17 Confinement in SUSY QCD: Part II

17.1 $F = N + 1$: Confinement without chiral symmetry breaking

For $F = N + 1$ the baryons are flavor anti-fundamentals:

$$B^i = \epsilon^{i_1,\ldots,i_N,i} B_{i_1,\ldots,i_N} \quad (17.1)$$

$$\overline{B}_i = \epsilon_{i_1,\ldots,1_N,i} \overline{B}^{i_1,\ldots,i_N} \quad (17.2)$$

The classical constraints are:

$$(M^{-1})^i_j \det M = B^i \overline{B}_j \quad (17.3)$$

$$M^j_i B^i = M^j_i \overline{B}_j = 0 \quad (17.4)$$

With quark masses turned on we have:

$$\langle M^j_i \rangle = (m^{-1})^j_i \left(\det m \Lambda^{2N-1} \right)^{\frac{1}{N}} \quad (17.5)$$

$$\langle B^i \rangle = \langle \overline{B}_j \rangle = 0 \quad (17.6)$$

So taking a determinant gives

$$(M^{-1})^i_j \det M = m^i_j \Lambda^{2N-1} \quad (17.7)$$

$$(17.8)$$

We see that the classical constraint is satisfied as $m^i_j \to 0$. Taking this limit in different ways we can cover the classical moduli space.

The most general superpotential allowed is

$$W = \frac{1}{\Lambda^{2N-1}} \left[\alpha B^i M^j_i \overline{B}_j + \beta \det M + \det M f \left(\frac{\det M}{B^i M^j_i \overline{B}_j} \right) \right] \quad (17.9)$$

Only $f = 0$ reproduces the classical constraints:

$$\frac{\delta W}{\delta M^j_i} = \frac{1}{\Lambda^{2N-1}} \left[\alpha B^i \overline{B}_j + \beta (M^{-1})^i_j \det M \right] = 0 \quad (17.10)$$

$$\frac{\delta W}{\delta B^i} = \frac{1}{\Lambda^{2N-1}} \alpha M^j_i \overline{B}_j = 0 \quad (17.11)$$

$$\frac{\delta W}{\delta \overline{B}_j} = \frac{1}{\Lambda^{2N-1}} \alpha B^i M^j_i = 0 \quad (17.12)$$

We also need $\beta = -\alpha$.

Consider adding a mass for one flavor.

$$W = \frac{\alpha}{\Lambda^{2N-1}}\left[B^i M_i^j \overline{B}_j - \det M\right] = mX \tag{17.13}$$

where

$$\begin{aligned} M &= \begin{pmatrix} M_j'^i & Z^i \\ Y_j & X \end{pmatrix} && (17.14)\\ B &= \left(U^i, B'\right) && (17.15)\\ \overline{B} &= \begin{pmatrix} \overline{U}_j \\ \overline{B}' \end{pmatrix} && (17.16) \end{aligned}$$

we have the following equations of motion:

$$\begin{aligned} \frac{\delta W}{\delta Y} &= \frac{\alpha}{\Lambda^{2N-1}}\left(B'\overline{U} - \mathrm{cof}(Y)\right) = 0 && (17.17)\\ \frac{\delta W}{\delta Z} &= \frac{\alpha}{\Lambda^{2N-1}}\left(U\overline{B}' - \mathrm{cof}(Z)\right) = 0 && (17.18)\\ \frac{\delta W}{\delta U} &= \frac{\alpha}{\Lambda^{2N-1}} Z\overline{B}' = 0 && (17.19)\\ \frac{\delta W}{\delta \overline{U}} &= \frac{\alpha}{\Lambda^{2N-1}} B'\overline{Y} = 0 && (17.20)\\ \frac{\delta W}{\delta X} &= \frac{\alpha}{\Lambda^{2N-1}}\left(B'\overline{B}' - \det M'\right) + m = 0 && (17.21)\\ & && (17.22) \end{aligned}$$

So

$$\begin{aligned} Y = Z = U = \overline{U} &= 0 && (17.23)\\ \det M' - B'\overline{B}' &= \frac{m\Lambda^{2N-1}}{\alpha} = \frac{1}{\alpha}\Lambda_{N,N}^{2N} && (17.24) \end{aligned}$$

This gives the correct quantum constraint for $F = N$ if $\alpha = 1$.

