & 0 \\ \epsilon & \epsilon & \epsilon \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$(\epsilon, \epsilon, 1)$ $(\epsilon, 1, 1)$	$(\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{4})$ $(\frac{\pi}{4}, 0, \epsilon)$ $(0, 0, 0)$ $(\frac{\pi}{4}, \epsilon, \epsilon)$

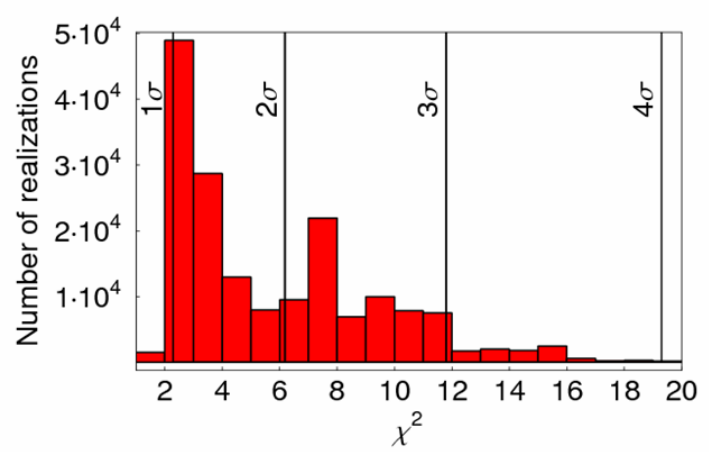
(FP,Seidl,Winter,2007)

Seesaw Procedure



Seesaw Performance





(FP,Seidl,Winter,2007)