4 Supersymmetry More or Less

4.1 More: Extended Supersymmetry

$$\{Q^a_\alpha, Q^\dagger_{\dot\alpha b}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu \delta^a_b \tag{4.1}$$

$$a = 1, \ldots, \mathcal{N} \tag{4.2}$$

allows an $SU(\mathcal{N})$ R symmetry
Massless multiplets:
choose the frame where $p_\mu = (E, 0, 0, E)$

$$\{Q^a_1, Q^\dagger_{1b}\} = 4E\delta^a_b, \tag{4.3}$$

$$\{Q^a_2, Q^\dagger_{2b}\} = 0. \tag{4.4}$$

so $Q^\dagger_{2b}$ produces states of zero norm.

state	helicity	degeneracy
$\lvert\Omega_\lambda\rangle$	λ	1
$Q^\dagger_{1a}\lvert\Omega_\lambda\rangle$	$\lambda + \frac{1}{2}$	$\mathcal{N}$
$Q^\dagger_{1a}Q^\dagger_{1b}\lvert\Omega_\lambda\rangle$	$\lambda + 1$	$\mathcal{N}(\mathcal{N}-1)/2$
$\vdots$	$\vdots$	$\vdots$
$Q^\dagger_{11}Q^\dagger_{12}\ldots Q^\dagger_{1\mathcal{N}}\lvert\Omega_\lambda\rangle$	$\lambda + \mathcal{N}/2$	1

(4.5)

$|\lambda| \leq 1$ and $|\lambda + \mathcal{N}/2| \leq 1 \Rightarrow \mathcal{N} \leq 4$

$\mathcal{N} = 2$: $-1 \leq \lambda \leq 0$
massless vector multiplet:

state	helicity	degeneracy
$\lvert\Omega_0\rangle$	0	1
$Q^\dagger\lvert\Omega_0\rangle$	$\frac{1}{2}$	2
$(Q^\dagger)^2\lvert\Omega_0\rangle$	1	1

(4.6)

+ CPT conjugate:

state	helicity	degeneracy
$\lvert\Omega_{-1}\rangle$	-1	1
$Q^\dagger\lvert\Omega_{-1}\rangle$	$-\frac{1}{2}$	2
$(Q^\dagger)^2\lvert\Omega_{-1}\rangle$	0	1

(4.7)

massless $\mathcal{N}=2$ vector multiplet is built from $\mathcal{N}=1$ vector and chiral multiplets

$$\begin{array}{rccc} \text{state} & \text{helicity} & \text{degeneracy} & \\ |\Omega_{-\frac{1}{2}}\rangle & -\frac{1}{2} & 1 & \chi_L^c \\ Q^\dagger|\Omega_{-\frac{1}{2}}\rangle & 0 & 2 & \phi \\ (Q^\dagger)^2|\Omega_{-\frac{1}{2}}\rangle & \frac{1}{2} & 1 & \psi_L \end{array} \tag{4.8}$$

gauge-invariant mass term: $\chi_L\psi_L \Rightarrow$ vector-like

$\mathcal{N}=3$: $-1 \leq \lambda \leq -\frac{1}{2}$
massless vector multiplet:

$$\begin{array}{rcc} \text{state} & \text{helicity} & \text{degeneracy} \\ |\Omega_{-1}\rangle & -1 & 1 \\ Q^\dagger|\Omega_{-1}\rangle & -\frac{1}{2} & 3 \\ (Q^\dagger)^2|\Omega_{-1}\rangle & 0 & 3 \\ (Q^\dagger)^3|\Omega_{-1}\rangle & \frac{1}{2} & 1 \end{array} \tag{4.9}$$

$$\begin{array}{rcc} \text{state} & \text{helicity} & \text{degeneracy} \\ |\Omega_{-\frac{1}{2}}\rangle & -\frac{1}{2} & 1 \\ Q^\dagger|\Omega_{-\frac{1}{2}}\rangle & 0 & 3 \\ (Q^\dagger)^2|\Omega_{-\frac{1}{2}}\rangle & \frac{1}{2} & 3 \\ (Q^\dagger)^3|\Omega_{-\frac{1}{2}}\rangle & 1 & 1 \end{array} \tag{4.10}$$

both vector-like

$\mathcal{N}=4$: $\lambda=-1$
massless vector multiplet:

