26 Anomaly Mediation Part I

26.1 "Supergravity" Mediation

Recall that we can write a supergravity theory using a spurion field Σ

$$
\begin{aligned}
\mathcal{L} &= e\left[\int d^4\theta f(Q^\dagger, e^V Q)\frac{\Sigma^\dagger\Sigma}{M_{\mathrm{Pl}}^2} + \int d^2\theta \frac{\Sigma^3}{M_{\mathrm{Pl}}^3} W(Q) - \int d^2\theta \frac{i\tau}{16\pi} W^\alpha W_\alpha + h.c.\right] \\
&\quad -\frac{e}{6} f(q^\dagger, q)\sigma^*\sigma R + \mathcal{L}_{\mathrm{aux}} + \mathcal{L}_{\mathrm{gravitino}} \qquad (26.1)
\end{aligned}
$$

where f is related to the Kahler potential

$$f = -3M_{\mathrm{Pl}}^2 e^{\frac{-K}{3M_{\mathrm{Pl}}^2}} \qquad (26.2)$$

and taking

$$\Sigma = \sigma = M_{\mathrm{Pl}} \qquad (26.3)$$

gives standard supergravity up to a rescaling of the metric.

In the supergravity mediated scenario one imagines that dynamical SUSY breaking in a hidden sector is communicated to the MSSM (the visible sector) purely through M_{Pl} suppressed operators. If X is a SUSY breaking field in the hidden sector, then in general the coupling functions take the form

$$
\begin{aligned}
W &= W_{\mathrm{hid}}(X) + W_{\mathrm{vis}}(Q) && (26.4) \\
f &= \left(1 - \frac{c_j^i}{M_{\mathrm{Pl}}^2} X^\dagger X\right) Q^{j\dagger} e^V Q_i + \ldots && (26.5) \\
\tau &= \frac{\theta_{\mathrm{YM}}}{2\pi} + i\frac{4\pi}{g^2} + i\frac{k}{M_{\mathrm{Pl}}} X + \ldots && (26.6)
\end{aligned}
$$

If we parametrize the SUSY breaking by

$$\langle X \rangle = M + F_X \theta^2 \qquad (26.7)$$

then we get induced squark and gluino masses:

$$(M_q^2)^i_j = c^i_j \frac{F_X^2}{M_{\mathrm{Pl}}^2} \qquad (26.8)$$

$$M_\lambda = k\frac{F_X}{M_{\mathrm{Pl}}} \; . \qquad (26.9)$$

Unfortunately there is no reason for the c^i_j interactions to respect flavor symmetries, so generically there would be FCNC's. Naively it was imagined that the Kahler potential might have a simple flavor blind form:

$$K = X^\dagger X + Q^{i\dagger} e^V Q_i \tag{26.10}$$

which implies

$$f = -3 + \frac{1}{M_{\rm Pl}^2}\left[X^\dagger X + \left(1 + \frac{X^\dagger X}{M_{\rm Pl}^2}\right) Q^{i\dagger} e^V Q_i + \ldots\right] \tag{26.11}$$

The interactions are flavor blind but we see that there are direct interactions induced by Planck scale (string) states which have been integrated out. (A better name for this scenario would be "string mediated".) These interactions should not be flavor blind since they must generate Yukawa couplings.

26.2 Extra Dimensions and Pure Supergravity Mediation

However in an extra-dimension scenario the SUSY breaking sector can be physically separated by a distance r from the MSSM by putting the two sectors on two different "branes". Interactions generated by Planck scale states will be supressed by

$$e^{-M_{\rm Pl} r} \tag{26.12}$$

If only supergravity states propagate in the bulk then setting $e^m_\mu = 0$ and $\Sigma = M_{\rm Pl}$ must decouple the two sectors. Thus

$$\begin{aligned} W &= W_{\rm hid} + W_{\rm vis} && (26.13)\\ f &= c + f_{\rm hid} + f_{\rm vis} && (26.14)\\ \tau W_\alpha^2 &= \tau_{\rm hid} W^2_{\alpha^2 {\rm hid}} + \tau_{\rm vis} W^2_{\alpha {\rm vis}}\ . && (26.15) \end{aligned}$$

So all the interactions between the two sectors are purely due to supergravity. This implies

$$K \propto \ln\left(1 - 3(f_{\rm hid} + f_{\rm vis})\right)\ . \tag{26.16}$$

We can easily integrate out the hidden sector, and get an effective theory that couples the MSSM to supergravity. Dropping Planck supressed interactions we have

$$\mathcal{L}_{\rm eff} = \int d^4\theta Q^\dagger e^V Q \frac{\Sigma^\dagger\Sigma}{M_{\rm Pl}^2} + \int d^2\theta \frac{\Sigma^3}{M_{\rm Pl}^3}(m_0 Q^2 + \lambda Q^3) - \frac{i}{16\pi}\int d^2\theta \tau W^\alpha W_\alpha + h.c. \tag{26.17}$$