So the effective superpotential is

$$\begin{aligned} W_{\text{eff}} &= \frac{X}{\Lambda^{2N-1}}\left(B'\overline{B}' - \det M' + m\Lambda^{2N-1}\right) && (17.25)\\ &= \frac{X}{\Lambda^{2N-1}}\left(B'\overline{B}' - \det M' + \Lambda_{N,N}^{2N}\right) && (17.26) \end{aligned}$$

Holding $\Lambda_{N,N}$ fixed as $m \to \infty$ implies that $\Lambda \to 0$, so X becomes a Lagrange multiplier field in this limit.

Since the point $M = B = \overline{B} = 0$ in on the quantum moduli space, we should worry about what singular behavior occurs there. Naively gluons and gluinos should become massless. What actually happens is that only the components of M, B, $\overline{B}$ become massless. That is we have confinement without chiral symmetry breaking. This is the type of theory that ‘t Hooft was searching for. The composites transform as:

	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
M	$\square$	$\overline{\square}$	0	$\frac{2}{F}$
B	$\overline{\square}$	$\mathbf{1}$	N	$\frac{N}{F}$
$\overline{B}$	$\mathbf{1}$	$\square$	$-N$	$\frac{N}{F}$

Some of the anomaly matchings go as follows:

$$\begin{array}{lcc} & \text{elem.} & \text{comp.} \\ SU(F)^3: & N & F-1 \\ U(1)SU(F)^2: & N\frac{1}{2} & N\frac{1}{2} \\ U(1)_R SU(F)^2: & -\frac{N}{F}\frac{N}{2} & \frac{2-F}{F}\frac{F}{2} + \frac{N-F}{F} \\ U(1)_R: & -N^2-1 & \frac{2-F}{F}F^2 + 2(N-F) \\ U(1)_R^3: & -\left(\frac{N}{F}\right)^3 2NF + N^2 - 1 & \left(\frac{2-F}{F}\right)^3 F^2 + \left(\frac{N-F}{F}\right)^3 2F \end{array} \tag{17.27}$$

which agree because $F = N + 1$.

17.2 Connection to theories with $F > N + 1$

Consider the dual theory for $F = N + 2$:

	$SU(2)$	$SU(N+2)$	$SU(N+2)$	$U(1)$	$U(1)_R$
q	$\square$	$\overline{\square}$	$\mathbf{1}$	$\frac{N}{2}$	$\frac{N}{N+2}$
$\overline{q}$	$\overline{\square}$	$\mathbf{1}$	$\square$	$-\frac{N}{2}$	$\frac{N}{N+2}$
M	$\mathbf{1}$	$\square$	$\overline{\square}$	0	$\frac{4}{N+2}$

Giving a mass to one flavor in the corresponding SUSY QCD theory produces a dual squark vev

$$\langle \overline{\phi}^F \phi_F \rangle = -\mu m \tag{17.28}$$

which completely breaks the $SU(2)$ gauge group.

The low-energy effective theory is:

	$SU(N+1)$	$SU(N+1)$	$U(1)$	$U(1)_R$
q'	$\overline{\square}$	$\mathbf{1}$	N	$\frac{N}{N+1}$
$\overline{q'}$	$\mathbf{1}$	$\square$	$-N$	$\frac{N}{N+1}$
M'	$\square$	$\overline{\square}$	0	$\frac{2}{N+1}$

So we should identify

$$q'^i = cB^i \tag{17.29}$$
$$\overline{q'}_j = \bar{c}\overline{B}_j \tag{17.30}$$

