Gravitational Lensing

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References: - Narayan + Bartelmann "Lectures on Gravitational Lensing"
- J. Wambsganss, "Gravitational Lensing in Astronomy"
- more on web site

Thanks: Salman Habib, Katrin Heitmann, Luis Teodoro

Outline

- History
- Lensing with point source
- Complications with extended objects
- Strong lensing
- Weak lensing
- Microlensing
- Conclusions

History

1804 – Soldner predicts 0.84 arcsec deflection by the Sun

1911 – Einstein independently gets the same result using SR

1916 – With GR, Einstein predicts 1.74" deflection by the Sun

1937 – Zwicky proposes clusters as lenses

1979 – 1st detection of lensed object, a quasar

1986 – Paczynski proposes MACHO search (results 1993 on)

1990 – 1st coherent weak lensing signal from clusters

2001 – Direct observation of micro-lens in the Milky Way

Refractive Index

$$n_{eff} = 1 + \frac{2}{c^2}|\Phi|$$

Φ: Newtonian gravitational potential

deflection angle:

$$\vec{\alpha} = \frac{2}{c^2}\int \vec{\nabla}_\perp \Phi \, dl$$

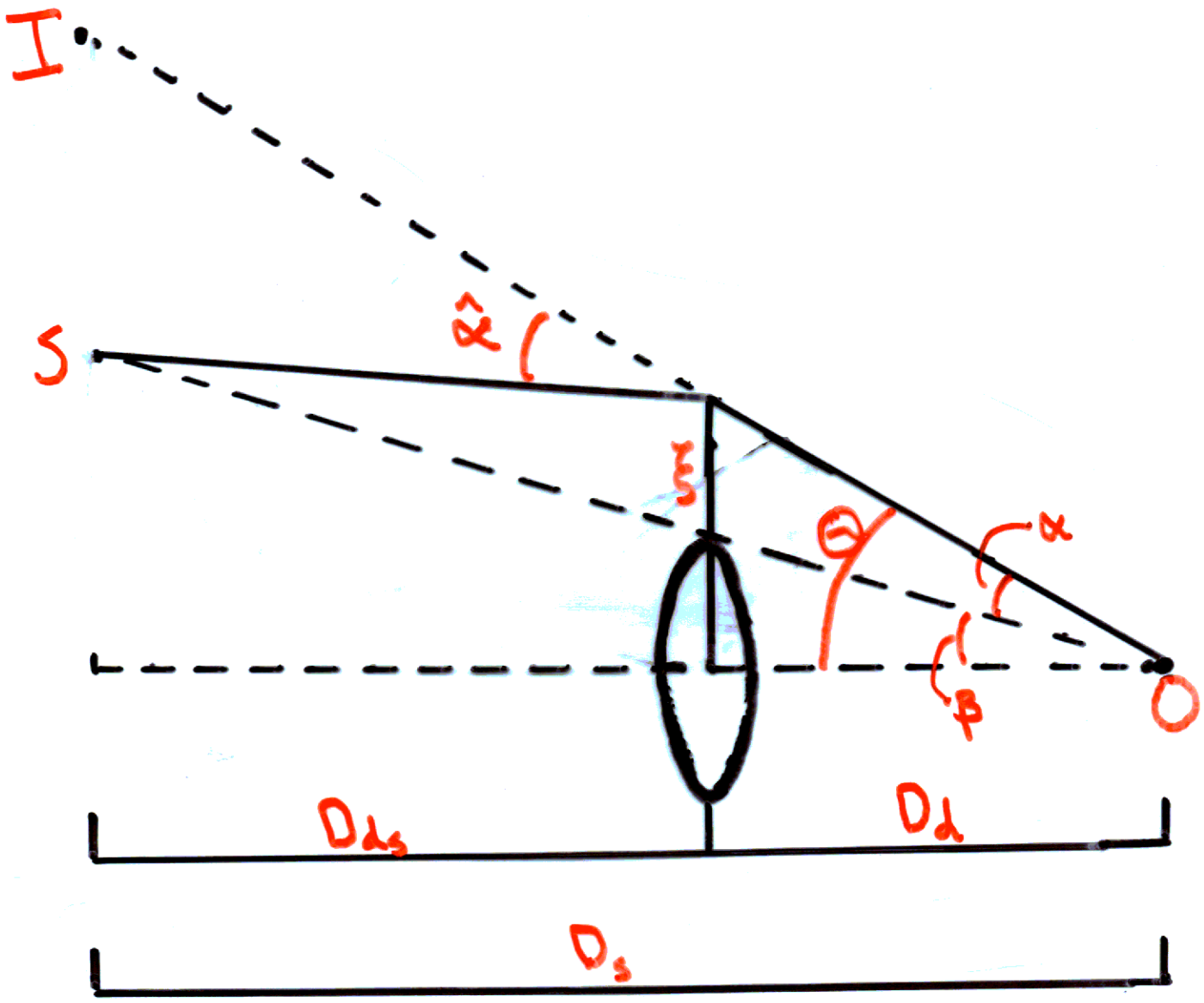
for $\Phi = -\frac{GM}{(b^2+z^2)^{1/2}}$,