Where the conformal weights of the fields determine

$$R[\Sigma] = \frac{2}{3},\ R[Q] = 0 \tag{26.18}$$

We can simplify the theory by rescaling

$$\frac{\Sigma Q}{M_{\rm Pl}} \to Q\ . \tag{26.19}$$

After this rescaling

$$R[Q] = \frac{2}{3} \tag{26.20}$$

and

$$\mathcal{L}_{\rm eff} = \int d^4\theta Q^\dagger e^V Q + \int d^2\theta (\frac{\Sigma}{M_{\rm Pl}} m_0 Q^2 + \lambda Q^3) - \frac{i}{16\pi}\int d^2\theta \tau W^\alpha W_\alpha + h.c. \tag{26.21}$$

If $m_0 = 0$, then the theory is classically scale and conformally invariant, and Σ classically decouples. However quantum corrections break scale invariance, in the supergravity literature this is referred to as the super-Weyl anomaly. In more prosaic terms couplings run. For example if we look at a two-point function we find

$$G = \frac{1}{p^2} h\left(\frac{p^2 M_{\rm Pl}^2}{\Lambda^2 \Sigma^\dagger \Sigma}\right)\ . \tag{26.22}$$

The effects of the scaling anomaly are determined by β functions and anomalous dimensions.

If we integrate our effective theory down to the scale μ we have:

$$\begin{aligned}\mathcal{L}_{\rm eff} \ &= \ \int d^4\theta Z\left(\frac{\mu M_{\rm Pl}}{\Lambda\Sigma}, \frac{\mu M_{\rm Pl}}{\Lambda\Sigma^\dagger}\right)\theta Q^\dagger e^V Q \\ &\quad + \int d^2\theta \lambda Q^3 - \frac{i}{16\pi}\int d^2\theta \tau W^\alpha W_\alpha + h.c.\end{aligned} \tag{26.23}$$

Since Z is real and R-invariant we must have

$$Z = Z\left(\frac{\mu M_{\rm Pl}}{\Lambda|\Sigma|}\right) \tag{26.24}$$

where

$$|\Sigma| = \left(\Sigma^\dagger \Sigma\right)^{1/2} \ . \tag{26.25}$$

We also know that with $\Sigma = M_{\rm Pl}$ the R symmetry is anomalous and $\theta_{\rm YM}$ shifts when the Q's are rephased, but if Σ is also rephased, this shift is canceled. Thus

$$\tau = i\frac{\widetilde{b}}{2\pi}\ln\left(\frac{\mu M_{\rm Pl}}{\Lambda\Sigma}\right) \ . \tag{26.26}$$

The μ dependence of this result detemines that $\widetilde{b}$ is actually equal to b the one-loop coefficient in the β function:

$$\frac{dg}{d\ln\mu} = -\frac{b}{16\pi^2}g^3 \ . \tag{26.27}$$

26.3 SUSY Breaking

In general if SUSY is broken in the hidden sector then SUSY breaking will be communicated to the the auxillary fields of supergravity, and we will have

$$\langle\Sigma\rangle = M_{\rm Pl} + F_\Sigma\theta^2 \tag{26.28}$$

This will induce a θ^2 term in τ, and as we saw in the case of gauge mediation, this will lead to a gaugino mass:

$$\begin{aligned} M_\lambda &= \frac{i}{2\tau}\frac{\partial\tau}{\partial\Sigma}|_{\Sigma=M_{\rm Pl}}F_\Sigma \\ &= \frac{bg^2}{16\pi^2}\frac{F_\Sigma}{M_{\rm Pl}} \end{aligned} \tag{26.29}$$

We can also Taylor expand Z in superspace:

$$Z = \left[Z - \frac{1}{2}\frac{\partial Z}{\partial\ln\mu}\left(\frac{F_\Sigma}{M_{\rm Pl}}\theta^2 + \frac{F_\Sigma^\dagger}{M_{\rm Pl}}\theta^{\dagger 2}\right) + \frac{1}{4}\frac{\partial^2 Z}{\partial(\ln\mu)^2}\frac{|F_\Sigma|^2}{M_{\rm Pl}^2}\theta^2\theta^{\dagger 2}\right]|_{\Sigma=M_{\rm Pl}} \tag{26.30}$$