The remanant of the dual superpotential is

$$W = \frac{c\bar{c}}{\mu} B^i M'^j_i \overline{B}_j \tag{17.31}$$

Since we have completely broken the dual gauge group we expect that instantons will generate extra terms in the superpotential. Indeed we find:

$$W_{\text{inst.}} = \frac{\widetilde{\Lambda}^{\tilde{b}}_{N,N+2}}{\langle \overline{\phi}^F \phi_F \rangle} \det\left(\frac{M'}{\mu}\right) \tag{17.32}$$
$$= -\frac{\widetilde{\Lambda}^{4-N}_{N,N+2}}{m} \frac{\det M'}{\mu^{N+2}} \tag{17.33}$$

So the effective superpotential is:

$$W_{\text{eff}} = \frac{1}{\Lambda^{2N-1}} \left[B^i M'^j_i \overline{B}_j - \det M' \right] \tag{17.34}$$

if

$$c\bar{c} = \frac{\mu}{\Lambda^{2N-1}} \tag{17.35}$$
$$\frac{\widetilde{\Lambda}^{4-N}_{N,N+2}}{\mu^{N+2} m} = \frac{1}{\Lambda^{2N-1}} \tag{17.36}$$

The second relation follows from a more general relation

$$\widetilde{\Lambda}^{3\widetilde{N}-F} \Lambda^{3N-F} = (-1)^{F-N} \mu^F \tag{17.37}$$

To see why this relation is true consider generic values of $\langle M \rangle$ in the dual of SUSY QCD. All the dual quarks are massive, so we have a pure $SU(F-N)$ gauge theory. The intrinsic scale of the effective theory is given by matching:

$$\widetilde{\Lambda} L^{3\widetilde{N}} = \widetilde{\Lambda}^{3\widetilde{N}-F} \det\left(\frac{M}{\mu}\right) \tag{17.38}$$

This theory has gaugino condensation:

$$\begin{aligned} W_L &= \widetilde{N}\widetilde{\Lambda}L^3 && (17.39)\\ &= (F-N)\left(\frac{\widetilde{\Lambda}^{3\widetilde{N}-F}\det M}{\mu}\right)^{\frac{1}{F-N}} && (17.40)\\ &= (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}} && (17.41) \end{aligned}$$

where we have used eq. (17.37). Adding a mass term $m^i_j M^j_i$ gives:

$$M^j_i = (m^{-1})^j_i \left(\det m \Lambda^{3N-F}\right)^{\frac{1}{N}} \tag{17.42}$$

which we have already seen is the correct result.

If we consider the dual of the dual of SUSY QCD eq. (17.37) implies

$$\Lambda^{3N-F}\widetilde{\Lambda}^{3\widetilde{N}-F} = (-1)^{F-\widetilde{N}}\widetilde{\mu}^F \tag{17.43}$$

So

$$\widetilde{\mu} = -\mu \tag{17.44}$$

Writing

$$N^i_j = \overline{\phi}^i \phi_j \tag{17.45}$$

The superpotential in the dual of the dual is

$$W_{dd} = \frac{N^j_i}{\widetilde{\mu}}\overline{d}^i d_j + \frac{M^i_j}{\mu} N^j_i \tag{17.46}$$

The equations of motion give

$$\begin{aligned} \frac{\delta W}{\delta M^i_j} &= \frac{1}{\mu} N^j_i = 0 && (17.47)\\ \frac{\delta W}{\delta N^j_i} &= \frac{1}{\widetilde{\mu}}\overline{d}^i d_j + \frac{1}{\mu} M^i_j = 0 && (17.48) \end{aligned}$$

So we can identify the original squarks with the dual-dual squarks:

$$\Phi_j = d_j \tag{17.49}$$

and the dual of the dual of SUSY QCD is just SUSY QCD.

References

[1] "Lectures on supersymmetric gauge theories and electric-magnetic duality," by K. Intriligator and N. Seiberg, hep-th/9509066.