$$\begin{array}{rcc} \text{state} & \text{helicity} & \text{degeneracy} \\ |\Omega_{-1}\rangle & -1 & 1 \\ Q^\dagger|\Omega_{-1}\rangle & -\frac{1}{2} & 4 \\ (Q^\dagger)^2|\Omega_{-1}\rangle & 0 & 6 \\ (Q^\dagger)^3|\Omega_{-1}\rangle & \frac{1}{2} & 4 \\ (Q^\dagger)^4|\Omega_{-1}\rangle & 1 & 1 \end{array} \tag{4.11}$$

vector-like

4.2 $\mathcal{N} = 1$ SUSY: Superspace

Superspace is a clever notational device for working with $\mathcal{N} = 1$ SUSY theories. One should note that it is not intrinsic to SUSY since it does not work for $\mathcal{N} > 1$ or in different numbers of dimensions. Since is just a notational device it gives us no new information, but since many people use it we need to be able to understand what their notation.

Introduce anticommuting (Grassmann) spinor "coordinates": $\theta_\alpha,\ \theta^\dagger_{\dot\alpha}$. Recall that for a single Grassmann variable we have:

$$\int d\theta = 0, \int \theta\, d\theta = 1. \tag{4.12}$$

Define:

$$d^2\theta \equiv -\frac{1}{4} d\theta^\alpha d\theta^\beta \epsilon_{\alpha\beta} \tag{4.13}$$

$$d^2\theta^\dagger \equiv -\frac{1}{4} d\theta^\dagger_{\dot\alpha} d\theta^\dagger_{\dot\beta} \epsilon^{\dot\alpha\dot\beta} \tag{4.14}$$

$$d^4\theta \equiv d^2\theta d^2\theta^\dagger \tag{4.15}$$

Then we have

$$\begin{aligned}
\int d^2\theta\, \theta^2 &= \int d^2\theta\, \theta^\sigma \theta_\sigma \\
&= -\frac{1}{4} \int d\theta^\alpha d\theta^\beta \epsilon_{\alpha\beta} \theta^\sigma \epsilon_{\sigma\tau} \theta^\tau \\
&= -\frac{1}{4} (\epsilon_{\alpha\beta} \delta^{\beta\sigma} \epsilon_{\sigma\tau} \delta^{\tau\alpha} - \epsilon_{\alpha\beta} \delta^{\alpha\sigma} \epsilon_{\sigma\tau} \delta^{\tau\beta}) \\
&= -\frac{1}{4} (\epsilon_{\alpha\beta} \epsilon_{\beta\alpha} + \epsilon_{\beta\alpha} \epsilon_{\alpha\beta}) \\
&= -\frac{1}{2} \epsilon_{\alpha\beta} \epsilon_{\beta\alpha} \\
&= 1
\end{aligned} \tag{4.16}$$

and

$$\int d^2\theta\, (\chi\theta)(\psi\theta) = -\frac{1}{2} (\chi\psi) \tag{4.17}$$

Define a new superspace "coordinate"

$$y^\mu = x^\mu - i\theta\sigma^\mu\theta^\dagger \tag{4.18}$$

Then we can assemble the fields of a chiral supermultiplet into a *chiral superfield*:

$$\begin{aligned}
\Phi(y) &\equiv \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \\
&= \phi(x) - i\theta\sigma^\mu\theta^\dagger\partial_\mu\phi(x) - \frac{1}{4}\theta^2\theta^{\dagger 2}\partial^2\phi(x) \\
&\quad + \sqrt{2}\theta\psi(x) + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi(x)\sigma^\mu\theta^\dagger + \theta^2 F(x) \qquad (4.19)
\end{aligned}$$

Now we can rewrite SUSY Lagrangians in superspace notation. Consider:

$$\begin{aligned}
\int d^4\theta\, \Phi^\dagger\Phi &= \int d^4\theta \begin{pmatrix} \phi^* + i\theta\sigma^\mu\theta^\dagger\partial_\mu\phi^* - \frac{1}{4}\theta^{\dagger 2}\theta^2\partial^2\phi^* \\ +\sqrt{2}\theta^\dagger\psi^\dagger - \frac{i}{\sqrt{2}}\theta^{\dagger 2}\theta\sigma^\mu\partial_\mu\psi^\dagger + \theta^{\dagger 2}F^* \end{pmatrix} \\
&\qquad \begin{pmatrix} \phi - i\theta\sigma^\mu\theta^\dagger\partial_\mu\phi - \frac{1}{4}\theta^2\theta^{\dagger 2}\partial^2\phi \\ +\sqrt{2}\theta\psi + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi\sigma^\mu\theta + \theta^2 F \end{pmatrix} \\
&= F^*F + \partial^\mu\phi^*\partial_\mu\phi + i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi \qquad (4.20) \\
&\quad -\frac{1}{4}\partial^\mu(\phi^*\partial_\mu\phi + \partial_\mu\phi^*\phi) + \frac{i}{2}\partial_\mu(\psi^\dagger\overline{\sigma}^\mu\psi) \qquad (4.21)
\end{aligned}$$

so

$$\int d^4x d^4\theta\, \Phi^\dagger\Phi = \int d^4x \mathcal{L}_{\text{free}} \qquad (4.22)$$

Now consider the superpotential as a function of the chiral superfield:

$$\begin{aligned}
\int d^2\theta\, W(\Phi) &= \int d^2\theta\, (W(\Phi)|_{\theta=0} + \theta W_1 + \theta^2 W_2) \\
&= \int d^2\theta\, \theta^2 W_2 \\
&= W_a F^a - \frac{1}{2}W^{ab}\psi_a\psi_b \\
&\quad -\partial_\mu(\frac{1}{4}W^a\theta^{\dagger 2}\partial^\mu\phi_a - \frac{i}{\sqrt{2}}W^a\psi_a\sigma^\mu\theta^\dagger) \qquad (4.23)
\end{aligned}$$

so

$$\int d^4x d^2\theta\, W(\Phi) + h.c. = \int d^4x \mathcal{L}_{\text{int}} \qquad (4.24)$$

4.3 Less: $\mathcal{N} = 0$ SUSY

SUSY guarantees the cancellation of quadratic divergences for scalar masses since they are put in supermultiplets with fermions. Chiral symmetries admit at most logarithmic divergences for fermion masses since the physical mass must vanish as the bare mass approaches 0.

$$m_f = m_0 + c\frac{\alpha}{16\pi^2} m_0 \ln\left(\frac{\Lambda}{m_0}\right) \tag{4.25}$$

However SUSY must be broken in the real world. There are lots of ways to do this: e.g. $W = E^a\phi_a$ gives $V = W_a^* W^a = E^a E_a^* \neq 0$ which breaks SUSY. As long as the relationships between dimensionless couplings are maintained, quadratic divergences will still cancel. i.e. we don't want

$$\delta m^2 \propto (\lambda - |\lambda_t|^2)\Lambda^2 \tag{4.26}$$

We want to parametrize our ignorance about SUSY breaking while maintaining good high-energy behavior, that is we want an effective theory of spontaneously broken SUSY with only soft breaking (dim < 4) terms.

$$\begin{aligned} \mathcal{L}_{\text{soft}} &= -\frac{1}{2}(M_\lambda \lambda^a \lambda^a + h.c.) - (m^2)^i_j \phi^{*j}\phi_i \\ &\quad -(\frac{1}{2} b^{ij}\phi_i\phi_j + \frac{1}{6} a^{ijk}\phi_i\phi_j\phi_k + h.c.) \\ &\quad -\frac{1}{2} c_i^{jk}\phi^*\phi_j\phi_k + e^i\phi_i + h.c. \end{aligned} \tag{4.27}$$

Adding a fermion mass is redundant, since it can be absorbed into a redefined superpotential. The c_i^{jk} term may introduce quadratic divergences if there is a gauge singlet multiplet in the model.

References

[1] J. Lykken, "Introduction to Supersymmetry," hep-th/9612114.

[2] S. Martin, "A Supersymmetry Primer," hep-ph/9709356.

[3] M. Drees, "An Introduction to Supersymmetry," hep-ph/9611409.

[4] L. Girardello and M.T. Grisaru *Nucl. Phys.* **B194**, 65 (1982).

[5] L.J. Hall and L. Randall, *Phys. Rev. Lett.* **65**, 2939 (1990).

Figure 1: Additional soft SUSY breaking interactions: gaugino mass M_λ, non-holomorphic mass m^2, holomorphic mass b^{ij}, holomorphic trilinear coupling a^{ijk}, non-holomorphic trilinear coupling c_i^{jk}, and tadpole e^i.