We can canonically normalize the kinetic terms by rescaling:

$$
\begin{aligned}
Q' &= Z^{1/2}\left(1-\frac{1}{2}\frac{\partial \ln Z}{\partial \ln \mu}\frac{F_\Sigma}{M_{\rm Pl}}\theta^2\right)|_{\Sigma=M_{\rm Pl}}Q \\
&= Z^{1/2}\left(1-\frac{1}{2}\gamma\frac{F_\Sigma}{M_{\rm Pl}}\theta^2\right)|_{\Sigma=M_{\rm Pl}}Q \qquad (26.31)
\end{aligned}
$$

(This rescaling actually introduces another higher order anomaly effect since the coefficient is already a one-loop effect.) We find:

$$
\begin{aligned}
ZQ^\dagger e^V Q &= \left[1+\frac{1}{4}\frac{\partial\gamma}{\partial\ln\mu}\frac{|F_\Sigma|^2}{M_{\rm Pl}^2}\theta^2\theta^{\dagger 2}\right]Q'^\dagger e^V Q' \\
&= \left[1+\frac{1}{4}\left(\frac{\partial\gamma}{\partial g}\beta_g+\frac{\partial\gamma}{\partial\lambda}\beta_\lambda\right)\frac{|F_\Sigma|^2}{M_{\rm Pl}^2}\theta^2\theta^{\dagger 2}\right]Q'^\dagger e^V Q' \qquad (26.32)
\end{aligned}
$$

So we find a squark or slepton mass squared:

$$
M_q^2 = -\frac{1}{4}\left(\frac{\partial\gamma}{\partial g}\beta_g+\frac{\partial\gamma}{\partial\lambda}\beta_\lambda\right)\frac{|F_\Sigma|^2}{M_{\rm Pl}^2} \qquad (26.33)
$$

To leading order we have

$$
\gamma = \frac{C_2(r)}{4\pi^2}g^2 - a\lambda^2 \qquad (26.34)
$$

$$
\beta_g = -bg^3 \qquad (26.35)
$$

$$
\beta_\lambda = \lambda(e\lambda^2 - fg^2) \qquad (26.36)
$$

so

$$
M_q^2 = \frac{1}{2}\left[\frac{C_2(r)}{4\pi^2}bg^4 + a\lambda^2(e\lambda^2-fg^2)\right]\frac{|F_\Sigma|^2}{M_{\rm Pl}^2}\ . \qquad (26.37)
$$

The first term is positive for asymptotically free gauge theories, and we see immediately that this gives a negative mass squared for sleptons. We will return to this problem later.

As with gauge medation, expanding $W(Q)$ after the rescaling gives trilinear scalar interactions with a coefficient

$$
A_{ijk} = \frac{1}{2}\left(\gamma_i+\gamma_j+\gamma_k\right)\lambda_{ijk}\frac{F_\Sigma}{M_{\rm Pl}}\ . \qquad (26.38)
$$

It is interesting to compare these results with those of the gauge mediation scenario. There the messengers have masses given by

$$\langle X \rangle = M \left(1 + \frac{F_X}{M} \theta^2 \right) \ . \tag{26.39}$$

With anomaly mediation the cutoff is given by

$$\Lambda \frac{\Sigma}{M_{\rm Pl}} = \Lambda \left(1 + \frac{F_\Sigma}{M_{\rm Pl}} \theta^2 \right) \tag{26.40}$$

which can be thought of as the mass of the regulator fields (e.g. Pauli-Villars fields). So in anomaly mediation, the regulator is the messenger.

It is also interesting to consider heavy SUSY thresholds. After rescaling the mass M of a SUSY threshold becomes $M\Sigma/M_{\rm Pl}$. So Z and τ in the low-energy effective theory become:

$$Z \left(\frac{\mu M_{\rm Pl}}{\Lambda |\Sigma|}, \frac{|M||\Sigma|}{\Lambda \Sigma} \right) \tag{26.41}$$

$$\tau \left(\frac{\mu M_{\rm Pl}}{\Lambda \Sigma}, \frac{M\Sigma}{\Lambda \Sigma} \right) \ , \tag{26.42}$$

so the gaugino and sfermion masses are independent of M. This is a reflection of the fact that the scaling anomaly is insensitive to UV physics, it is determined by the low-energy effective theory. The SUSY breaking masses only depend on β_g, β_λ, $\partial\gamma/\partial g$, and $\partial\gamma/\partial\lambda$ at the weak scale.

References

[1] L. Randall and R. Sundrum, "Out of this world supersymmetry breaking," hep-th/9810155.

[2] G.F. Giudice, M.A. Luty, H. Murayama and R. Rattazzi, "Gaugino mass without singlets," JHEP **12** (1998) 027 hep-ph/9810442.