$$\hat{\alpha} = \frac{4GM}{c^2 b}$$

b: impact parameter

z: distance from pt. of closest approach

Point source + simple lens



Assume lens is a surface, then

$$\hat{\alpha} = \frac{4GM(\xi)}{c^2 \xi}$$

Also, since α is what we want, not $\hat{\alpha}$

$$\alpha = \frac{D_{ds}}{D_s}\hat{\alpha}$$

- lens equation:

$$\theta D_s = \beta D_s - \hat{\alpha} D_{ds}$$

- Constant surface mass density Σ

$$\alpha(\theta) = \frac{\Sigma}{\Sigma_{cr}} \theta$$

Σ_{cr}: critical surface mass density

$$\Sigma_{cr} = \frac{c^2}{4\pi G} \frac{D_s}{D_d D_{ds}}$$

- Einstein radius θ_E

$$\theta_E = \left[\frac{4GM(\theta)}{c^2} \frac{D_{ds}}{D_d D_s} \right]^{1/2}$$

$\Sigma = \Sigma_{cr}$ inside θ_E (on average)

Characteristic separation for multiple images is $2\theta_E$

Complications

lensing potential:

$$\psi(\vec{\theta}) = \frac{D_{ls}}{D_l D_s} \frac{2}{c^2} \int \Phi(D_l \vec{\theta}, z)\, dz$$

then

$$\vec{\nabla}_\theta \psi = D_l \vec{\nabla}_\xi \psi = \vec{\alpha}$$, the deflection $\angle$

and

$$\nabla^2_\theta \psi = 2\kappa(\vec{\theta})$$, where κ is called the convergence

