Precision Cosmology: Scalar and Tensor Perturbations from Inflation

Salman Habib (T-8), Andreas Heinen (University of Dortmund), Katrin Heitmann (ISR-1), and Gerard Jungman (T-6); ***habib@lanl.gov***

The idea of cosmological inflation is a central component of the present theoretical picture of cosmology. Inflation not only directly addresses and solves the fundamental weaknesses of the older Big Bang picture—the flatness and horizon problems—it also provides an elegant mechanism for the creation of primordial fluctuations, essential for explaining the observed structure of the present-day universe. As the universe inflates, microscopic quantum scales are stretched to cosmologically macroscopic scales. The inevitably present quantum vacuum fluctuations provide the initial seeds that are amplified by the gravitational instability leading to the formation of structure in the universe.

The primordial fluctuations associated with inflation are, primarily, of a very simple type. In "standard" inflationary models, they arise from the fluctuations of an effectively free scalar field, and are hence Gaussian random fields, completely characterized by two-point statistics, such as the power spectrum. The basic task, then, is to determine the scalar and tensor perturbations (vector perturbations being naturally suppressed) in terms of the associated power spectra. It turns out that power spectra from standard inflation models are conveniently parameterized by a spectral index and its (weak) variation with scale—the "running" of the spectral index. In addition, inflation couples the physics of the generation of scalar and tensor fluctuations, so that, if both are independently measured, "compatibility" relations characteristic of inflation can be put to observational test. It should be noted that complicated inflationary models can be constructed to produce baroque "designer" power spectra. Present observations do not require the existence of such models.

Observational constraints on the primordial power spectra arise from measurements of temperature anisotropies in the cosmic microwave background radiation (CMBR)—sensitive to both scalar and tensor modes—and measurements of the scalar (density) power spectrum from surveys of the large-scale mass distribution in the universe. Ground and satellite-based observations of the CMBR anisotropy have yielded results very consistent with the essential adiabatic and Gaussian nature of inflationary perturbations, with a value of the scalar spectral index very close to unity. The tensor fluctuations are expected to be much lower in amplitude and have not been observed. Scalar perturbations seed structure formation and hence can be measured by observing the large-scale distribution of galaxies and neutral hydrogen, as in ongoing redshift surveys and Ly-alpha observations. This second set of independent measurements provides information on smaller scales than the CMBR, yet there is an overlap region where both measurements have been shown to be consistent.

As measurements continue to improve, tests of the inflationary paradigm and specific models will become more stringent, especially if accurate measurements of the running of the spectral index are performed and tensor perturbations are observed. In addition, an inverse analysis of the observational data may then be attempted, in an effort to directly measure the inflationary "equation of state" [1]. In keeping with the remarkable improvement of observational capabilities, the quality of theoretical predictions has necessarily to be addressed. This issue is the central theme of this project.

Primordial power spectra and the associated spectral indices and their running can be calculated either exactly in a direct numerical approach or via approximate schemes. The most common approximation is the so called "slow-roll" approximation, which is based on the idea that the inflation field undergoes a phase of rolling slowly down a very shallow potential hill. Most inflationary models lead to such a "slow-roll" phase to insure that the inflationary phase lasts long enough to solve the flatness and horizon problems. As it is based on Taylor expansions, the main weakness of the slow-roll approximation is that the errors are uncontrolled. While the approximation may work very well for certain models it can unpredictably break down for others. In two recent papers [2, 3] we have implemented a new approach to calculate the power spectra and spectral indices of primordial perturbations based on the mathematically controlled, and systematically extendable, uniform approximation. Our method leads to simple expressions for the power spectra and spectral indices with calculable error bounds.

