21 Dynamical SUSY Breaking: Part I

21.1 A Rule of Thumb for SUSY Breaking

A theory that has no flat directions and spontaneously breaks a continuous global symmetry generally breaks SUSY. This is because there must be a Goldstone boson (which has no interactions in the potential), and by SUSY it must have a scalar partner (a modulus), but if there are no flat directions this is impossible. (Unless the modulus is also a Goldstone). In the early days people looked for theories that had no classical flat directions (assuming that quantum corrections would not cancel the classical potential) and tried to make them break global symmetries in the perturbative regime. This resulted in a handful of theories. With duality we can find many examples of dynamical SUSY breaking. An important twist is that we will find that non-perturbative quantum effects can lift flat directions.

21.2 The 3-2 Model

Affleck, Dine, and Seiberg found the simplest model of dynamical SUSY breaking:

	$SU(3)$	$SU(2)$
Q	$\Box$	$\Box$
L	$\mathbf{1}$	$\Box$
$\bar{U}$	$\overline{\Box}$	$\mathbf{1}$
$\bar{D}$	$\overline{\Box}$	$\mathbf{1}$

we will write $\overline{Q} = (\overline{U}, \overline{D})$. For $\Lambda_3 \gg \Lambda_2$, instantons give:

$$W_{\rm dyn} = \frac{\Lambda_3^7}{\det(\overline{Q}Q)} \tag{21.1}$$

which has a runaway vacuum. Adding

$$W = \lambda\, Q\bar{U}L \tag{21.2}$$

removes classical flat directions and produces a stable minimum. The L equation of motion tries to set $\det\overline{Q}Q$ to zero, so the potential can't have a zero minimum. SUSY is broken.

We can estimate the vacuum energy for $\lambda \ll 1$ by minimizing

$$V \approx \frac{\Lambda_3^{14}}{\phi^{10}} + \lambda^2\phi^4 \tag{21.3}$$

so

$$\langle\phi\rangle \approx \Lambda_3 \lambda^{\frac{-1}{7}} \tag{21.4}$$

so the vacuum is weakly coupled for small λ and

$$V \approx \lambda^{\frac{-10}{7}} \Lambda_3^4 \tag{21.5}$$

With duality we can consider consider $\Lambda_2 \gg \Lambda_3$ (Intriligator, Thomas hep-th/9608046). The $SU(2)$ gauge group has 4 doublets which is equivalent to 2 flavors, so we have confinement with chiral symmetry breaking. The $SU(3)$ gauge group is generically broken. It is simpler to consider $SU(2)$ as an SU group rather than an Sp group, so we write:

$$\begin{array}{rcl} B & \sim & Q_1 Q_2 \\ \bar{B} & \sim & Q_3 L \\ M & \sim & \begin{pmatrix} LQ_1 & LQ_2 \\ Q_3Q_1 & Q_3Q_2 \end{pmatrix} \end{array}$$

In this notation the effective superpotential is

$$W = A\left(\det M - B\bar{B} - \Lambda_2^4\right) + \lambda\left(M_{1\mathrm{i}}\bar{U}^{\mathrm{i}} + \bar{B}\bar{U}^3\right) \tag{21.6}$$

The constraint means that at least one of M_{11}, M_{12}, or $\overline{B}$ is non-zero, so we see that SUSY is broken at tree-level in the dual description. We can estimate the vacuum energy as

$$V \approx \lambda^2 \Lambda_2^4 \tag{21.7}$$

Without making the approximation that one gauge group is much stronger than the other we should consider the full superpotential

$$W = A\left(\det M - B\bar{B} - \Lambda_2^4\right) + \frac{\Lambda_3^7}{\det(\overline{Q}Q)} + \lambda\, Q\bar{U}L \tag{21.8}$$

which still breaks SUSY.

21.3 The SU(5) Model

Another simple model due to Affleck, Dine, and Seiberg is $SU(5)$ with $\overline{\square}+\begin{array}{c}\square\\[-4pt]\square\end{array}$ This theory has no classical flat directions, since there are no gauge invariant operators that we can write down. People who tried to match the anomalies in order to find a confined description found only “bizarre”, “implausible” solutions. This lead people to believe that the at least one of the global $U(1)$'s was broken and that therefore SUSY was broken. Adding extra flavors ($\Box + \overline{\Box}$) with masses one finds that SUSY is broken, but taking the masses to ∞ takes the theory to a strongly coupled regime. With duality we can see that SUSY is indeed broken.

We know that for four flavors

	$SU(5)$	$SU(4)$	$SU(5)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	1	1	0	9	0
$\overline{Q}$	$\overline{\Box}$	1	$\Box$	4	-3	0
Q	$\Box$	$\Box$	1	-5	-3	$\frac{1}{2}$

the theory confines:

	$SU(4)$	$SU(5)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$Q\overline{Q}$	$\Box$	$\Box$	-1	-6	$\frac{1}{2}$
$A\overline{Q}^2$	1	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	8	3	0
A^2Q	$\Box$	1	-5	15	$\frac{1}{2}$
AQ^3	$\overline{\Box}$	1	-15	0	$\frac{3}{2}$
$\overline{Q}^5$	1	1	20	-15	0

with a superpotential

$$\begin{aligned} W_{dyn} &= \frac{1}{\Lambda^4}\Big[(A^2Q)(Q\overline{Q})^3(A\overline{Q}^2) + (AQ^3)(Q\overline{Q})(A\overline{Q}^2)^2 + \qquad (21.9)\\ &\quad (\overline{Q}^5)(A^2Q)(AQ^3)\Big] \qquad (21.10)\end{aligned}$$

We can add mass terms and Yukawa couplings for the extra flavors:

$$\Delta W = \sum_{i=1}^{4} mQ_i\overline{Q}_i + \sum_{i,j\leq 4}\lambda_{ij}A\overline{Q}_i\overline{Q}_j \qquad (21.11)$$

which lift all the flat directions.

The equations of motion give

$$\begin{aligned} \frac{\delta W}{(\overline{Q}^5)} &= (A^2Q)(AQ^3) = 0 \qquad (21.12)\\ \frac{\delta W}{(Q\overline{Q})} &= 3(A^2Q)(Q\overline{Q})^2(A\overline{Q}^2) + (AQ^3)(A\overline{Q}^2)^2 + m \qquad (21.13)\end{aligned}$$

Multiplying the second equation by (A^2Q) (and keeping track of antisymmetrizations) we find $(A^2Q) = 0$, so

$$m = -(AQ^3)(A\overline{Q}^2)^2 \ . \tag{21.14}$$

Multiplying by (AQ^3) we find that the right hand side vanishes dues to antisymmetrizations, so $(AQ^3) = 0$ but this contradicts eq. (21.14), so the equations of motion cannot be satisfied, and SUSY is broken at tree level in the dual.

References

[1] I. Affleck, M. Dine and N. Seiberg, "Calculable Nonperturbative Supersymmetry Breaking," Phys. Rev. Lett. **52** (1984) 1677.

[2] I. Affleck, M. Dine and N. Seiberg, "Dynamical Supersymmetry Breaking In Four-Dimensions And Its Phenomenological Implications," Nucl. Phys. **B256** (1985) 557.

[3] K. Intriligator and S. Thomas, "Dynamical Supersymmetry Breaking on Quantum Moduli Spaces," Nucl. Phys. **B473** (1996) 121 hep-th/9603158.