The other derivatives of ψ are the shear

$$\gamma_1(\vec{\theta}) = \gamma(\vec{\theta}) \cos[2\phi(\vec{\theta})]$$

$$\gamma_2(\vec{\theta}) = \gamma(\vec{\theta}) \sin[2\phi(\vec{\theta})]$$

This can all be described by

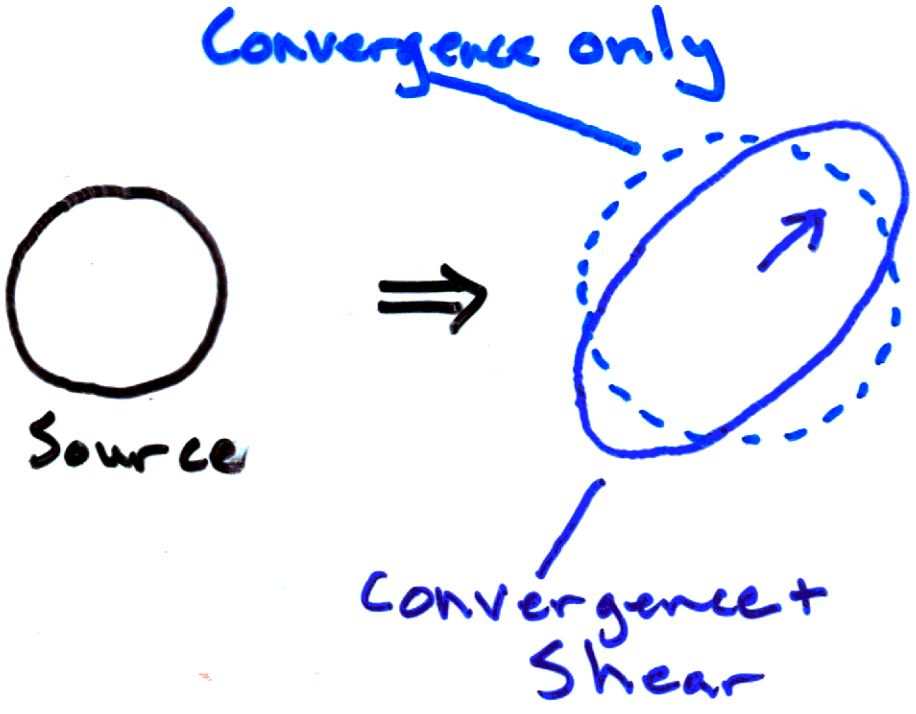
$$A = (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix}$$

magnification μ

$$\mu = \frac{1}{\det A} = \frac{1}{(1-\kappa)^2 - \gamma^2}$$

net magnification of all images

$$\mu_+ + \mu_- > 1$$



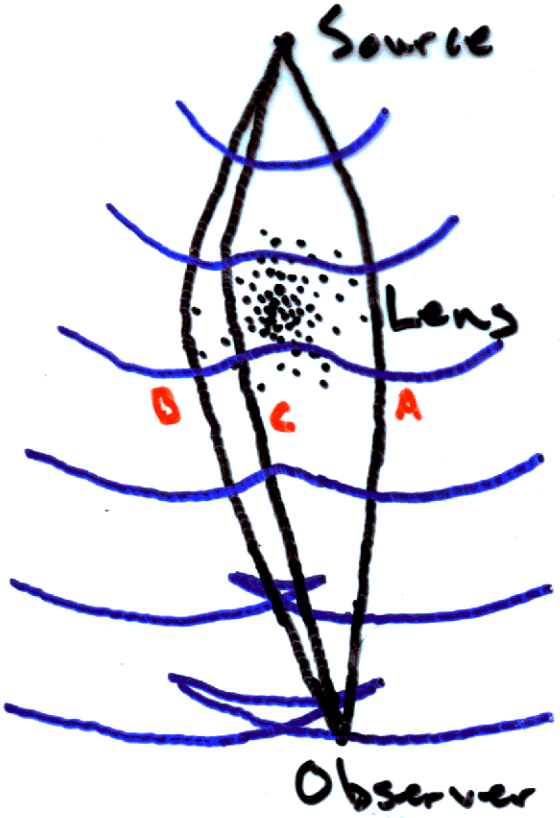
Time delay

$$t(\vec{\theta}) = \frac{1+z_d}{c}\frac{D_d D_s}{D_{ds}}\left[\frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \psi(\vec{\theta})\right]$$