In our most recent work [4] we have implemented a direct numerical approach as well as the slow-roll and uniform approximations with the aim of characterizing the errors associated with each method and understanding the associated advantages and disadvantages. The main technical advances are a robust and numerically efficient strategy for the mode-by-mode integration, simple and useful error estimates for the uniform approximation, and a simple improvement strategy for power spectra amplitudes for the uniform approximation at leading order. We have shown that a simple improvement of the leading order uniform approximation based on our recent results leads to very good accuracy for the spectral indices and their running, as well as for the power spectra amplitudes and the ratio of tensor to scalar perturbations. For the most part, the accuracy of the results obtained is better than 0.1%. The primordial flucutations are connected to observations via computation of the radiation and matter transfer functions using Boltzmann codes such as CMBFAST. These codes also have a demonstrated accuracy of 0.1%. This level of accuracy is sufficient for the analysis of next-generation CMBR observations such as those from the PLANCK satellite.

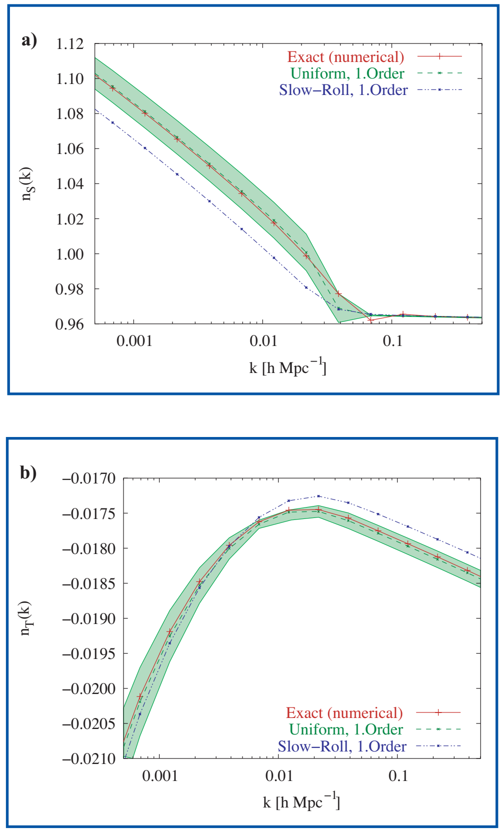

Figure 1—
(a) Scalar spectral index $n_S(k)$. Red solid line with crosses: exact numerical results, green dashed lines: uniform approximation, blue dashed dotted lines: slow-roll; the green band is the error estimate for the uniform approximation.
(b) Tensor spectral index $n_T(k)$ for an inflationary potential which combines a fast- and a slow-roll phase.

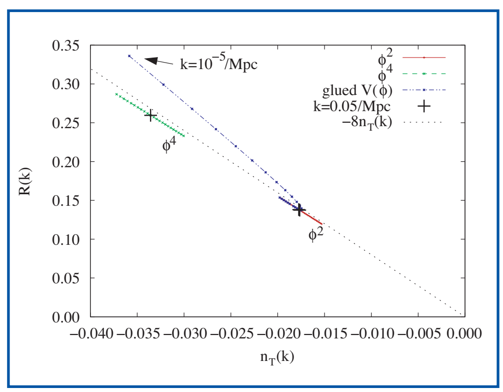

Figure 2—
Tensor to scalar ratio $R(k)$ against the tensor spectral index $n_T(k)$ for different inflationary models. The dotted line denotes $\simeq -8n_T$, a prediction from slow-roll inflation models. However, fitting the ϕ^2 and the ϕ^4 model results show that these models lie on a straight line $R = -(7.757 +/- 0.004)n_T$.

[1] S. Habib, K. Heitmann, and G. Jungman, "Inverse Scattering and Density Perturbations from Inflation," contribution on p. 88 in this volume.
[2] S. Habib, K. Heitmann, G. Jungman, and C. Molina-Paris, *Phys. Rev. Lett.* **89**, 281301 (2002).
[3] S. Habib, A. Heinen, K. Heitmann, G. Jungman, and C. Molina-Paris, *Phys. Rev. D* **70**, 083507 (2004).
[4] S. Habib, A. Heinen, K. Heitmann, and G. Jungman, LA-UR-04-8611 (2004), *Phys. Rev. D* (in press).