$$= t_{geom} + t_{grav}$$

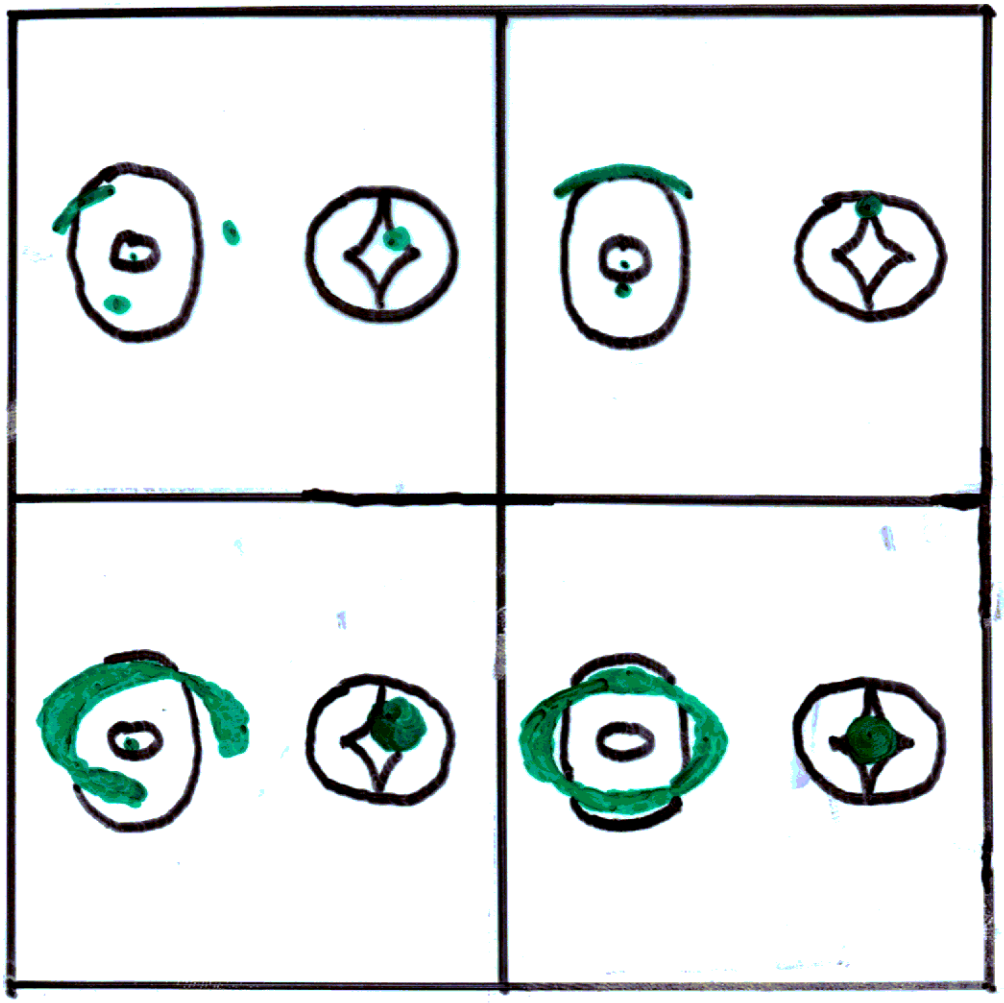
t_{geom}: path length diff. to unperturbed path

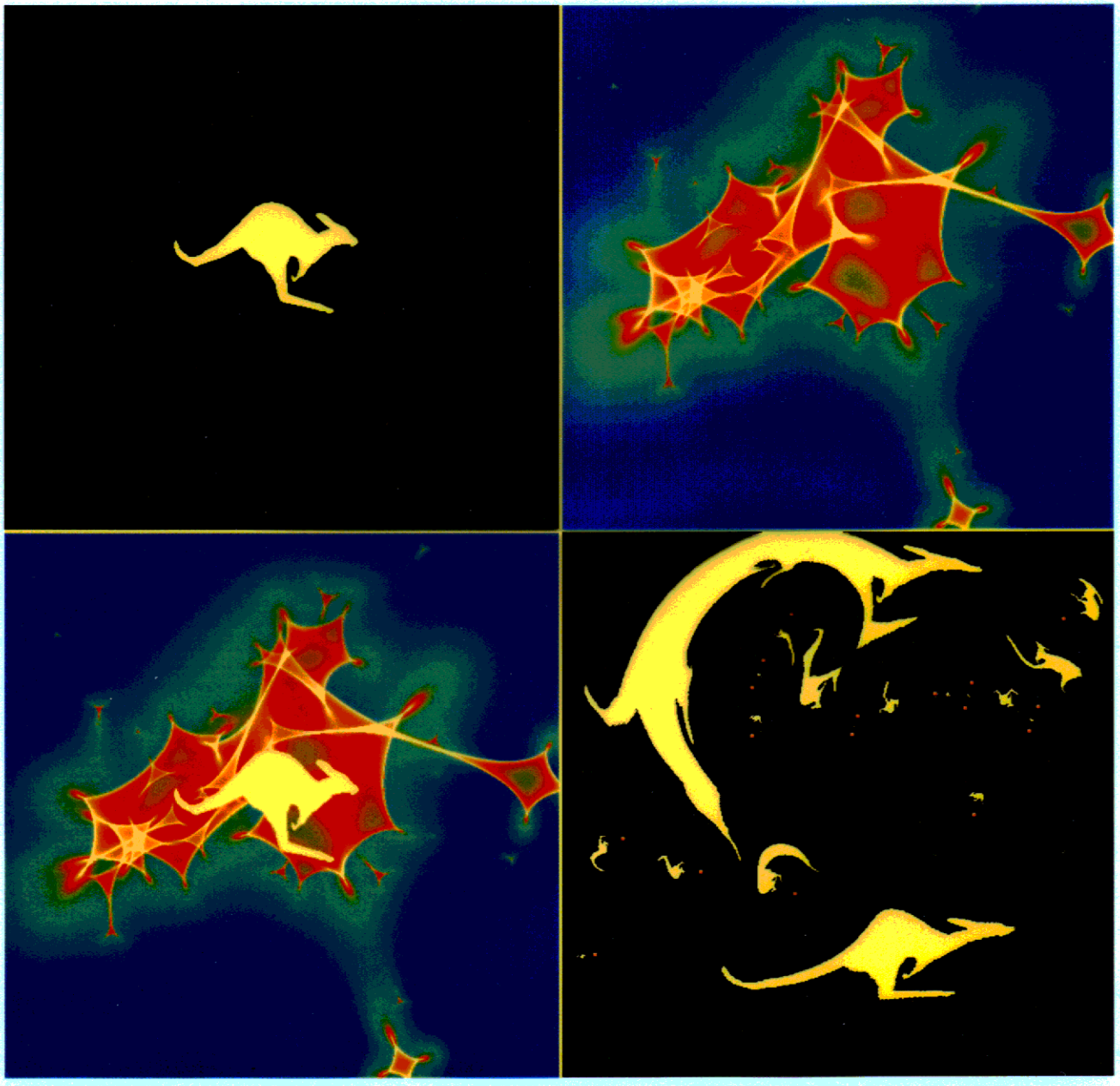
t_{grav}: delay due to increased index of refraction



Lensing by galaxies

Different galaxy models (isothermal spheres, ellipses, disks with cores) produce different effects on source images





WAMBSGANSS

Lensing by clusters

Strong lensing: multiply-imaged lensed objects, often forming giant luminous arcs

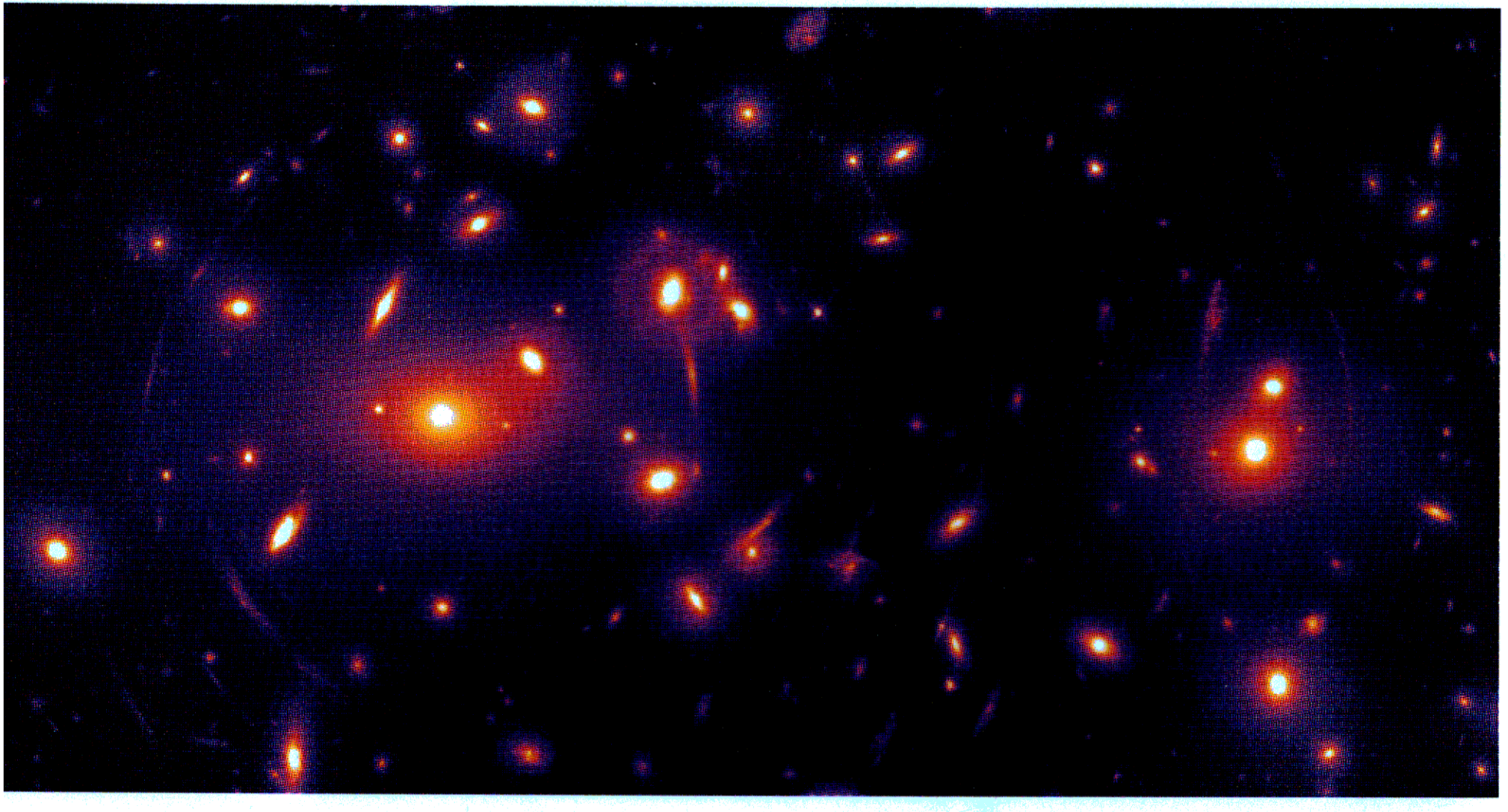
Σ within an arc $= \Sigma_{cr}$

$\Rightarrow$ arc radius $= \theta_E$

$\Rightarrow M(\theta) = \Sigma_{cr} \pi (D_d \theta)^2$

$\approx 1.1 \times 10^{14} M_\odot \left(\frac{\theta}{30''}\right)^2 \left(\frac{D}{1\,Gpc}\right)$

Cluster	Mass ($M_\odot$)	σ (km/s)
A370	$5 \times 10^{13}\ h^{-1}$	~1350
A2390	$8 \times 10^{13}\ h^{-1}$	~1250
MS 2137-23	$3 \times 10^{13}\ h^{-1}$	~1100



ABEL 2218 HST

[KNEIB et al 1996] [EBBELS et al 1996]

Hubble constant determination

Time delay $\propto$ path length $\propto H_0^{-1}$

$H_0 \Delta t$ depends only on the lens + geometry of the system

Grogin + Narayan (1996) put upper limit $H_0 < 88$ km s^{-1} Mpc^{-1}

Measure κ from weak lensing to remove degeneracies

$$H_0 = 62^{+12}_{-13} \text{ km s}^{-1} \text{ Mpc}^{-1}$$

κ from velocity dispersion

$$H_0 = 66 \pm 7 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Weak lensing

Weak lensing: statistical method for studying coherent distortions of background galaxies

Density of background galaxies:
50-100/arcsec^2

$$\langle \epsilon \rangle = \left\langle \frac{\gamma}{1-\kappa} \right\rangle \quad \kappa \ll 1, \gamma \ll 1 \Rightarrow$$

$$\langle \epsilon \rangle \sim \langle \gamma \rangle$$

$$\kappa(\vec{\theta}) = \frac{1}{\pi} \int d^2\theta' \, \mathrm{Re}\left[\mathcal{D}^*(\vec{\theta} - \vec{\theta}') \gamma(\vec{\theta}') \right]$$

can be solved iteratively

$$\Sigma(\vec{\theta}) = \Sigma_{cr} \kappa(\vec{\theta})$$

Complications / Practicalities

- Degeneracy in calculation of κ from shear

- Selection effects
  - # density a function of lensin
  - not all sources at same z

- "weight function" depends on assumptions about distribution, measuring apparatus

LSS implications

- mass follows light?

  mass smoother than light
  generally tracks well

- Ω_{matter}?

  $\Omega_m \sim 0.28$, consistent with other techniques

- halos?

  measure clumpiness of dark matter, compare w/ simulations

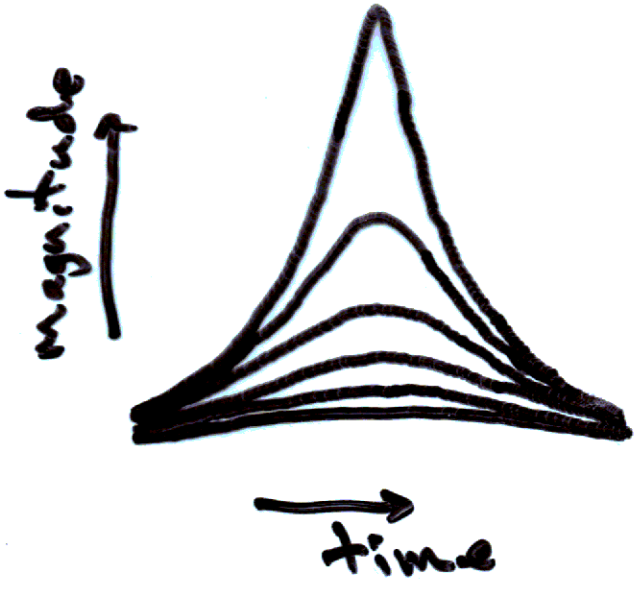
- Λ?

  COBE-normalized CDM ruled out at 3σ, ΛCDM consistent

Microlensing

lensing guaranteed to magnify the source

first observed with quasars



Signatures: time symmetric
not color dependent

Microlensing in the Milky Way

Surveys look for events in MW or between here and LMC, SMC

Attempt to constrain what fraction of Ω_b is in MACHOs

MACHO collaboration: 8%-50% of halo is MACHOs

recently - observed lens of one of their events - dim red star in the MW, not in halo

side benefit: new variable stars, new kinds of variables

Things I didn't talk about

- Cosmological lensing + lensing of the CMB

- Models of lenses
  - isothermal sphere: $\rho \propto \frac{1}{r^2}$
  - NFW: $\rho \propto \frac{1}{x(1+x)^2}$, $x = \frac{r}{r_s}$
  - others

- distribution of strong lenses

  there are fewer observed than straight CDM would predict; consistent with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$

Conclusions

- lensing helps pin down
  - $\Omega_m \sim 0.3$, $\Omega_\Lambda \sim 0.7$
  - $H_0 \sim 65$ km s^{-1} Mpc^{-1}
  - mass distribution in galaxies, clusters
  - fraction of Ω_b in MACHOs

- future results will improve understanding of systematics, and bigger sample size will iron out inconsistencies, reduce error bars